Social Network Analysis for Computer Scientists

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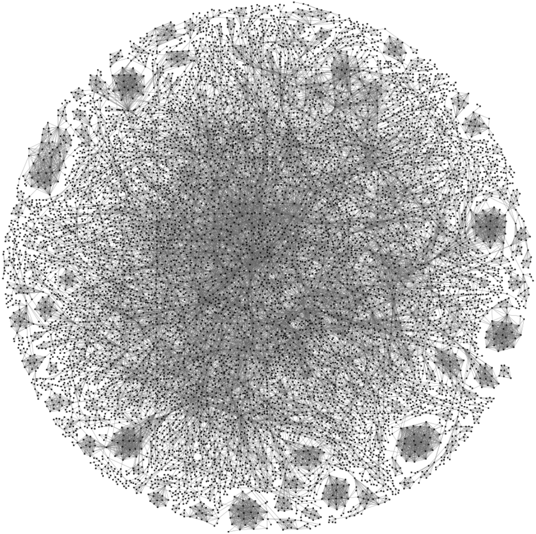
Lecture 2 — Small world phenomenon, Advanced network concepts and Centrality

Recap

Networks

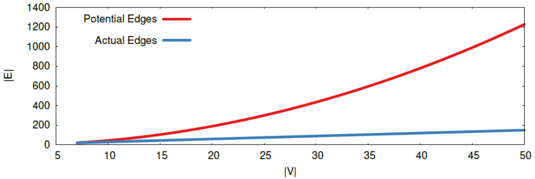


Notation

Concept	**Symbol**
▪ Network (graph)	$G = (V, E)$
▪ Nodes (objects, vertices, ...)	V
▪ Links (ties, relationships, ...)	E
▪ Directed — $E \subseteq V \times V$ — "links"	
▪ Undirected — "edges"	
▪ Number of nodes — $\|V\|$	n
▪ Number of edges — $\|E\|$	m
▪ Degree of node u	$deg(u)$
▪ Distance from node u to v	$d(u, v)$

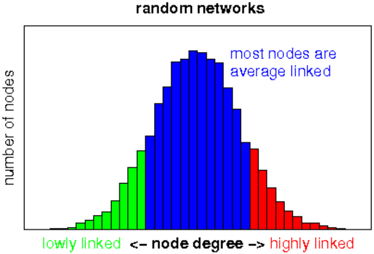
Density

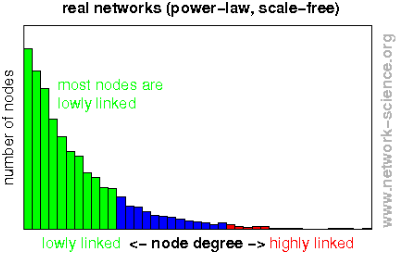
- Maximum number of edges $m_{\max}$
  - $m_{\max} = n(n-1)$ for directed graphs
  - $m_{\max} = \frac{1}{2}n(n-1)$ for undirected graphs
- **Density**: $\frac{m}{m_{\max}}$



- Real-world networks are typically **sparse**, i.e., $m \ll m_{\max}$
- Density is particularly relevant when comparing networks

Degree distribution





Giant component

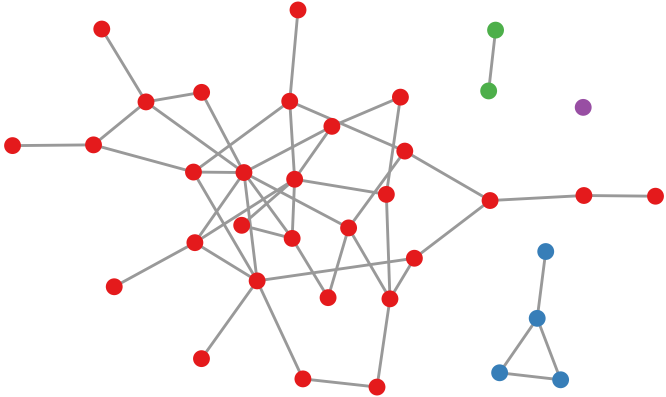


Figure: Each component has its own color.

Unique properties of real-world networks

- Five interesting metrics that distinguish real-world networks from trivial graph structures:
  1. Density
  2. Degree
  3. Components
- To be discussed today:
  4. Distance
  5. Clustering coefficient

Today

- Two additional properties of real-world networks
  4. Distance
  5. Clustering coefficient
- Advanced network concepts
- Graph traversal
- Centrality

Small world experiment

- Stanley Milgram
- Starts with 96 random people in Omaha
- Ask them to get a letter to a stock-broker in Boston by passing it through to a closer acquaintance.
- How many steps did it take?

Small world experiment

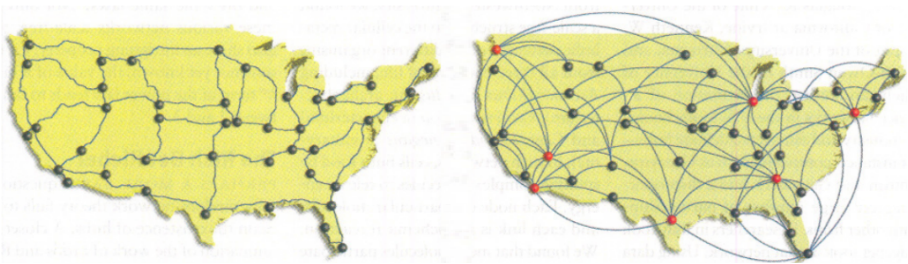
- Stanley Milgram
- Starts with 96 random people in Omaha
- Ask them to get a letter to a stock-broker in Boston by passing it through to a closer acquaintance.
- How many steps did it take?
- Letters arrived after on average 5.9 steps
- Total of 18 chains completed

J. Travers and S. Milgram, "An Experimental Study of the Small World Problem", Sociometry 32(4): 425-443, 1969

Yahoo small world experiment

Hubs

- **Hubs** are highly connected amongst each other
- Peripheral nodes have a very small distance to the hubs



Barabasi, Scientific American, May 2003

Distance

- **Average distance** $\overline{d} = \frac{1}{n(n-1)} \sum_{v,w \in V} d(v,w)$
- **Distance distribution**: how often each distance value occurs (computed over all node pairs).

Dataset	Nodes	Links	Average degree	Average distance
AstroPhys	17,903	396K	21	4.15
Enron	33,696	362K	10	4.07
Web	855,802	8.64M	10	6.30
YouTube	1,134,890	5.98M	5.3	5.32
Skitter	1,696,415	22.2M	13	5.08
Wikipedia	2,213,236	23.5M	11	4.81
Orkut	3,072,441	234M	76	4.16
LiveJournal	5,189,809	97.4M	19	5.48
Hyves	8,057,981	871M	112	4.75

F.W. Takes and W.A. Kosters, Determining the Diameter of Small World Networks, In CIKM, pp. 1191-1196, 2011.

Distance distribution

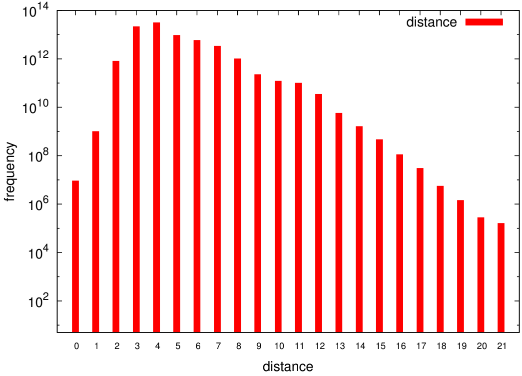


Figure: Distance distribution of the HYVES network (sampled over node pairs).

Erdős number

- Scientific collaboration network
- Edges between scientists who wrote a paper together
- Erdős number: the distance of a scientist (node) to Erdős
- `https://mathscinet.ams.org/mathscinet/collaborationDistance.html`

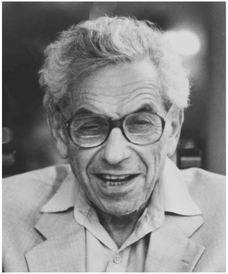
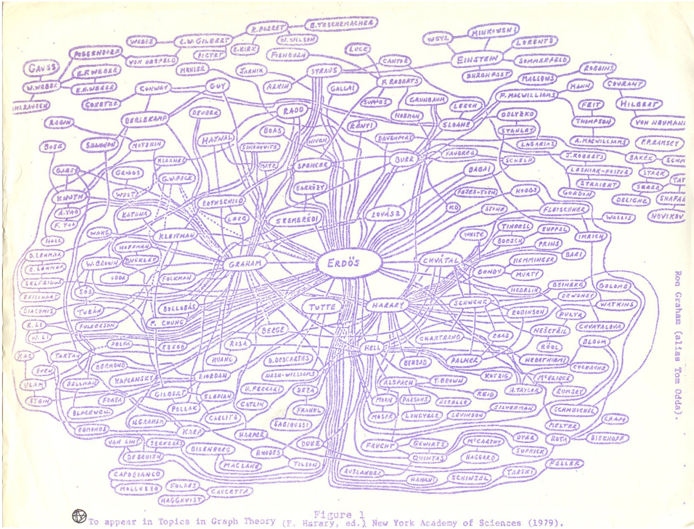
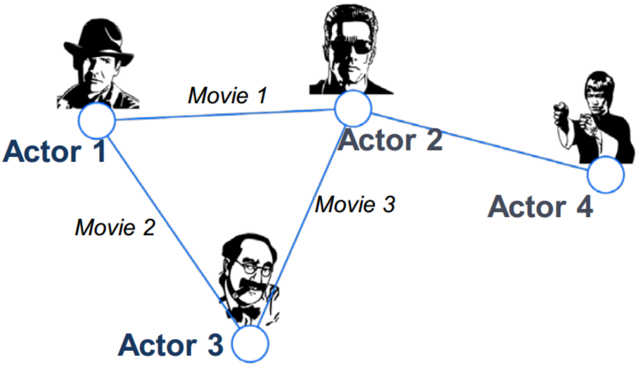


Figure: Paul Erdős (1913-1996)

Erdős number



Movie actor network



Source: `http://web.stanford.edu/class/cs224w`

Six degrees of Kevin Bacon

- Actor collaboration network based on co-starring actors
- Variant of "Six degrees of Separation"
- Edges between actors indicate they played in a movie together
- Try finding a path of length longer than six at `https://oracleofbacon.org`

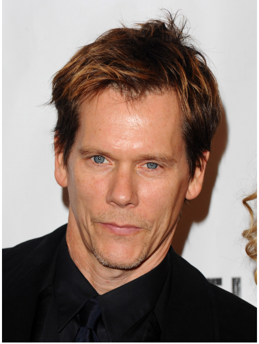
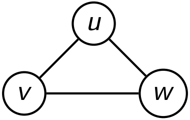


Figure: Kevin Bacon (1958)

The Wiki Game

Expected number of triangles



- **Triangle**: for nodes $u, v, w \in V$ we have $(u, v), (v, w), (w, u) \in E$

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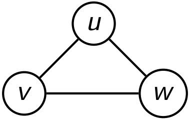

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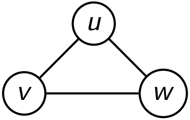

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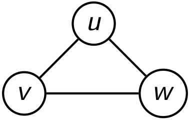

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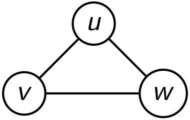

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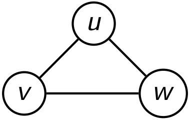
Expected number of triangles



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- **Expected triangles**: $(8m^3/n^6)(n^3/6) = \frac{4}{3}(m/n)^3$
- For $n = 1000$ and $m = 8000$, we would expect 683 triangles.

Triangles

Network	Nodes	Edges	Expected	Real	Difference
Facebook (WOSN)	63,731	817,035	2,809	3,500,542	1,246×
Epinions	75,879	508,837	402	162,448	404×
Amazon (TWEB)	403,394	3,387,388	789	3,986,507	5,049×
Baidu	415,641	3,284,387	658	14,287,651	21,718×
Youtube links	1,138,499	4,942,297	109	3,049,419	27,957×
Flickr	2,302,925	33,140,017	3,973	837,605,842	210,806×
LiveJournal links	5,204,176	49,174,464	1,125	310,876,909	276,367×
Twitter (MPI)	52,579,682	1,963,263,821	69,410	55,428,217,664	798,565×

Table: Expected vs. real triangle counts in real-world networks.

Triads and triangles

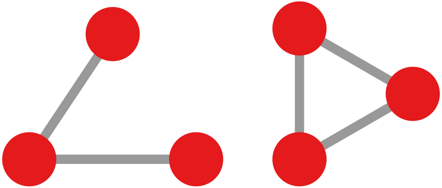

Figure: Triad/wedge (left) and Triangle (right).

- **Structural hole**: common neighbors that are not connected to each other

Global clustering coefficient

- **Global clustering coefficient** for a graph G:

$$CC'(G) = \frac{3 \cdot \text{number of triangles}}{\text{number of triads}}$$

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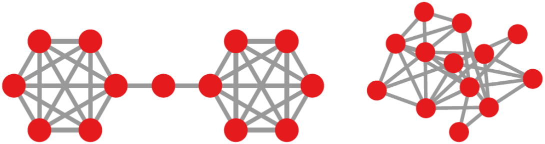

Figure: High clustering (left) and lower clustering (right).

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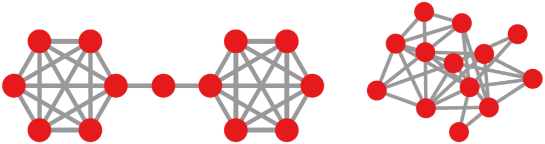


Figure: High clustering (left) and lower clustering (right).

- Measure of **global transitivity**
- Small world networks: **high clustering coefficient** compared to a random graph with the same number of nodes

Local clustering coefficient

- **Local clustering coefficient**: extent to which a node v forms triangles with its neighbors
- Node clustering coefficient for a node $v \in V$:

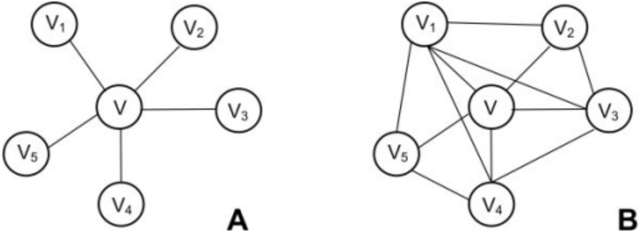
$$CC(v) = \frac{|\{(u,w) \in E : (u,v) \in E \wedge (v,w) \in E\}|}{\frac{1}{2} deg(v) \cdot (deg(v) - 1)}$$

(where $deg(v) > 1$ is the degree of node v)

$$CC(v) = \frac{\text{edges between neighbors of } v}{\text{maximum number of such edges}}$$

- Measure of **local transitivity**

Local clustering coefficient



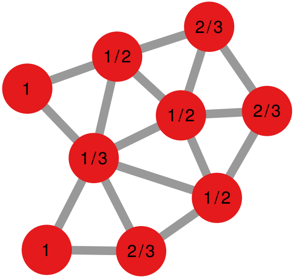
- Situation A: v has a clustering coefficient of 0
- Situation B: v has a clustering coefficient of $\frac{7}{10} = 0.7$

Image: G.A. Pavlopoulos et al., "Using graph theory to analyze biological networks", in BioData Mining 4(1), 2011.

Average local clustering coefficient

- **Average local clustering coefficient** for a graph G:

$$CC(G) = \frac{1}{n} \cdot \sum_{v \in V} CC(v)$$



- Small world networks: **high clustering coefficient** compared to a random graph with the same number of nodes

Real-world networks

- Five interesting properties that distinguish real-world networks from trivial graph structures:
  1. Sparse networks — density
  2. Fat-tailed power-law degree distribution — degree
  3. Giant component — components
  4. Low pairwise node-to-node distances — distance
  5. Many triangles — clustering coefficient
- Many examples: social networks, communication networks, citation networks, collaboration networks (Erdős, Kevin Bacon), protein interaction networks, information networks (Wikipedia), webgraphs, financial networks (Bitcoin) ...

Advanced concepts

Advanced concepts

- Reciprocity
- Complete graph, complement of a graph
- Planar graph, lattice, regular graph
- Tree and dag
- Subgraphs
- Ego network
- Spanning trees
- Diameter
- Bridges

Reciprocity

- **Reciprocity**: measure of the likelihood of nodes in a directed network to be mutually linked
- Let $m_{<->}$ be the number of links in the directed network for which there also exists a symmetric counterpart:

$$m_{<->} = |\{(u, v) \in E : (v, u) \in E\}|$$

- Reciprocity r is then the fraction of links that is symmetric:

$$r = \frac{m_{<->}}{m}$$

- Measures the extent to which relationships are mutual
- Useful to compare between networks

Simple vs. complex networks

- **Simple networks** can be fully described analytically, typically using some formula or definition

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- **Simple networks** can be fully described analytically, typically using some formula or definition
  Not to be confused with the **simple graph** (yes, this is a bit confusing, also in the Coscia book)
- **Complex networks**: topological features cannot be described analytically, formulas can at best approximate salient characteristics

Complete graphs

- In **complete graphs**, all pairs of nodes are connected
- The number of edges m is equal to $\frac{1}{2} \cdot n \cdot (n-1)$

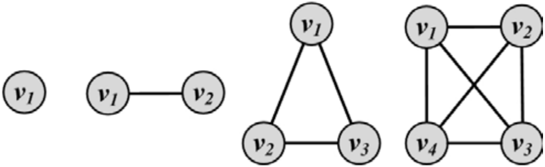


Figure: Complete graphs of size 1, 2, 3 and 4

Image: Zafarani et al., Social Media Mining, 2014.

Complement of a graph

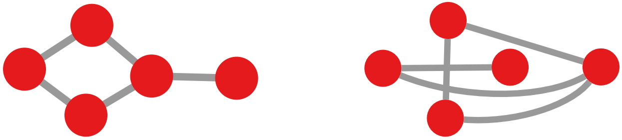
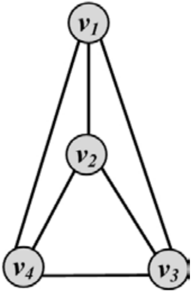


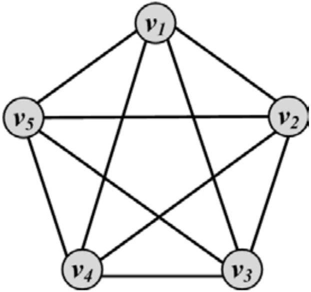
Figure: A simple graph and its complement.

Planar graphs

- **Planar graphs** can be visualized such that no two edges cross each other



(a) Planar Graph



(b) Non-planar Graph

Image: Zafarani et al., Social Media Mining, 2014.

Regular graphs and lattices

- **Regular graph**: graph in which each node has the same degree
- **Lattice**: graph drawn in an Euclidean space to form a tiling or pattern

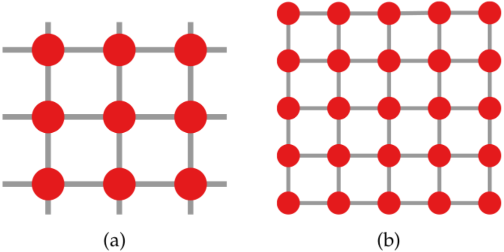


Figure: An infinite and a finite (2D) lattice.

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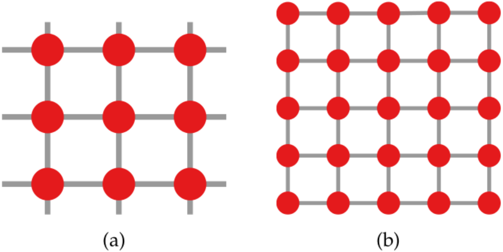

Figure: An infinite and a finite (2D) lattice.

- Which lattices are regular?

Other lattices

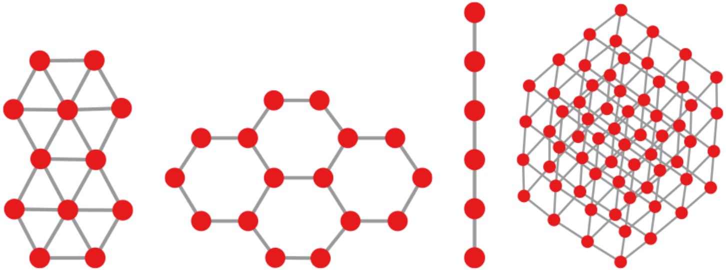

Figure: 2D triangular and 2D hexagonal lattices, 1D and 3D lattices.

Other lattices

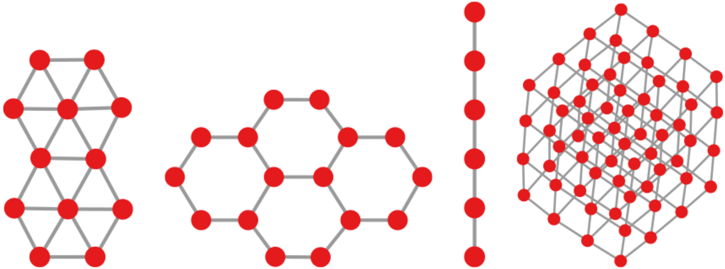

Figure: 2D triangular and 2D hexagonal lattices, 1D and 3D lattices.

- Which lattices are planar?

Trees

- A **tree** is a connected graph without cycles
- A set of disconnected trees is called a **forest**
- A tree with n nodes has $m = n - 1$ edges

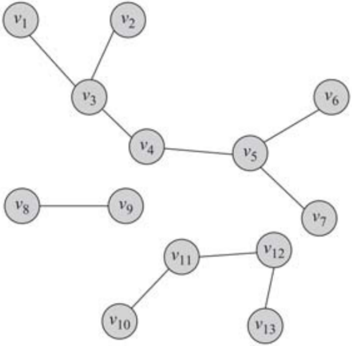

Image: Zafarani et al., Social Media Mining, 2014.

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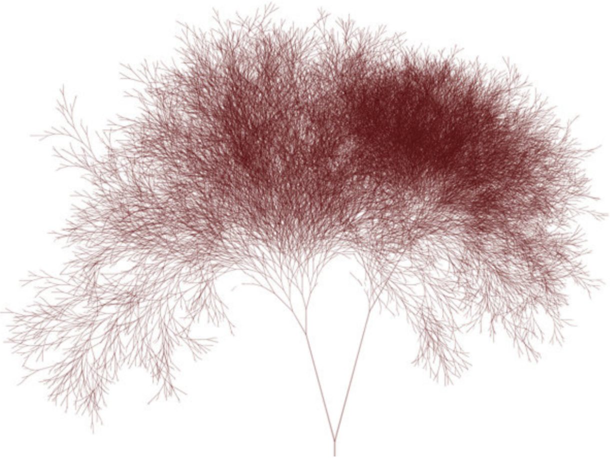

Image: M. Lima, Book of trees: Visualizing branches of knowledge, 2014.

Directed acyclic graph

- An undirected tree has no cycles
- **Directed acyclic graph**: directed graph which does not contain a directed cycle (but may have cycles if you ignore edge direction)

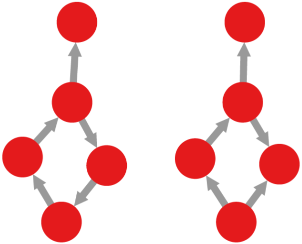

Figure: Directed cyclic graph and directed acyclic graph (dag).

Subgraphs

- Given a graph $G = (V, E)$, there are two ways in which subgraphs are commonly studied:
  1. **Subgraph** $G' = (V', E')$ with $V' \subseteq V$ and $E' \subseteq (E \cap (V' \times V'))$ (subset of the nodes and edges of the original network, commonly used when defining communities or clusters)

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- Special subgraphs:
  - Ego network
  - Spanning tree

Ego network

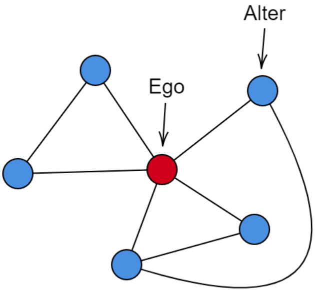


Figure: The **ego network** of a given node in a network consists of the set of nodes containing that node ("Ego") and its direct neighbors ("Alters"), and all edges present between the nodes in this set

Image: Wikipedia "Egocentric network.png", accessed 2022.

Spanning trees

- A **spanning tree** is a tree and subgraph of a graph that covers all nodes of the graph
- In weighted graphs, a **minimal** spanning tree is one of minimal edge weight

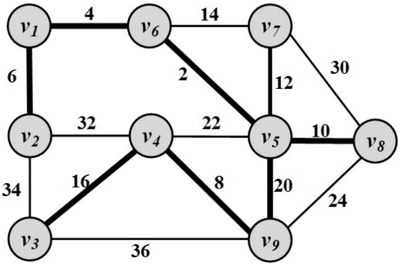


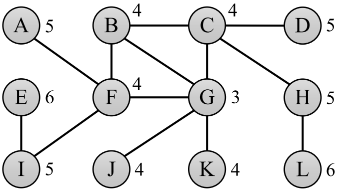
Image: Zafarani et al., Social Media Mining, 2014.

Diameter

- Distance $d(u, v)$ = length of shortest path from u to v
- **Diameter** $D(G) = \max_{u,v \in V} d(u, v)$ = maximal distance

Diameter

- Distance $d(u, v)$ = length of shortest path from u to v
- **Diameter** $D(G) = \max_{u,v \in V} d(u, v)$ = maximal distance
- **Eccentricity** $e(u) = \max_{v \in V} d(u, v)$ = length of longest shortest path from u
- Diameter $D(G) = \max_{u \in V} e(u)$ = maximal eccentricity
- **Radius** $R(G) = \min_{u \in V} e(u)$ = minimal eccentricity



Bridges

- **Bridge**: an edge whose removal will result in an increase in the number of connected components
- Also called **cut edges**, with applications in community detection

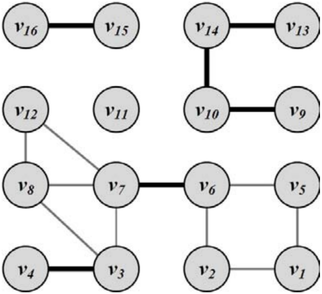


Image: Zafarani et al., Social Media Mining, 2014.

Graph traversal

Graph traversal

- Given a network, how can we explore it?
- Specifically: exploration starting from a particular source
- Node **adjacency**: two nodes are adjacent if there is an edge connecting them
- **Neighborhood**: set of nodes adjacent to a node $v \in V$:

$$N(v) = \{w \in V : (v, w) \in E\}$$

- Techniques to iteratively explore neighborhoods: Depth-First Search (DFS) and Breadth-First Search (BFS)

Graph traversal: DFS and BFS

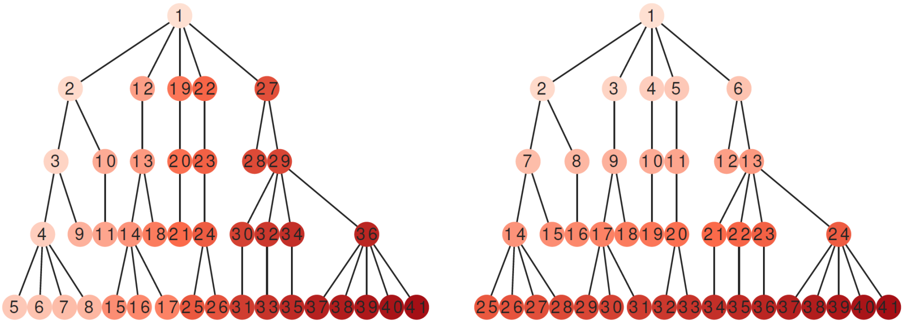


Figure: DFS and BFS

Graph traversal: BFS

- **Breadth First Search** (BFS)
- From source node, create a rooted spanning tree of the graph
- Graph traversal in level-order
- Often implemented using a queue
- BFS considers traversing each of the m edges once, so $O(m)$
- Important for computing various centrality measures

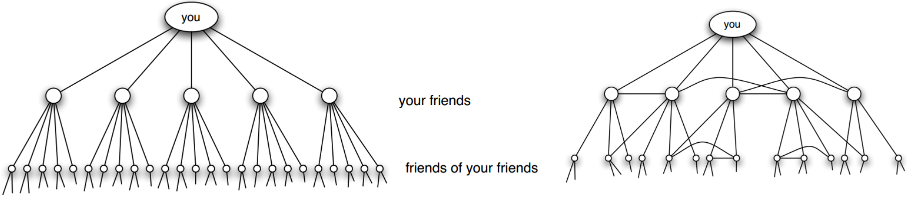


Image: Zafarani et al., Social Media Mining, 2014.

Shortest path algorithms

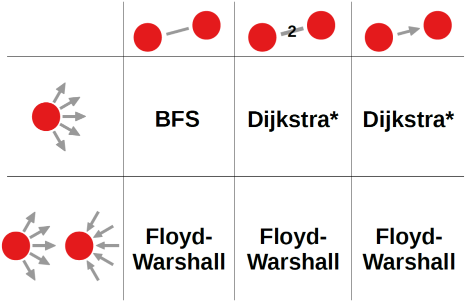


Figure: Columns: Undirected, weighted and directed graphs. Rows: single-source or all-source shortest paths.

(Many faster approaches exist; see *Network Science Seminar*)

Centrality

Centrality

- Given a social network, which person is most important?
- What is the most important page on the web?
- Which protein is most vital in a biological network?
- Who is the most respected author in a scientific citation network?
- What is the most crucial router in an internet topology network?

Centrality

- **Node centrality**: the importance of a node with respect to the other nodes based on the structure of the network
- **Centrality measure**: computes the centrality value of all nodes in the graph
- For all $v \in V$ a measure M returns a value $C_M(v) \in [0;1]$
- $C_M(v) > C_M(w)$ means that node v is more important than w


GAME OF THRONES
Watermark

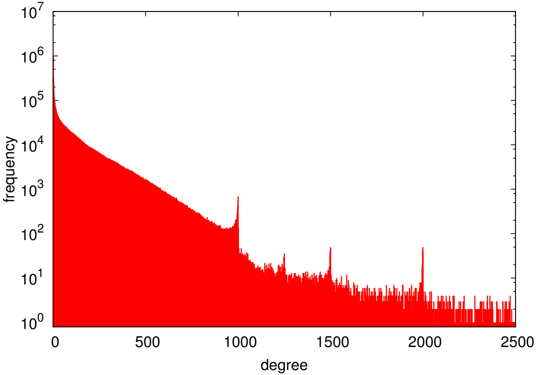
Degree centrality

- Undirected graphs – **degree centrality**: measure the number of adjacent nodes

$$C_d(v) = \frac{deg(v)}{n-1}$$

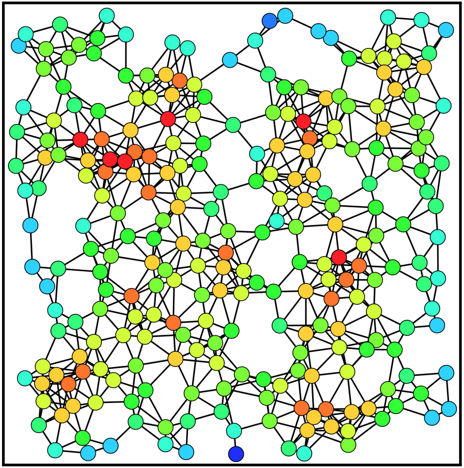
- Directed graphs — indegree centrality and outdegree centrality
- Local measure
- $O(1)$ time to compute

Degree distribution

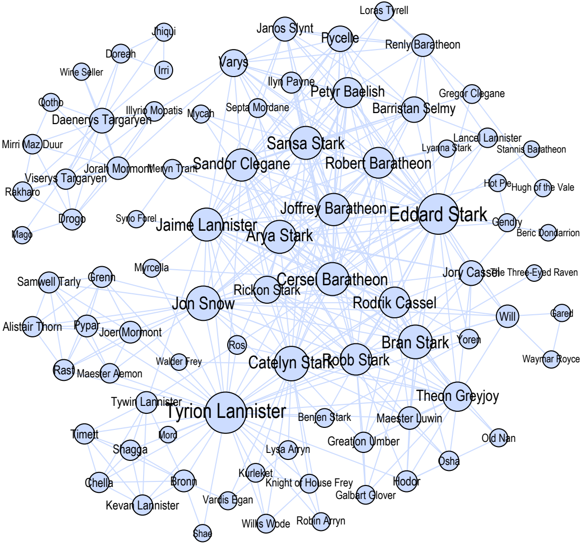


- Not so many distinct values in the lower ranges

Degree centrality



Degree centrality



Closeness centrality

- **Closeness centrality**: based on the average distance to all other nodes
$$C_c(v) = \left(\frac{1}{n-1} \sum_{w \in V} d(v,w) \right)^{-1}$$
- Global distance-based measure
- $O(mn)$ to compute: one BFS in $O(m)$ for each of the n nodes

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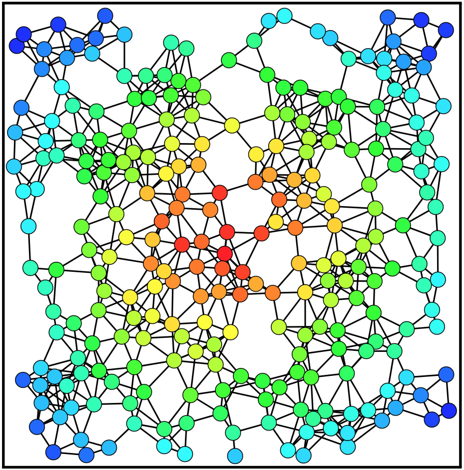
- Global distance-based measure
- $O(mn)$ to compute: one BFS in $O(m)$ for each of the n nodes
- Connected component(s)...
- **Harmonic centrality**: variant of closeness that can deal with multiple components (not normalized)

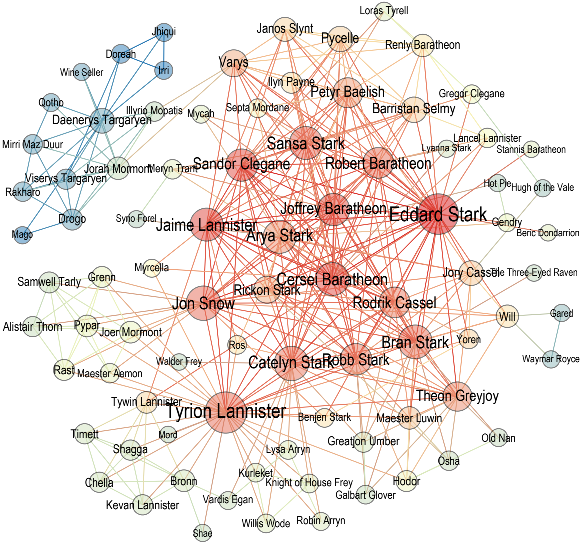
$$C_h(v) = \sum_{w \in V} \frac{1}{d(w,v)}$$

Sabidussi, G., The centrality index of a graph, *Psychometrika* 31(4): 581–603, 1966.

Closeness centrality



Degree vs. closeness centrality



Betweenness centrality

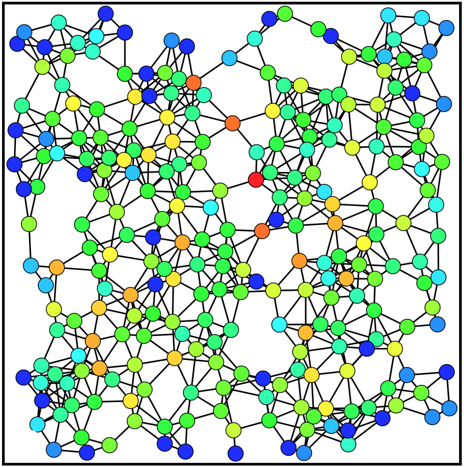
- **Betweenness centrality**: measure the number of shortest paths that run through a node

$$C_b(u) = \sum_{\substack{v,w \in V \\ v \neq w, u \neq v, u \neq w}} \frac{\sigma_u(v,w)}{\sigma(v,w)}$$

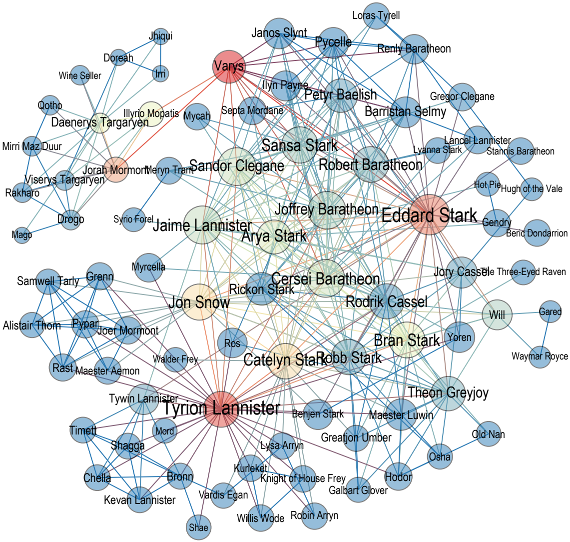
- $\sigma(v,w)$ is the number of shortest paths from v to w
- $\sigma_u(v,w)$ is the number of such shortest paths that run through u
- Divide by largest value to normalize to $[0;1]$
- Global path-based measure
- $O(2mn)$ time to compute (two "BFSes" for each node)

U. Brandes, "A faster algorithm for betweenness centrality", Journal of Mathematical Sociology 25(2): 163–177, 2001

Betweenness centrality



Degree vs. betweenness centrality



Centrality measures compared

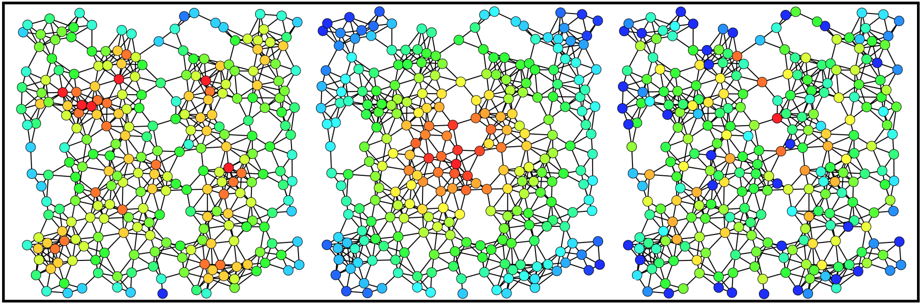


Figure: Degree, closeness and betweenness centrality

Source: "Centrality"' by Claudio Rocchini, Wikipedia `File:Centrality.svg`

Eccentricity centrality

- Node **eccentricity**: length of a longest shortest path (distance to a node furthest away)

$$e(v) = \max_{w \in V} d(v, w)$$

- **Eccentricity centrality**:

$$C_e(v) = \frac{1}{e(v)}$$

- Worst-case variant of closeness centrality
- $O(mn)$ to compute: one BFS in $O(m)$ for each of the n nodes

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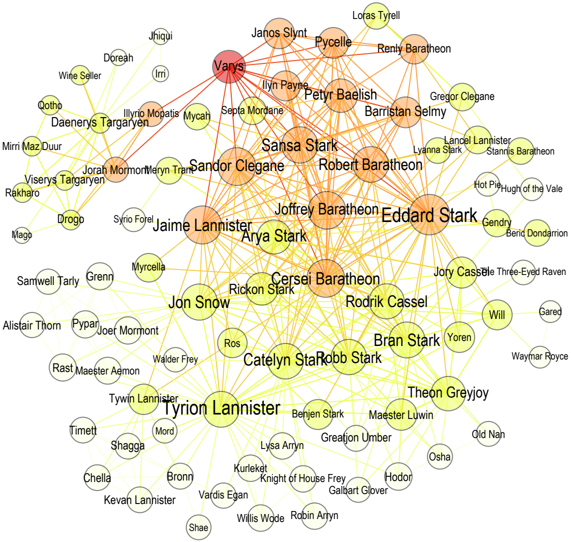
- Worst-case variant of closeness centrality
- $O(mn)$ to compute: one BFS in $O(m)$ for each of the n nodes
  - Large optimizations possible using lower and upper bounds, see F.W. Takes and W.A. Kosters, Computing the Eccentricity Distribution of Large Graphs, *Algorithms*, vol. 6, nr. 1, pp. 100-118, 2013.

Eccentricity centrality

Degree vs. eccentricity centrality



Centrality measures

- Distance/path-based measures:
  - Degree centrality $O(n)$
  - Closeness centrality $O(mn)$
  - Betweenness centrality $O(mn)$
  - Eccentricity centrality $O(mn)$

  (complexity is for computing centralities of all n nodes)
- Many more: Eigenvector centrality, Katz centrality, ...
- Approximating these measures is also possible (see, e.g., *Network Science Seminar*)
- Also: propagation-based centrality measures like PageRank (coming in a next lecture)

Periodic table of centrality

Periodic Table of Network Centrality

	1 IA	2 IIA	3 IIIA	4 IVB	5 VB	6 VIB	7 VIIB	8 VIIIB	9 VIIIB	10 VIIIB	11 IB	12 IIB	13 IIIA	14 IVA	15 VA	16 VIA	17 VIIA	18 VIIIA
1	8000 1979 **DC** Degree																	518 1989 **IC** Information C
2	224 1971 **BC** Betweenness	239 2008 **EBC** Endpoint BC											26 1989 **kPC** kPath C.	275 2002 **EGO** Ego	51 2004 **HYPER** Hypergraphs	279 1997 **AFF** Affiliation C.	399 2 001 **α-C** α-Cent.	178 1995 **ECC** Eccentricity
3	942 1966 **CC** Closeness	239 2008 **PBC** Proxy BC											9068 1999 **HITS** Hubs/Authority	573 2006 **g-kPC** geodesic kPath	296 1999 **GROUP** Groups/Classes	80 2006 **HYPSC** Hyperg. SC	34 2010 **t-SC** t-Subgraph	116 1998 **RAD** Radiality
4	1279 1972 **EC** Eigenvector	239 2008 **LSBC** LscaledBC	224 1971 **EBC** Edge BC	53 2009 **CBC** Commun. BC	236 2007 **ΔC** Delta Cent.	5 2010 **MDC** MD Cent.	0 2015 **EYC** Entropy C.	2 2013 **CAC** Comm. Ability	56 2007 **EPTC** Entropy PC	281 1971 **CCoef** Clust. Coef.	42 2012 **PeC** PeC	427 2007 **BN** Bottleneck	43 2009 **EI** Essentiality I.	573 2006 **e-kPC** e-disjoint kPC	573 2006 **v-kPC** v-disjoint kPC	505 2010 **WEIGHT** Weighted C.	17 2013 **TCom** Total Comm.	116 1998 **INT** Integration
5	1306 1953 **KS** Katz Status	239 2008 **DBBC** DBounded BC	979 2005 **RWBC** RWalk BC	477 1991 **TEC** Total Effects	42 2009 **LI** Lobby Index	11 2008 **MC** Mod Cent.	0 2014 **COMCC** Community C.	45 2012 **ECCoef** ECCoef	0 2015 **SMD** Super Mediat.	1 2014 **UCC** United Comp.	4 2012 **WDC** WDC	119 2008 **MNC** MNC	43 2009 **KL** Clique Level	179 2005 **BIP** Bipartivity	426 1988 **GPI** GPI Power	116 1991 **kRPC** Reachability	58 2007 **SCodd** odd Subgraph	586 2004 **RWCC** RWalk CC
6	8053 1999 **PR** Page Rank	239 2008 **DSBC** DScaled BC	291 1953 **σ** Stress	477 1991 **IEC** Immediate Eff.	1 2014 **DM** Degree Mass	10 2012 **LAPC** Laplacian C.	0 2012 **ABC** Attentive BC	1699 2001 **STRC** Straightness C	0 2015 **SNR** Silent Node R.	15 2011 **HPC** Harm. Prot.	26 2011 **LAC** Local Average	119 2008 **DMNC** DMNC	3 2013 **LR** Lurker Rank	2457 1987 **β-C** β Cent.	X X **HYP** Hyperbolic C.	27 2012 **kEPC** k-edge PC	13 2007 **FC** Functional C.	0 2014 **HCC** Hierar. CC
7	484 2005 **SC** Subgraph	613 1991 **FBC** Flow BC	14 2012 **RLBC** RLimited BC	477 1991 **MEC** Mediative Eff.	69 2010 **LEVC** Leverage Cent.	35 2010 **TC** Topological C.	X X **SDC** Sphere Degree	15 2012 **ZC** Zonal Cent.	14 2013 **CI** Collab. Index	11 2013 **CoEWC** CoEWC	45 2012 **NC** NC	108 2010 **MLC** Moduland C.	X X **RSC** Resolvent SC	1 2014 **SWIPD** SWIPD	36 2009 **XXXX** LinComb	0 2014 **BCPR** BCPR	0 2014 **TPC** Tunable PC	0 2015 **EDCC** Effective Dist.

citations	year
C	
Name	

8000 1979 **Freeman** Conceptual	942 1966 **Sabidussi** Axiomatic	573 2006 **Borgatti/Everett** Conceptual	1130 2005 **Borgatti** Conceptual	24 2014 **Boldi/Vigna** Axiomatic	252 1974 **Nieminen** Axiomatic	6 1981 **Kishi** Axiomatic	3 2012 **Kitti** Axiomatic	3 2009 **Garg** Axiomatic
2065 1934 **Moreno** Historic	1546 1950 **Bavelas** Historic	780 1948 **Bavelas** Historic	1475 1951 **Leavitt** Historic	297 1992 **Borgatti/Everett** Conceptual	3649 2001 **Jeong et al.** Empirical	4167 1998 **Tsai/Ghoshal** Empirical	961 1993 **Ibarra** Empirical	71 2008 **Valente** Empirical

- "Traditional"
- Betweenness-like
- Friedkin Measures
- Miscellaneous
- Path-based
- Specific Network Type
- Spectral-based
- Closeness-like

Upcoming lab session & "Homework" for next week

- Lab session: Introduction to NetworkX, then work on Assignment 1
- Ask your questions
- "Homework": make serious progress with Assignment 1
- Make choice of participation in course explicit. Un-enroll if you wish to drop the course

Credits

- Several images are gratefully reused from
  Michele Coscia, The Atlas for the Aspiring Network Scientist, v3 of
  `https://arxiv.org/abs/2101.00863`