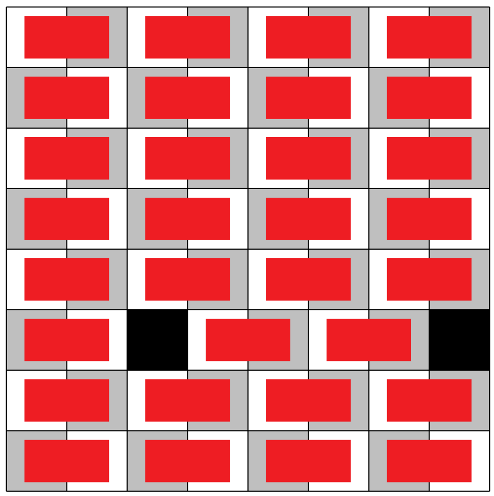
**ALGORITMIEK: some solutions to problems from exercise class 1**

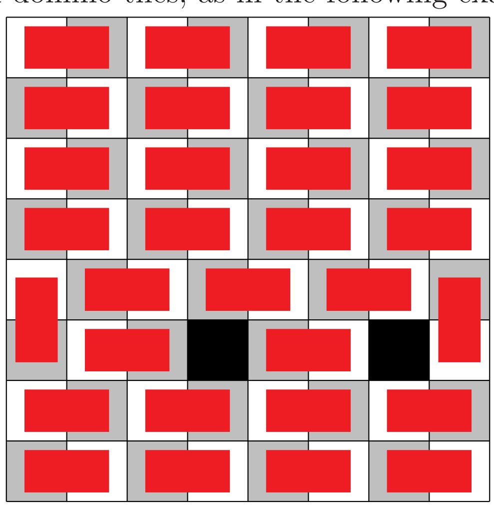
1. There are two possibilities:

   - The two deleted squares are in the same row. In this case, because one of the deleted squares is black, and the other one is white, the number of squares inbetween is even (in particular: 0, 2, 4 or 6). These squares can be filled by horizontal domino tiles. Now there are two subcases:

     – Both the number of squares to the left of the leftmost deleted square and the number of squares to the right of the rightmost deleted square is even. In that case, the entire row can be filled by horizontal domino tiles. All other rows can also be filled by horizontal domino tiles, as in the following example:

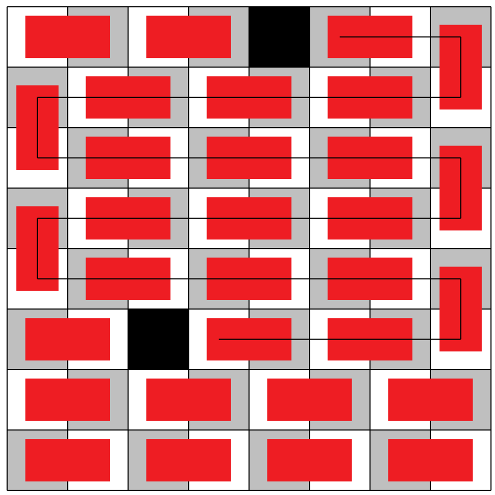
     

     – Both the number of squares to the left of the leftmost deleted square and the number of squares to the right of the rightmost deleted square is odd. In that case, we need an adjacent row (either the row above or the row below the row containing the deleted squares), and two vertical domino tiles to fill the row containing the deleted squares. All other rows can be filled by horizontal domino tiles, as in the following example:

   - The two deleted squares are not in the same row. Consider the lower deleted square (the one in the lower row). Either the number of squares to the left or the number of squares to the right of this square is even, because their sum is 7.

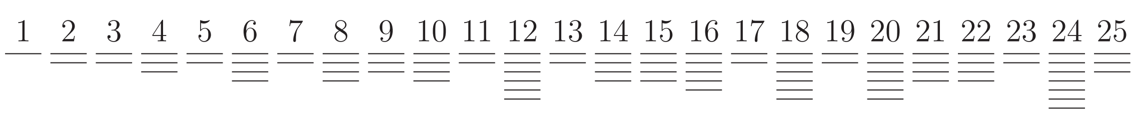
Without loss of generality, assume the number of squares to the left of the deleted square is even. Then we fill this row from left to right with horizontal domino tiles, until we hit the deleted square. Assume this deleted square is black. Then we continue placing domino tiles in a snake-like fashion, where every domino 'begins on a white square and ends on a black square'. The snake moves up from row to row until it hits the second deleted square. As this deleted square is white, the snake consists of an integer number of tiles. Now, we can fill the rest of the chessboard with horizontal tiles, as in the following example



Actually, the case with both deleted squares in the same row can be solved with this snake-like solution, as well, simply by filling the chessboard column-by-column, instead of row-by-row. Obviously, two squares cannot be both in the same row and in the same column.

2. After the first iteration, all doors are open. After the second iteration, only the odd-numbered doors are open (the even-numbered doors have just been closed). In the third iteration, the multiples of 3 are toggled, etc.

   We consider the result for an example, in particular the case $n = 25$. Every time we toggle a door, we add an underline:




   Note that the number of times door $m$ is toggled (i.e., the number of lines under door $m$) equals the number of divisors of $m$. When this number is odd, the door is open. When this number is even, the door is closed.

   We observe that doors 1, 4, 9, 16 and 25 are open, exactly the squares. This will be the case for every $n$: the square-numbered doors $\leq n$ will be open.

   This can be explained as follows: for every number $m$, both 1 and $m$ divide $m$, so door $m$ is toggled (a.o.) in iterations 1 and $m$ (two times, unless $m = 1$). Whenever $k$ divides $m$, also $\frac{m}{k}$ divides $m$, so divisors of $m$ come in pairs. Therefore, in general a number $m$ has an even number of divisors, which means that door $m$ will is closed at the end.

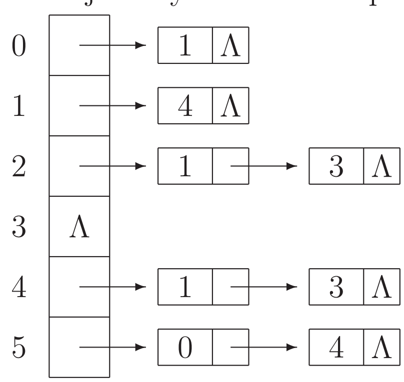
The only exception are the squares $m = k^2$. For such $m$, $k$ equals $\frac{m}{k}$, so this particular divisor comes alone. Hence, the total number of divisors of a square $m$ is odd, which means that door $m$ is open at the end.

6. There are four cases to be considered:

**Directed graph, adjacency matrix.** In this case, `graaf[i][j]` $= 1$, if and only if there is a *directed* edge from $i$ to $j$. The matrix `graaf` is not necessarily symmetric. The adjacency matrix for example graph 2 is:

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

**Directed graph, adjacency list.** In this case, `graaf[i]` must be interpreted as the header of a list of nodes $j$ for which there is a *directed* edge from $i$ to $j$. The adjacency list for example graph 2 is:



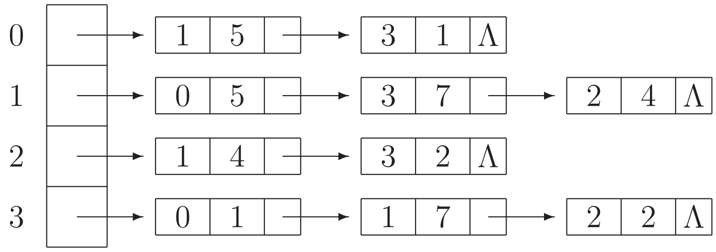

**Weighted graph, adjacency matrix.** In this case, `graaf[i][j]` is the weight of the edge between $i$ and $j$ (or from $i$ to $j$). Hence, the matrix `graaf` can hold values different from 0 and 1. We should choose a proper default value for edges that are missing in the graph. Depending on the situation, this might for instance be: 0, $\infty$, $-1$ or $-\infty$. When we use $\infty$ as the default value, the adjacency matrix for example graph 3 is:

$$\begin{pmatrix} \infty & 5 & \infty & 1 \\ 5 & \infty & 4 & 7 \\ \infty & 4 & \infty & 2 \\ 1 & 7 & 2 & \infty \end{pmatrix}$$

**Weighted graph, adjacency list.** In this case, `graaf[i]` is the header of a list of objects of an extended class `buur`, which also contains the weight of the edge represented:

```
class buur
{ public:
    int knoopnummer;
    int gewicht;
    buur* volgende;
};  // buur
```

The adjacency list for example graph 3 is:




7.

```
void telvoorEuler ( buur* graaf[n] ) {
  int totaal = 0;
  for ( int i=0; i<n; i++ ) {
    teller = 0;
    buur* hulp = graaf[i];
    while ( hulp != NULL ) {  // buurlijst aflopen
      teller ++;
      hulp = hulp->volgende;
    }
    if ( teller%2 == 1 )  // oneven
      totaal ++;
  }  // for
  if ( totaal <= 2 )
    cout << "hooguit twee..." << endl;
  else
    cout << "meer dan twee..." << endl;
}
```

8. (a)

```
int takken ( buur* graaf[n] ) {
  int count = 0;
  for ( int i=0; i<n; i++ ) {
    buur* hulp = graaf[i];
    while ( hulp != NULL ) {  // buurlijst aflopen
      count++;
      hulp = hulp->volgende;
    }
  }
    // alles ‘per ongeluk’ dubbel geteld -> corrigeren
  return count/2;
}
```

(b)

```
int takken2 ( int graaf[n][n] ) {
  int count = 0;
  for ( int i=0; i<n; i++ ) {
    for ( int j=0; j<n; j++ ) {
      if ( graaf[i][j] > 0 ) {
        count++;
      }
    }
  }
    // alles ‘per ongeluk’ dubbel geteld -> corrigeren
  return count/2;
```

```
}
```

Variant: alleen deel rechtsboven de diagonaal van de adjacency matrix aflopen, of juist alleen het deel linksonder de diagonaal. Dan hoef je na afloop niet door 2 te delen.

(c) De adjacency list representatie is het meest efficiënt, omdat je daarbij alleen maar kijkt naar de bestaande takken. Bij de adjacency matrix representatie loop je ook niet-bestaande takken af.

9. (a) Omkeren van een pijl $(i, j)$ met adjacency matrix `int graaf[n][n]` .
Er loopt tevoren een tak $(i, j)$, maar geen tak $(j, i)$, dus `graaf[i][j] = 1` en `graaf[j][i] = 0`. Dit draaien we om.

```
void draaiom ( int graaf[n][n], int i, int j ) {
  graaf[i][j] = 0;
  graaf[j][i] = 1;
}
```

(b) Omkeren van een pijl $(i, j)$ met adjacency list `buur* graaf[n]` .
De lijsten zijn niet per se gesorteerd. Er loopt tevoren een tak $(i, j)$, maar geen tak $(j, i)$, dus $j$ komt voor in lijst `graaf[i]` en $i$ komt niet voor in lijst `graaf[j]` .

Idee voor algoritme:
- j in buurlijst van i opzoeken
- j uit buurlijst van i verwijderen
- i vooraan in buurlijst van j plaatsen.

```
void draaiom2 ( buur* graaf[n], int i, int j ) {
  buur* hulp = graaf[i];
  buur* vorige = NULL;
  while ( hulp->knoopnummer != j ) { // knoop j zoeken
    vorige = hulp;
    hulp = hulp->volgende;
  }
    // haal buur j uit lijst
  if ( vorige == NULL ) { // j is eerste buur van i
    graaf[i] = hulp->volgende;
  }
  else { // j is niet eerste buur van i
    vorige->volgende = hulp->volgende;
  }
  delete hulp;  // gooi buur j weg

    // voeg nu i vooraan in de buurlijst van j toe
  hulp = new buur;
  hulp->knoopnummer = i;
  hulp->volgende = graaf[j];
  graaf[j] = hulp;
}
```