

1. **[0.5 point]** If $\Sigma = \{a, b, c\}$ and $L - \Sigma^* = \emptyset$, which of the following languages can be L ?
 - (a) \emptyset
 - (b) $\{\Lambda, b^{100}\}$
 - (c) Σ^*
 - (d) $\{a^n b^n c^n \mid n > 0\}$
2. **[1 point]** Give a *regular expression* for each of the following languages:
 - (a) $L = \{a^m b^n \mid n + m \text{ is even}\}$
 - (b) $L = \{w \in \{0, 1\}^* \mid w \text{ contains exactly one pair of consecutive 0's}\}$
3. **[1 point]** Suppose language L represented by the *regular expression* $(00)^*01$. Give a maximal set S of pairwise L -distinguishable strings. From the set S , what can we say about the number of states of a DFA M that accepts L ?
4. **[1 point]** Languages L_1 and L_2 are represented by a^*baa^* and ab^* , respectively. Construct a DFA that accepts $L_1 - L_2$.
5. **[1 points]** Give a context-free grammar G generating the following language L :

$$L = \{a^n b^m c^k \mid k = |n - m|\}$$

6. **[1 point]** Prove the following grammars are ambiguous:
 - (a)

$$\begin{aligned} S &\rightarrow T \mid Sa \mid a \\ T &\rightarrow ab \mid \lambda \end{aligned}$$

(b)

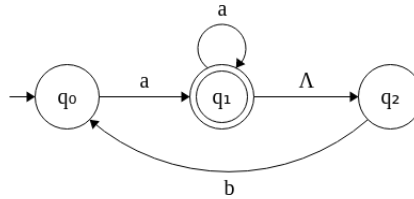
$$S \rightarrow bSc \mid bbSc \mid a$$

7. **[2 points]** Suppose the following context-free grammar G :

$$\begin{aligned} S &\rightarrow aSU \mid X \\ X &\rightarrow bXU \mid \Lambda \\ U &\rightarrow aY \\ Y &\rightarrow aY \mid \Lambda \end{aligned}$$

(a) Give $L(G)$.

- (b) Give a grammar in *Chomsky Normal Form* that generates $L(G) - \{\Lambda\}$.
8. [1 point] Construct a DFA that accepts the same language as the following NFA. Explain the steps.
Note: eliminate the Λ -transition and the resulting non-determinism.



9. [1.5 points] Suppose the following PDA M , where q_0 is the initial state and q_2 is the accepting state:

$$\sigma(q_0, a, Z_0) = \{(q_1, AZ_0), (q_2, Z_0)\}$$

$$\sigma(q_1, b, A) = \{(q_1, B)\}$$

$$\sigma(q_1, b, B) = \{(q_1, B)\}$$

$$\sigma(q_1, a, B) = \{(q_2, \Lambda)\}$$

- (a) give $L(M)$.
- (b) is M deterministic or not? Why?