Homework 2 Automata Theory 2023

Published on: Monday 16 October 2023.
Deadline for submission: Monday 6 November 2023, 23.59.
The assignment must be completed individually. A total of 100 points can be earned. Answers to be submitted via Brightspace. Submit a single file, e.g., a pdf or possibly a zip. Please include your name and student number in your submission. You may either type your answers or hand-write them. In the latter case, please hand in an easy-to-read scan / photos.

1. [35 pt] This exercise is about the following language:
   $L_1 = \{x \in \{a,b\}^* \mid \forall \text{ prefix } z \text{ of } x \text{ it holds that } n_a(z) \geq n_b(z) \geq n_a(z) - 2\}$

   In other words: the number of b's is never greater, but also never much less than the number of a's.

   (a) List the first five elements of L_1 in canonical (shortlex) order.

   (b) Find a regular expression corresponding to the language L_1. And explain why your expression describes the language L_1.

   Hint: Ask yourself what could be the first letter of an element x of L_1, and what could be the second letter.

2. [40 pt] Let $L \subseteq \{a,b\}^*$ be the language corresponding to the regular expression $(a+b)a^*$.

   (a) Use Thompson's construction (without possible simplifications), i.e., the construction in Section 3.4 from the book, to systematically construct a non-deterministic finite automaton M_1 with Λ-transitions, such that $L(M_1) = L$.

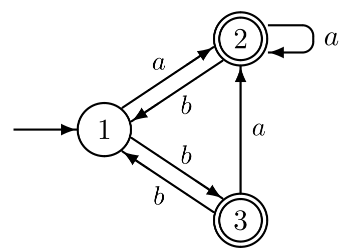
   Give only the resulting automaton M_1 as your answer.

   (b) Remove the Λ-transitions from M_1. Use the construction from lecture 5, i.e., the simpler variant than that of Theorem 3.17 in the book, to construct a non-deterministic finite automaton M_2 without Λ-transitions, such that $L(M_2) = L(M_1) = L$.

   Draw a table where for each state q in M_1 you give the values of the transition function and Λ-closure $\Lambda(\{q\})$, and draw the resulting automaton M_2.

   Remark: Do not remove unreachable states.

3. [25 pt] Use the state elimination algorithm of Brzozowski and McCluskey to find a regular expression corresponding to the finite automaton M below:



   In addition to the regular expression, give the order in which you eliminate the states and draw the intermediate automata.