ALGORITMIEK: antwoorden werkcollege 13

Problem 1.
a. The all-matrix version: Repeat the following operation n times. Select the smallest element in the unmarked rows and columns of the cost matrix and then mark its row and column.
The row-by-row version: Starting with the first row and ending with the last row of the cost matrix, select the smallest element in that row which is not in a previously marked column. After such an element has been selected, mark its column to prevent selecting another element from the same column.
b. Neither of the two versions always yields an optimal solution. The following cost matrix is a simple counterexample:

	job 1	job 2
Anna	1	2
Bob	2	100

Problem 2.
a. Let 1, 2, 5 and 10 be labels representing the men from the problem, and let f represent the flashlight's location. The following sequence of states solves the problem:

left side	time elapsed	right side
$1, 2, 5, 10, f$	0	
$5, 10$	2	$1, 2, f$
$1, 5, 10, f$	3	2
1	13	$2, 5, 10, f$
$1, 2, f$	15	$5, 10$
	17	$1, 2, 5, 10, f$

b. Repeat the following steps $n-2$ times: Send to the other side the pair of two fastest remaining persons and then return the flashlight with the (single) fastest person. Finally, send the remaining two people together. The total crossing time will be equal to

$$(t_2 + t_1) + (t_3 + t_1) + \cdots + (t_{n-1} + t_1) + t_n = \sum_{i=2}^{n} t_i + (n-2) \cdot t_1 = \sum_{i=1}^{n} t_i + (n-3) \cdot t_1$$

The solution to the instance from part **a.** shows that this greedy algorithm does not always yield the minimal crossing time for $n > 3$. No smaller counterexample can be given, as a simple exhaustive check for $n = 3$ demonstrates. (The obvious solution for $n = 2$ is the one generated by the greedy algorithm as well.)
Note: for an algorithm that always yields a minimal crossing time, see Günter Rote, "Crossing the bridge at night," *EATCS Bulletin*, vol. 78 (October 2002), 241–246.

Problem 6. (Travelling Salesman Problem) This problem has been worked out in the slides of lecture 13.