Automata Theory 2024
Homework 3

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Deadline for submission: Wednesday 27 November 2024, 23:59.

The assignment must be completed individually. Submit your answers via Brightspace. Submit a single file, e.g., a pdf or possibly a zip. Please include your name and student number in your submission. You may either type your answers or hand-write them. Make sure that your solutions are readable.

1. [25 pt] Let $L = \{a^i b^j c^k \mid j > i + k + 2\}$, and let L' be the complement of L, i.e., the set of all strings over alphabet $\{a, b, c\}$ that are not in L. For example, Λ, bc, $abbbbc$ and $cbbbbb$ are elements of L'.
   (a) Give a context-free grammar G, such that $L(G) = L'$.
   (b) Explain why G indeed generates exactly L'. You do not need to prove this formally, but do explain the role of the different variables and productions of G in generating L'.
2. [20 pt] Let $L = \{a^i b^j \mid i \neq j\}$. Each of the following context-free grammars generates L from start variable S. For each of them, indicate whether or not the grammar is ambiguous. If so, then give a string x in L and two different derivation trees for x in the grammar.
   (a) $S \to aSb \mid A \mid B$ $\quad A \to aA \mid a$ $\quad B \to bB \mid b$
   (b) $S \to aSb \mid aAb \mid aBb \mid aA \mid a \mid Bb \mid b$ $\quad A \to aA \mid a$ $\quad B \to bB \mid b$
   (c) $S \to A \mid B$ $\quad A \to aAb \mid aA \mid a$ $\quad B \to aBb \mid Bb \mid b$
   (d) $S \to AQ \mid QB$ $\quad A \to aA \mid a$ $\quad B \to bB \mid b$ $\quad Q \to aQb \mid \Lambda$
   (e) $S \to aA \mid Bb$ $\quad A \to aA\hat{B} \mid \Lambda$ $\quad \hat{B} \to b \mid \Lambda$ $\quad B \to \hat{A}Bb \mid \Lambda$ $\quad \hat{A} \to a \mid \Lambda$
3. [25 pt] Let $L = \{a^i b^j \mid 0 \leq i \leq j \leq 2i\}$.
   (a) Give the first six elements in the canonical (shortlex) order of L.
   (b) Let G be the context-free grammar with start variable (and only variable) S, and the following productions:

   $$S \to aSb \mid aSbb \mid \Lambda$$

   Use induction on i to prove that each string $a^i b^j$ with $0 \leq i \leq j \leq 2i$ can be generated by G, i.e., that $L \subseteq L(G)$.
4. [30 pt] Let G be the context-free grammar with start variable S and the following productions:

   $$S \to AQ \mid QB \qquad A \to aA \mid a \qquad B \to bB \mid b \qquad Q \to aQQb \mid \Lambda$$

   Convert G step-by-step into Chomsky normal form. That is, answer the following questions, using the constructions discussed in lecture 10.
   (a) Give the set of nullable variables in G.
   (b) Give a context-free grammar G_1 resulting from G by eliminating Λ-productions.
   (c) For each variable X in G_1, give the set of X-derivable variables.
   (d) Give a context-free grammar G_2 resulting from G_1 by eliminating unit productions.
   (e) Give the context-free grammar G_3 resulting from G_2 by introducing for every terminal symbol σ a variable X_σ (with a corresponding production), and substituting this variable for occurrences of σ where necessary in the righthand side of productions.
   (f) Give the context-free grammar G_4 resulting from G_3 by splitting the righthand side of productions which are too long.