Deadline homework 3: Wednesday, 29 November, 23.59

$L = \{ a^i b^j \mid i \neq j \}$

$S \to X \mid Y$ (choice!)

$X \to aXb \mid aX \mid a$ $(i > j)$

$Y \to aYb \mid Yb \mid b$ $(i < j)$

$S \Rightarrow X \Rightarrow aXb \Rightarrow aaXb \Rightarrow aaaXbb \Rightarrow aaaabb$

$L = \{ a^i b^j \mid i \neq j \}$

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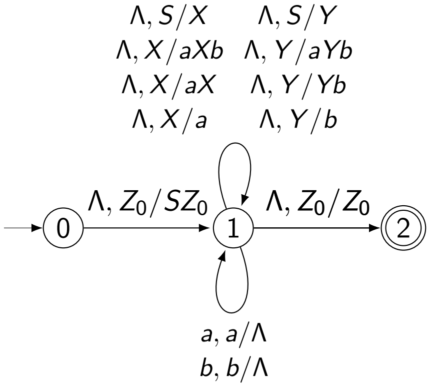
$Y \rightarrow aYb \mid Yb \mid b$ $(i < j)$



CFG $G = (V, \Sigma, S, P)$

Definition (Nondeterministic Top-Down PDA)

$NT(G) = (Q, \Sigma, \Gamma, q_0, Z_0, A, \delta)$, as follows:

- $Q = \{q_0, q_1, q_2\}$
- $A = \{q_2\}$
- $\Gamma = V \cup \Sigma \cup \{Z_0\}$
- start $\delta(q_0, \Lambda, Z_0) = \{(q_1, SZ_0)\}$
- *expand* $\delta(q_1, \Lambda, A) = \{(q_1, \alpha) \mid A \to \alpha \text{ in } P\}$ for $A \in V$
- *match* $\delta(q_1, \sigma, \sigma) = \{(q_1, \Lambda)\}$ for $\sigma \in \Sigma$
- finish $\delta(q_1, \Lambda, Z_0) = \{(q_2, Z_0)\}$ check empty stack

[M] Def 5.17

From lecture 8:

$AeqB = \{\ x \in \{a, b\}^* \mid n_a(x) = n_b(x)\ \}$

$aaabbb, ababab, aababb, \ldots$

$$S \to \Lambda \mid aB \mid bA$$
$$A \to aS \mid bAA \qquad A \text{ generates } n_a(x) = n_b(x) + 1$$
$$B \to bS \mid aBB \qquad B \text{ generates } n_a(x) + 1 = n_b(x)$$

$S \Rightarrow aB \Rightarrow aaBB \Rightarrow aabSB \Rightarrow \ldots$ (different options)

(1) $aabB \Rightarrow aabaBB \Rightarrow aababSB \Rightarrow aababB \Rightarrow aababbS \Rightarrow aababb$

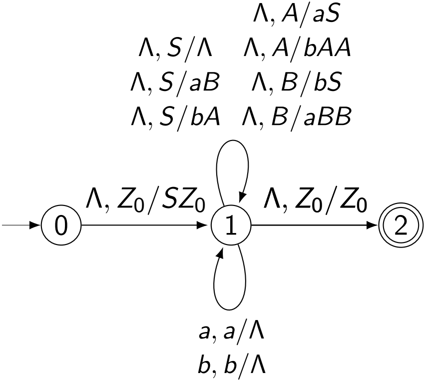
(2) $aaba\underline{B}B \Rightarrow aabab\underline{S}B \Rightarrow aabab\underline{B} \Rightarrow aababb\underline{S} \Rightarrow aababb$

(2') $aabaB\underline{B} \Rightarrow aaba\underline{B}bS \Rightarrow aababSb\underline{S} \Rightarrow aabab\underline{S}b \Rightarrow aababb$

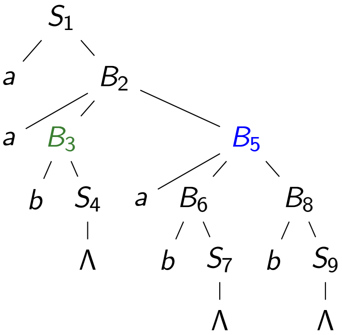
[M] E 4.8

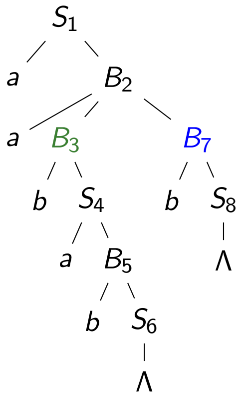
$$AeqB = \{\ x \in \{a, b\}^* \mid n_a(x) = n_b(x)\ \}$$

$$S \to \Lambda \mid aB \mid bA$$
$$A \to aS \mid bAA$$
$$B \to bS \mid aBB$$



From lecture 8:



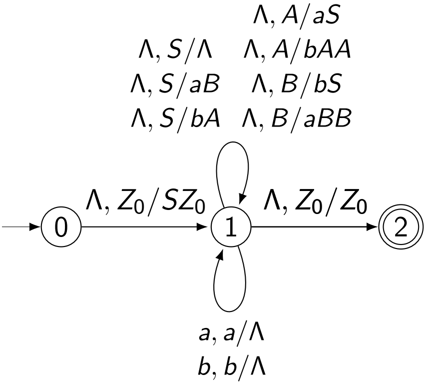


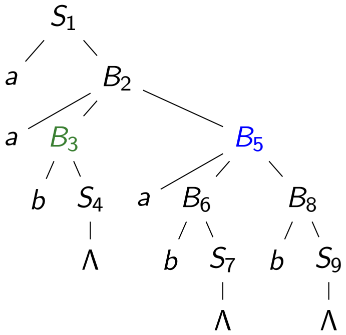
$S \Rightarrow aB \Rightarrow aaBB \Rightarrow aabSB \Rightarrow aabB \Rightarrow aabaBB \Rightarrow aababSB \Rightarrow aababB \Rightarrow aababbS \Rightarrow aababb$

$S \Rightarrow aB \Rightarrow aaBB \Rightarrow aabSB \Rightarrow aabaBB \Rightarrow aababSB \Rightarrow aababB \Rightarrow aababbS \Rightarrow aababb$

$AeqB = \{\ x \in \{a, b\}^* \mid n_a(x) = n_b(x)\ \}$

$$S \to \Lambda \mid aB \mid bA$$
$$A \to aS \mid bAA$$
$$B \to bS \mid aBB$$





$q_0 \quad aababb \quad Z_0$

$\ldots$

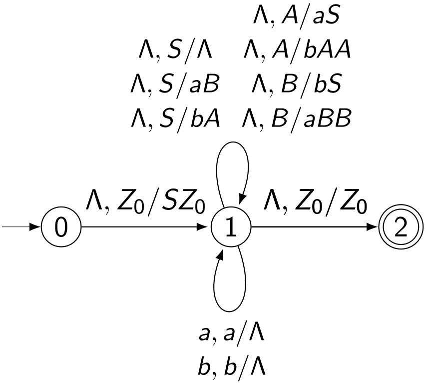
Top-down = expand-match

$AeqB = \{\ x \in \{a, b\}^* \mid n_a(x) = n_b(x)\ \}$

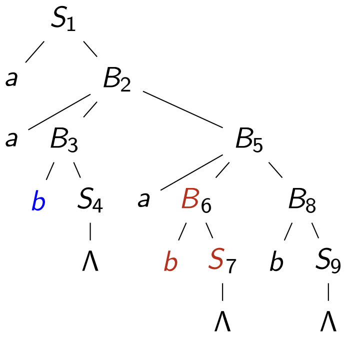
$S \to \Lambda \mid aB \mid bA$

$A \to aS \mid bAA$

$B \to bS \mid aBB$



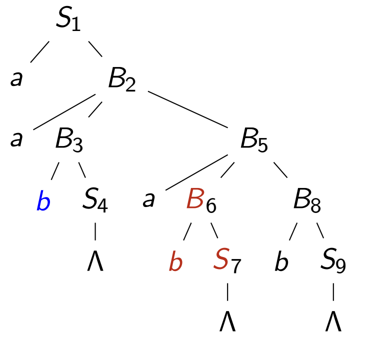
q_0	$aababb$	Z_0	
q_1	$aababb$	$S\ Z_0$	$1 : S \to aB$
q_1	$aababb$	$aB\ Z_0$	match a
q_1	$a\ ababb$	$B\ Z_0$	$2 : B \to aBB$
q_1	$a\ ababb$	$aBB\ Z_0$	match a
q_1	$aa\ babb$	$BB\ Z_0$	$3 : B \to bS$
q_1	$aa\ babb$	$bSB\ Z_0$	match b
q_1	$aab\ abb$	$SB\ Z_0$	$4 : S \to \Lambda$
q_1	$aab\ abb$	$B\ Z_0$	$5 : B \to aBB$
q_1	$aab\ abb$	$aBB\ Z_0$	match a
q_1	$aaba\ bb$	$BB\ Z_0$	$6 : B \to bS$
q_1	$aaba\ bb$	$bSB\ Z_0$	match b
q_1	$aabab\ b$	$SB\ Z_0$	$7 : S \to \Lambda$
q_1	$aabab\ b$	$B\ Z_0$	$8 : B \to bS$
q_1	$aabab\ b$	$bS\ Z_0$	match b
q_1	$aababb$	$S\ Z_0$	$9 : S \to \Lambda$
q_1	$aababb$	Z_0	
q_2	$aababb$	Z_0	



preorder: leftmost

$S \stackrel{\ell}{\Rightarrow} aB \Rightarrow aaBB \Rightarrow aabSB \Rightarrow$
$aabB \Rightarrow aabaBB \Rightarrow aababSB \Rightarrow$
$aababB \Rightarrow aababbS \Rightarrow aababb$

q_0	$aababb$	Z_0	
q_1	$aababb$	$S\,Z_0$	$1 : S \to aB$
q_1	$aababb$	$aB\,Z_0$	match a
q_1	$a\ ababb$	$B\,Z_0$	$2 : B \to aBB$
q_1	$a\ ababb$	$aBB\,Z_0$	match a
q_1	$aa\ babb$	$BB\,Z_0$	$3 : B \to bS$
q_1	$aa\ babb$	$bSB\,Z_0$	match b
q_1	$aab\ abb$	$SB\,Z_0$	$4 : S \to \Lambda$
q_1	$aab\ abb$	$B\,Z_0$	$5 : B \to aBB$
q_1	$aab\ abb$	$aBB\,Z_0$	match a
q_1	$aaba\ bb$	$BB\,Z_0$	$6 : B \to bS$
q_1	$aaba\ bb$	$bSB\,Z_0$	match b
q_1	$aabab\ b$	$SB\,Z_0$	$7 : S \to \Lambda$
q_1	$aabab\ b$	$B\,Z_0$	$8 : B \to bS$
q_1	$aabab\ b$	$bS\,Z_0$	match b
q_1	$aababb$	$S\,Z_0$	$9 : S \to \Lambda$
q_1	$aababb$	Z_0	
q_2	$aababb$	Z_0	



preorder: leftmost

$S \stackrel{\ell}{\Rightarrow} aB \Rightarrow aaBB \Rightarrow aabSB \Rightarrow$
$aabB \Rightarrow aabaBB \Rightarrow aababSB \Rightarrow$
$aababB \Rightarrow aababbS \Rightarrow aababb$

q_0	$aababb$	Z_0	
q_1	$aababb$	$S\ Z_0$	$1 : S \to aB$
q_1	$aababb$	$aB\ Z_0$	match a
q_1	$a\ ababb$	$B\ Z_0$	$2 : B \to aBB$
q_1	$a\ ababb$	$aBB\ Z_0$	match a
q_1	$aa\ babb$	$BB\ Z_0$	$3 : B \to bS$
q_1	$aa\ babb$	$bSB\ Z_0$	match b
q_1	$aab\ abb$	$SB\ Z_0$	$4 : S \to \Lambda$
q_1	$aab\ abb$	$B\ Z_0$	$5 : B \to aBB$
q_1	$aab\ abb$	$aBB\ Z_0$	match a
q_1	$aaba\ bb$	$BB\ Z_0$	$6 : B \to bS$
q_1	$aaba\ bb$	$bSB\ Z_0$	match b
q_1	$aabab\ b$	$SB\ Z_0$	$7 : S \to \Lambda$
q_1	$aabab\ b$	$B\ Z_0$	$8 : B \to bS$
q_1	$aabab\ b$	$bS\ Z_0$	match b
q_1	$aababb$	$S\ Z_0$	$9 : S \to \Lambda$
q_1	$aababb$	Z_0	
q_2	$aababb$	Z_0	

Theorem

If G is a context-free grammar, then the nondeterministic top-down PDA $\mathrm{NT}(G)$ *accepts the language* $L(G)$.

The details of the proof of this result do not have to be known for the exam.

Intuition $L(G) \subseteq L(NT(G))\ldots$:

- wish to simulate derivation on stack
- match terminals when on top of stack
- topmost variable first, so leftmost derivation

[M] Th 5.18

One leftmost derivation step:

$$y_i A_i \alpha_i \Rightarrow y_i \beta_i \alpha_i = y_i x_{i+1} A_{i+1} \alpha_{i+1} \qquad \text{with } y_i, x_{i+1} \in \Sigma^*$$

With $y_i = x_0 x_1 \ldots x_i$:

$$x_0 x_1 \ldots x_i A_i \alpha_i \Rightarrow x_0 x_1 \ldots x_i \beta_i \alpha_i = x_0 x_1 \ldots x_i x_{i+1} A_{i+1} \alpha_{i+1}$$

Complete leftmost derivation:

$$\begin{aligned} S &= x_0 A_0 \alpha_0 \\ &\Rightarrow x_0 x_1 A_1 \alpha_1 \\ &\Rightarrow x_0 x_1 x_2 A_2 \alpha_2 \\ &\Rightarrow \ldots \\ &\Rightarrow x_0 x_1 x_2 \ldots x_m A_m \alpha_m \\ &\Rightarrow x_0 x_1 x_2 \ldots x_m \beta_m \alpha_m = x \end{aligned}$$

☒Proof $L(G) \subseteq L(NT(G))$

Use induction on i to prove that for $i = 0, 1, \ldots, m$, in $NT(G)$,

$$(q_0, x, Z_0) = (q_0, x_0 x_1 \ldots x_m \beta_m \alpha_m, Z_0) \vdash^* (q_1, x_{i+1} \ldots x_m \beta_m \alpha_m, A_i \alpha_i Z_0)$$

That is, $NT(G)$ can perform steps that read as input $x_0 x_1 \ldots x_i$ and leave $A_i \alpha_i Z_0$ on stack.

Then prove that

$$(q_1, \beta_m \alpha_m, A_m \alpha_m Z_0) \vdash^* (q_2, \Lambda, Z_0)$$

Bottom-up = shift-reduce

$$S \to abc$$

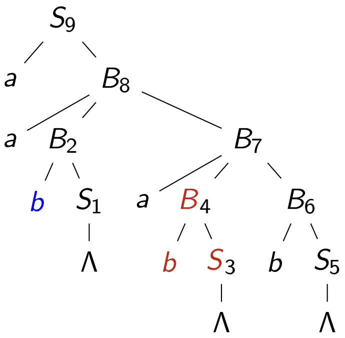
Bottom-up = shift-reduce

$AeqB = \{\ x \in \{a, b\}^* \mid n_a(x) = n_b(x)\ \}$

$S \to \Lambda \mid aB \mid bA$

$A \to aS \mid bAA$

$B \to bS \mid aBB$



	stack^r	input	
q_0	Z_0	$aababb$	shift a
...			

$$AeqB = \{ x \in \{a,b\}^* \mid n_a(x) = n_b(x) \}$$

$S \to \Lambda \mid aB \mid bA$

$A \to aS \mid bAA$

$B \to bS \mid aBB$

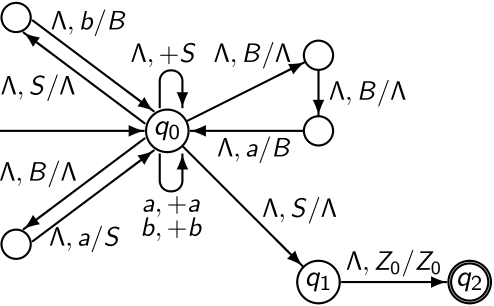
	stack^r	input	
q_0	Z_0	$aababb$	shift a
q_0	$Z_0\ a$	$a\ ababb$	shift a
q_0	$Z_0\ aa$	$aa\ babb$	shift b
q_0	$Z_0\ aab$	$aab\ abb$	$1 : S \to \Lambda$
q_0	$Z_0\ aabS$	$aab\ abb$	$2 : B \to bS$
q_0	$Z_0\ aaB$	$aab\ abb$	shift a
q_0	$Z_0\ aaBa$	$aaba\ bb$	shift b
q_0	$Z_0\ aaBab$	$aabab\ b$	$3 : S \to \Lambda$
q_0	$Z_0\ aaBabS$	$aabab\ b$	$4 : B \to bS$
q_0	$Z_0\ aaBaB$	$aabab\ b$	shift b
q_0	$Z_0\ aaBaBb$	$aababb$	$5 : S \to \Lambda$
q_0	$Z_0\ aaBaBbS$	$aababb$	$6 : B \to bS$
q_0	$Z_0\ aaBaBB$	$aababb$	$7 : B \to aBB$
q_0	$Z_0\ aaBB$	$aababb$	$8 : B \to aBB$
q_0	$Z_0\ aB$	$aababb$	$9 : S \to aB$
q_0	$Z_0\ S$	$aababb$	
q_1	Z_0	$aababb$	
q_2	Z_0	$aababb$	

$AeqB = \{\ x \in \{a,b\}^* \mid n_a(x) = n_b(x)\ \}$

$S \to \Lambda \mid aB \mid bA$

$A \to aS \mid bAA$

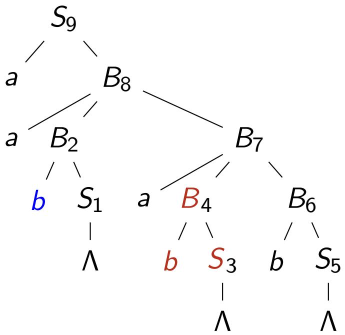
$B \to bS \mid aBB$



+ states/transitions for other productions

	stack^r	input	
q_0	Z_0	$aababb$	shift a
q_0	$Z_0\ a$	$a\ ababb$	shift a
q_0	$Z_0\ aa$	$aa\ babb$	shift b
q_0	$Z_0\ aab$	$aab\ abb$	$1 : S \to \Lambda$
q_0	$Z_0\ aabS$	$aab\ abb$	$2 : B \to bS$
q_0	$Z_0\ aaB$	$aab\ abb$	shift a
q_0	$Z_0\ aaBa$	$aaba\ bb$	shift b
q_0	$Z_0\ aaBab$	$aabab\ b$	$3 : S \to \Lambda$
q_0	$Z_0\ aaBabS$	$aabab\ b$	$4 : B \to bS$
q_0	$Z_0\ aaBaB$	$aabab\ b$	shift b
q_0	$Z_0\ aaBaBb$	$aababb$	$5 : S \to \Lambda$
q_0	$Z_0\ aaBaBbS$	$aababb$	$6 : B \to bS$
q_0	$Z_0\ aaBaBB$	$aababb$	$7 : B \to aBB$
q_0	$Z_0\ aaBB$	$aababb$	$8 : B \to aBB$
q_0	$Z_0\ aB$	$aababb$	$9 : S \to aB$
q_0	$Z_0\ S$	$aababb$	
q_1	Z_0	$aababb$	
q_2	Z_0	$aababb$	

Bottom-up = shift-reduce

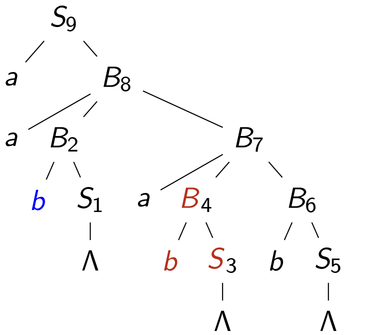


postorder: rightmost

$S \overset{r}{\Rightarrow}_9 aB \Rightarrow_8 aaBB \Rightarrow_7 aaBaBB \Rightarrow_6 aaBaBbS \Rightarrow_5 aaBaBb \Rightarrow_4 aaBabSb \Rightarrow_3 aaBabb \Rightarrow_2 aabSabb \Rightarrow_1 aababb$

	stack^r	input	
q_0	Z_0	$aababb$	shift a
q_0	$Z_0\ a$	$a\ ababb$	shift a
q_0	$Z_0\ aa$	$aa\ babb$	shift b
q_0	$Z_0\ aab$	$aab\ abb$	$1 : S \to \Lambda$
q_0	$Z_0\ aabS$	$aab\ abb$	$2 : B \to bS$
q_0	$Z_0\ aaB$	$aab\ abb$	shift a
q_0	$Z_0\ aaBa$	$aaba\ bb$	shift b
q_0	$Z_0\ aaBab$	$aabab\ b$	$3 : S \to \Lambda$
q_0	$Z_0\ aaBabS$	$aabab\ b$	$4 : B \to bS$
q_0	$Z_0\ aaBaB$	$aabab\ b$	shift b
q_0	$Z_0\ aaBaBb$	$aababb$	$5 : S \to \Lambda$
q_0	$Z_0\ aaBaBbS$	$aababb$	$6 : B \to bS$
q_0	$Z_0\ aaBaBB$	$aababb$	$7 : B \to aBB$
q_0	$Z_0\ aaBB$	$aababb$	$8 : B \to aBB$
q_0	$Z_0\ aB$	$aababb$	$9 : S \to aB$
q_0	$Z_0\ S$	$aababb$	
q_1	Z_0	$aababb$	
q_2	Z_0	$aababb$	

Bottom-up = shift-reduce



postorder: rightmost, in reverse

$S \overset{r}{\Rightarrow}_9 aB \Rightarrow_8 aaBB \Rightarrow_7 aaBaBB \Rightarrow_6 aaBaBbS \Rightarrow_5 aaBaBb \Rightarrow_4 aaBabSb \Rightarrow_3 aaBabb \Rightarrow_2 aabSabb \Rightarrow_1 aababb$

	stack^r	input	
q_0	Z_0	$aababb$	shift a
q_0	$Z_0\ a$	$a\ ababb$	shift a
q_0	$Z_0\ aa$	$aa\ babb$	shift b
q_0	$Z_0\ aab$	$aab\ abb$	$1 : S \to \Lambda$
q_0	$Z_0\ aabS$	$aab\ abb$	$2 : B \to bS$
q_0	$Z_0\ aaB$	$aab\ abb$	shift a
q_0	$Z_0\ aaBa$	$aaba\ bb$	shift b
q_0	$Z_0\ aaBab$	$aabab\ b$	$3 : S \to \Lambda$
q_0	$Z_0\ aaBabS$	$aabab\ b$	$4 : B \to bS$
q_0	$Z_0\ aaBaB$	$aabab\ b$	shift b
q_0	$Z_0\ aaBaBb$	$aababb$	$5 : S \to \Lambda$
q_0	$Z_0\ aaBaBbS$	$aababb$	$6 : B \to bS$
q_0	$Z_0\ aaBaBB$	$aababb$	$7 : B \to aBB$
q_0	$Z_0\ aaBB$	$aababb$	$8 : B \to aBB$
q_0	$Z_0\ aB$	$aababb$	$9 : S \to aB$
q_0	$Z_0\ S$	$aababb$	
q_1	Z_0	$aababb$	
q_2	Z_0	$aababb$	

ABOVE

To write down the construction of the shift-reduce PDA for a given CFG, we have two technical problems.

Consider a production $A \to \alpha$

First the stack (in standard notation) now contains the string α in reverse.

Second, we pop α, that is, several symbols, rather than exactly one. This can be simulated by popping the symbols one-by-one, using $|\alpha|$ separate transitions.

shift $\delta(q_0, \sigma, X) = \{(q_0, \sigma X)\}$ for $\sigma \in \Sigma$, $X \in \Gamma$

reduce '$\delta^*(q_0, \Lambda, \alpha^r) \ni (q_0, A)$' for $A \to \alpha$ in P

Special case: $\alpha = \Lambda$

Definition

The Nondeterministic Bottom-Up PDA $NB(G)$

Let $G = (V, \Sigma, S, P)$ be a context-free grammar.

The nondeterministic bottom-up PDA corresponding to G is $NB(G) = (Q, \Sigma, \Gamma, q_0, Z_0, A, \delta)$, defined as follows:

Q contains the initial state q_0, the state q_1, and the (only) accepting state q_2, together with other states to be described shortly.

$\Gamma = \ldots$

[M] D 5.22

Definition

The Nondeterministic Bottom-Up PDA $NB(G)$

Let $G = (V, \Sigma, S, P)$ be a context-free grammar.

The nondeterministic bottom-up PDA corresponding to G is $NB(G) = (Q, \Sigma, \Gamma, q_0, Z_0, A, \delta)$, defined as follows:

Q contains the initial state q_0, the state q_1, and the (only) accepting state q_2, together with other states to be described shortly.

$\Gamma = V \cup \Sigma \cup \{Z_0\}$

[M] D 5.22

Definition

The Nondeterministic Bottom-Up PDA $NB(G)$ (continued)

For every $\sigma \in \Sigma$ and every $X \in \Gamma$, $\delta(q_0, \sigma, X) = \{(q_0, \sigma X)\}$. This is a *shift* move.

For every production $B \to \alpha$ in G, and every nonnull string $\beta \in \Gamma^*$,
$(q_0, \Lambda, \alpha^r \beta) \vdash^* (q_0, \Lambda, B\beta)$,
where this *reduction* is a sequence of one or more moves in which, if there is more than one, the intermediate configurations involve other states that are specific to this sequence and appear in no other moves of $NB(G)$.

One of the elements of $\delta(q_0, \Lambda, S)$ is (q_1, Λ),
and $\delta(q_1, \Lambda, Z_0) = \{(q_2, Z_0)\}$.

[M] D 5.22

Theorem

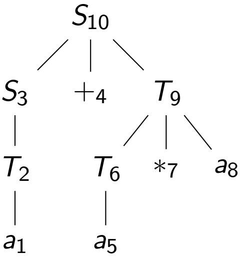
If G is a context-free grammar, then the nondeterministic bottom-up PDA $\mathrm{NB}(G)$ *accepts the language* $L(G)$.

The details of the proof of this result do not have to be known for the exam.

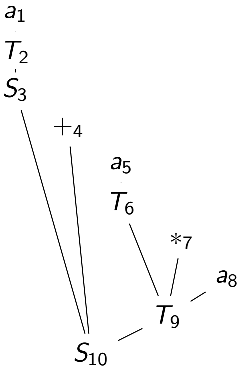
[M] Th 5.23

shift-reduce

post-order reduction $\equiv$ rightmost derivation, bottom-up



stack [reverse]	input
Z_0	$a + a * a$
$Z_0\ a_1$	$+a * a$
$Z_0\ T_2$	$+a * a$
$Z_0\ S_3$	$+a * a$
$Z_0\ S_3\ +_4$	$a * a$
$Z_0\ S_3\ +_4\ a_5$	$*a$
$Z_0\ S_3\ +_4\ T_6$	$*a$
$Z_0\ S_3\ +_4\ T_6\ *_7$	a
$Z_0\ S_3\ +_4\ T_6\ *_7\ a_8$	
$Z_0\ S_3\ +_4\ T_9$	
$Z_0\ S_{10}$	
$-$	



Due to Chomsky, Evey, and Schützenberger (1962/3).

Theorem

Context-free grammars and Pushdown automata are equivalent.

$\hookrightarrow$(1) PDA acceptance by empty stack

$\hookrightarrow$(2) triplet construction, CFG nonterminals $[p, A, q]$ for PDA computations

References

N. Chomsky. Context-free grammars and push-down storage. Quarterly Progress Report No. 65, Research Laboratory of Electronics, M.I.T., Princeton, New Jersey (1962)

R.J. Evey. The theory and application of pushdown store machines. In Mathematical Linguistics and Automatic Translation, NSF-IO, pages 217–255. Harvard University, May 1963,
and

M. P. Schützenberger. On context-free languages and pushdown automata. Inform. and Control, 6:217–255, 1963. doi:10.1016/S0019-9958(63)90306-1

From lecture 10:

$M = (Q, \Sigma, \Gamma, q_0, Z_0, A, \delta)$

configuration (q, x, α) $\quad q \in Q,\ x \in \Sigma^*,\ \alpha \in \Gamma^*$

state, remaining input, stack with top left

step $(p, ax, B\alpha) \vdash_M (q, x, \beta\alpha)$ $\quad$ when $(q, \beta) \in \delta(p, a, B)$

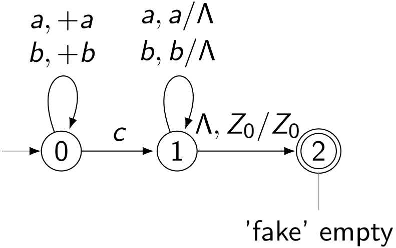
$\vdash_M^n \qquad \vdash_M^* \qquad \vdash \qquad \vdash^n \qquad \vdash^*$

Definition

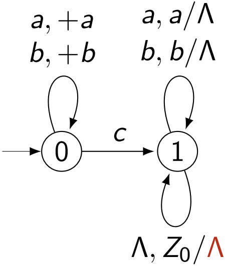
String x accepted by M (by *final state*), if
$(q_0, x, Z_0) \vdash^* (q, \Lambda, \alpha)$ for some $q \in A$, and some $\alpha \in \Gamma^*$
Language accepted by M (by *final state*)
$L(M) = \{\ x \in \Sigma^* \mid x \text{ accepted by } M\ \}$

read complete input, end in accepting state, some path

[M] D 5.2



check state



check stack

ABOVE

On many cases the PDA moves to the accepting state after checking that the stack is empty, when the topmost symbol is a special Z_0 that always has been at the bottom of the stack.

It may be more natural to accept directly by looking at the stack rather than by looking at the state. This leads to the notion of the *empty stack* language of a PDA.

$M = (Q, \Sigma, \Gamma, q_0, Z_0, A, \delta)$

Definition

Language accepted by M by *empty stack*
$L_e(M) = \{ \ x \in \Sigma^* \mid (q_0, x, Z_0) \vdash^* (q, \Lambda, \Lambda) \text{ for some state } q \in Q \ \}$

[M] D 5.27

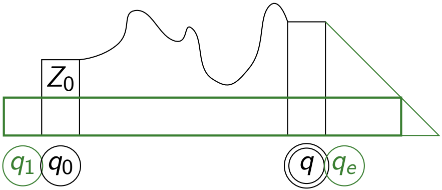
Theorem

If M is a PDA then there is a PDA M_1 such that $L_e(M_1) = L(M)$.

Sketch of proof...
[M] Th 5.28

Simulate $M = (Q, \Sigma, \Gamma, q_0, Z_0, A, \delta)$

Simulate $M = (Q, \Sigma, \Gamma, q_0, Z_0, A, \delta)$
– empty stack 'at' final state
– prohibit early empty stack

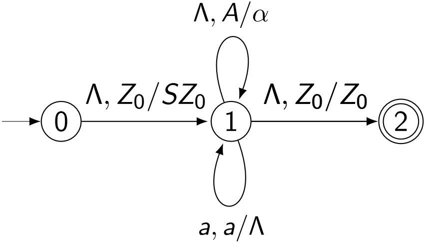


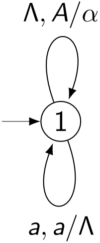
Construction PDA $M_1 = (Q_1, \Sigma, \Gamma_1, q_1, Z_1, A_1, \delta_1)$ such that $L_e(M_1) = L(M)$

– $Q_1 = Q \cup \{q_1, q_e\}$
– $\Gamma_1 = \Gamma \cup \{Z_1\}$
– new instructions:

$\delta_1(q_1, \Lambda, Z_1) = \{(q_0, Z_0 Z_1)\}$
$\delta_1(q, \Lambda, X) \ni (q_e, X)$ for $q \in A$, and $X \in \Gamma_1$
$\delta_1(q_e, \Lambda, X) = \{(q_e, \Lambda)\}$ for $X \in \Gamma_1$

$A \to \alpha \in P$, $a \in \Sigma$





Theorem

For every CFL L there exists a single state PDA M such that $L_e(M) = L$.

ABOVE

Now that we have empty stack acceptance we can reconsider the expand-match technique. In fact we do not need two extra states to introduce a bottom of stack symbol, and can make a single state PDA.

BELOW

The expand-match method can be used for any CFG. If we slightly restrict the grammars, we can combine each match with the expand step just before, that introduced the terminal. This gives a very direct translation between grammar and its leftmost derivation, and a single state PDA and its computation.

On this normal form each production is of the form $A \to a\alpha$, where $a \in \Sigma \cup \{\Lambda\}$ can be the only terminal at the right. That means that any terminal pushed on the stack will be on top, and immediately will be matched.

cfg G	$\Longleftrightarrow$	1-pda M	
$A \to \alpha$		$\delta(-, \Lambda, A) \ni (-, \alpha)$	expand
		$\delta(-, a, a) = \{(-, \Lambda)\}$	match

normal form $\alpha \in (\Sigma \cup \{\Lambda\}) \cdot V^*$

$A \to a\alpha$ $\delta(-, a, A) \ni (-, \alpha)$ combined

SimplePal: $S \to aSA \mid bSB \mid c$ $A \to a$ $B \to b$

leftmost derivation $\Longleftrightarrow$ computation

	S		$(-, abbcbba, S)$
$\Rightarrow$	$a\, SA$	$\vdash$	$(-, bbcbba, SA)$
$\Rightarrow$	$ab\, SBA$	$\vdash$	$(-, bcbba, SBA)$
$\Rightarrow$	$abb\, SBBA$	$\vdash$	$(-, cbba, SBBA)$
$\Rightarrow$	$abbc\, BBA$	$\vdash$	$(-, bba, BBA)$
$\Rightarrow$	$abbcb\, BA$	$\vdash$	$(-, ba, BA)$
$\Rightarrow$	$abbcbb\, A$	$\vdash$	$(-, a, A)$
$\Rightarrow$	$abbcbba$	$\vdash$	$(-, \Lambda, \Lambda)$

In this case: deterministic PDA

Exercise 5.21.
Prove the converse of Theorem 5.28:
If there is a PDA $M = (Q, \Sigma, \Gamma, q_0, Z_0, A, \delta)$ accepting L by empty stack (that is, $x \in L$ if and only if $(q_0, x, Z_0) \vdash_M^* (q, \Lambda, \Lambda)$ for some state q), then there is a PDA M_1 accepting L by final state (i.e., the ordinary way).