Automata Theory 2024
Homework 1

Deadline: Monday 7 October 11:00

Clearly show your step-by-step solutions; only a final answer will not score any points. Cooperation to share ideas is allowed and encouraged, but *write* your solutions *individually.* Hand in your solutions via Brightspace in *one* file. Make sure that your solutions are readable.

Exercise 1

(a) [10 pt.] Consider the language $L_1 = \{x \in \{a,b\}^* \mid x \text{ starts with } ba\}$. Construct a deterministic finite automaton that accepts L_1.

(b) [10 pt.] Consider the language $L_2 = \{x \in \{a,b\}^* \mid n_b(x) \geqslant 2\}$. Construct a deterministic finite automaton that accepts L_2.

(c) [15 pt.] Use the product construction (Theorem 2.15) to obtain a deterministic finite automaton that accepts $L_1 - L_2$. Consequently, remove all states that cannot be reached from the initial state. Do not simplify the automaton in any other way yet.

(d) [15 pt.] Minimize the amount of states in the automaton obtained in (c) using Algorithm 2.40. Clearly show which states are marked in which pass, and indicate in which order you loop through the pairs of states.

Exercise 2

Consider the language $L = \{a^i b^j c^k \mid k = i + j\} \subseteq \{a,b,c\}^*$.

(a) [20 pt.] Prove that the language L cannot be accepted by a finite automaton by using the pumping lemma for regular languages Theorem 2.29.

(b) [20 pt.] Prove that the language L cannot be accepted by a finite automaton by finding infinitely many pairwise L-distinguishable strings, and showing that these are indeed distinguishable.

(c) [10 pt.] Use the result from (a) or (b) to conclude that the language

$$\{x \in \{a,b,c\}^* \mid n_a(x) + n_b(x) = n_c(x)\}$$

cannot be accepted by a finite automaton.