Automata Theory 2024
Homework 4

Published on: Monday 2 December 2024.
Deadline for submission: Monday 16 December 2024, 23:59.

The assignment must be completed individually. Submit your answers via Brightspace. Submit a single file, e.g., a pdf or possibly a zip. Please include your name and student number in your submission. You may either type your answers or hand-write them. Make sure that your solutions are readable.

1. [35 pt] Let

   $$L = \{a^i b^j c^k \mid \ i, j, k \geq 0 \text{ and } j \leq i \text{ and } i - j \leq k\}$$

   (a) Give the first seven elements in the canonical (shortlex) order of L.

   (b) Give a pushdown automaton M, such that $L(M) = L$.
   M should be based directly on properties of L. It must not be the result of applying a standard construction, e.g., to convert a context-free grammar into a pushdown automaton.
   Try to ensure that M is deterministic and has no Λ-transitions. If you do not succeed in this, you lose 10 points. If M has only one of these two properties, then you lose 5 points
   Hint: Realise that a^i may be written as $a^{i-j}a^j$.

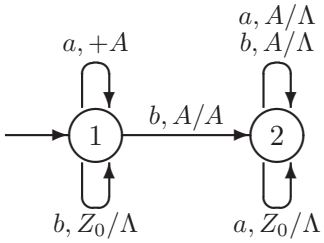
   (c) Explain how M uses its states and/or stack to accept exactly L.

2. [10 pt] Let G be the context-free grammar with start variable S and the productions:

   $$S \to ST \mid \Lambda \qquad T \to aS \mid bT \mid b$$

   Give the nondeterministic top-down pushdown automaton $NT(G)$.

3. [55 pt] Consider the following pushdown automaton M_1 with initial stack symbol Z_0:



   (a) What is $L_e(M_1)$ (the empty-stack language) for this automaton M_1? Express (in words or in formulas, but at least clearly and completely) what are the elements of this concrete language. Explain why it is exactly these strings that are accepted by M_1 with empty stack.

   (b) Choose a string $x \in L_e(M_1)$, such that $|x| = 4$ or $|x| = 5$. Give a succesful computation in M_1 for this string x, i.e., a computation from the initial configuration $(1, x, Z_0)$ to an accepting configuration.

   (c) In lecture 13, we have discussed the construction of a context-free grammar G from a given pushdown automaton M, such that $L(G) = L_e(M)$.
   Apply this construction to our pushdown automataon M_1. That is, give all variables in the resulting grammar, with all their productions. Also give variables and productions (following from the construction) that are not useful to derive elements of $L_e(M_1)$.

In your description of variables and productions, it is allowed to use general states $p, q, \ldots$ and stack symbols $X, Y, \ldots$. If you do so, mention explicitly what the possibilities are for $p, q, \ldots$ and $X, Y, \ldots$, respectively.

(d) Which of the variables in your answer to part (c) are not useful to derive elements of $L_e(M_1)$? Explain your answer. You do not have to explain why the other variables *are* useful.

(e) Give a leftmost derivation in your grammar from part (c) for your string x from part (b).