



# Social Network Analysis for Computer Scientists

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## Lecture 5 — Network dynamics and evolution

## Assignment feedback

- Please study and utilize detailed assignment feedback
- Grade  $< 5.0$ : insufficient; compensate with extra assignment
- $5.0 \leq \text{Grade} < 5.5$ : insufficient, unless compensated with Assignment 2 to average of two assignments  $\geq 5.5$
- Grade  $\geq 5.5$ : sufficient
- Questions? Ask your grader during the upcoming lab session (name or initials present on your graded work)

## Assignment feedback

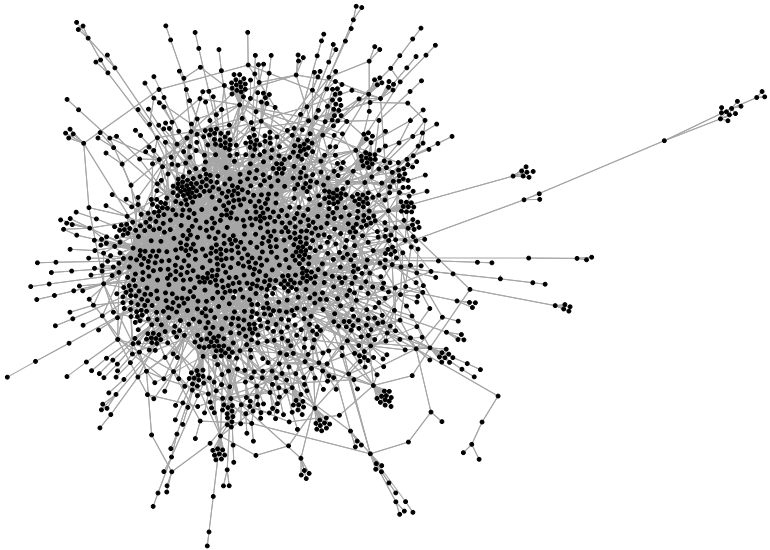
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- Please stay away from plagiarism.

# Today

- Recap
- Temporal networks
- Network models
- Network dynamics and evolution
- Data science lab
- Structure of the web
- Network science challenges

# Recap

# Networks



# Notation

## Concept

- Network (graph)
- Nodes (objects, vertices, ...)
- Links (ties, relationships, ...)
  - Directed —  $E \subseteq V \times V$  — "links"
  - Undirected — "edges"
- Number of nodes —  $|V|$
- Number of edges —  $|E|$
- Degree of node  $u$
- Distance from node  $u$  to  $v$

## Symbol

$G = (V, E)$

$V$

$E$

$n$

$m$

$\deg(u)$

$d(u, v)$

# Real-world networks

- |   |  |                        |
|---|--|------------------------|
| 1 | Sparse networks                          | density                |
| 2 | Fat-tailed power-law degree distribution | degree                 |
| 3 | Giant component                          | components             |
| 4 | Low pairwise node-to-node distances      | distance               |
| 5 | Many triangles                           | clustering coefficient |

# Real-world networks

- 1 Sparse networks density
- 2 Fat-tailed power-law degree distribution degree
- 3 Giant component components
- 4 Low pairwise node-to-node distances distance
- 5 Many triangles clustering coefficient
- Many examples: communication networks, citation networks, collaboration networks (Erdős, Kevin Bacon), protein interaction networks, information networks (Wikipedia), webgraphs, financial networks (Bitcoin) ...

## Advanced concepts

- Assortativity, homophily
- Reciprocity
- Power law exponent
- Planar graphs
- Complete graphs
- Subgraphs
- Trees
- Spanning trees
- Diameter, eccentricity
- Bridges
- Graph traversal: DFS, BFS

# Centrality measures

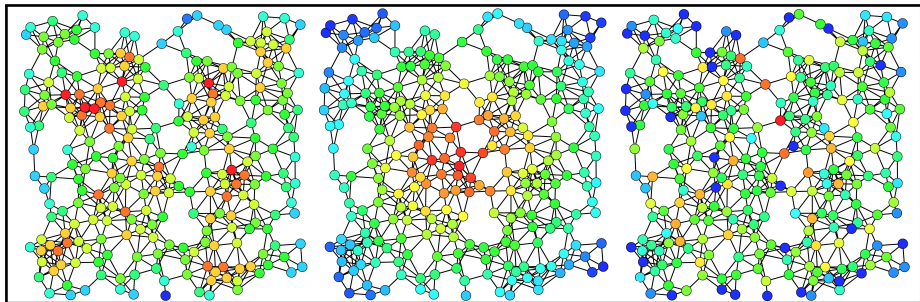
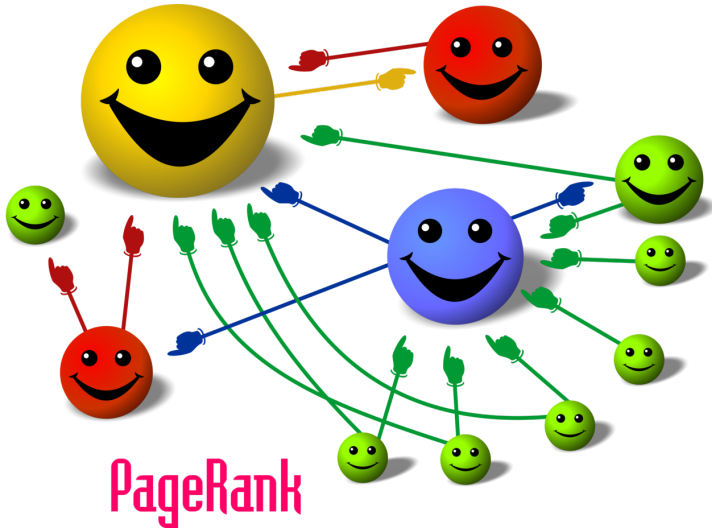


Figure: Degree, closeness and betweenness centrality

Source: "Centrality" by Claudio Rocchini, Wikipedia File:Centrality.svg

# Centrality measures: PageRank



# Centrality measures

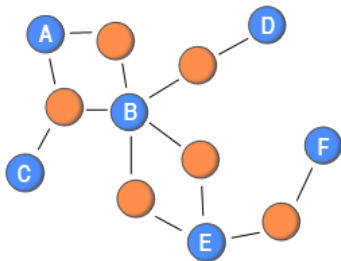
## ■ Distance/path-based measures:

- Degree centrality  $O(n)$
- Closeness centrality  $O(mn)$
- Betweenness centrality  $O(mn)$
- Eccentricity centrality  $O(mn)$

## ■ **Propagation-based** measures:

- Hyperlink Induced Topic Search (HITS)  $O(m)$
- PageRank  $O(m)$

# Network projection



# Network projection

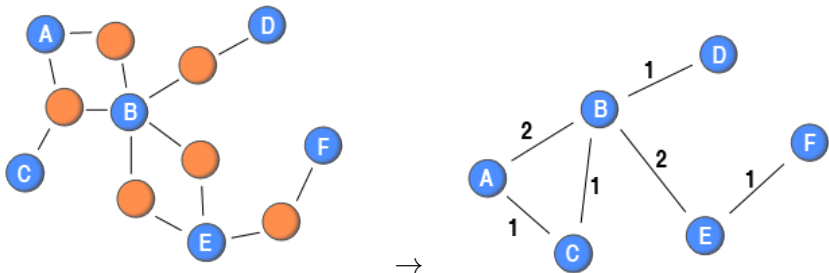
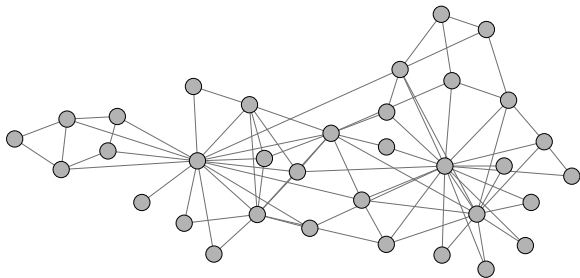


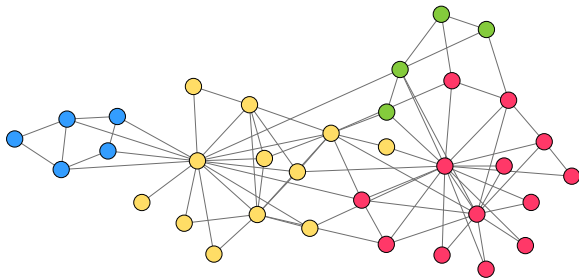
Image: <http://toreopsahl.com>

# Community detection



**Figure:** Communities: node subsets connected more strongly with each other

# Community detection



**Figure:** Communities: node subsets connected more strongly with each other

# Temporal networks

# Temporal network analysis

- Graphs **evolve** over time
  - Social networks: users join the network and create new friendships
  - Webgraphs: new pages and links to pages appear on the internet
  - Scientific networks: new papers are being co-authored and new citations are made in these papers
- Interesting: small world properties emerge and are preserved during evolution!

# Temporal networks

- Graph  $G^t = (V^t, E^t)$
- Time window  $0 \leq t \leq T$
- Usually at  $t = 0$ , either
  - $V^0 = \emptyset$  and a new edge may bring new nodes, or
  - $V^0 = V^T$  and only edges are added at each timestamp
- Timestamp on node  $v \in V$ :  
 $\tau(v) \in [0; T]$
- Timestamp on edge  $e \in E$ :  
 $\tau(e) \in [0; T]$ , or as common input format:  
 $e = (u, v, t)$  with  $u, v \in V$  and  $t \in [0, T]$   
u v t as line contents of an edge list file

# Two schools

## ■ **Synthetic graphs**

model-driven

- Model or algorithm to generate graphs from scratch
- Tune parameters to obtain a graph similar to an observed network
- Statistical analysis

## ■ **Real-world graphs**

data-driven

- Obtain data from an actual network
- Compute and derive properties and determine similarity with other networks
- Computational analysis

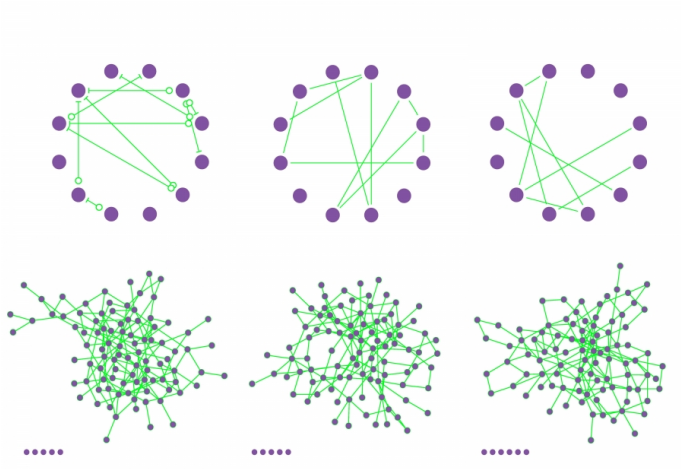
# Three models

- Random graphs (Erdős-Rényi)
- Barabási-Albert model
- Watts-Strogatz model

## Random graphs (1959)

- Initially,  $n$  nodes and 0 edges
- Add edges at random
- **Edgar Gilbert / Erdős-Rényi**: a random graph  $G(n, p)$  has  $n$  nodes and each undirected edge exists with probability  $0 < p < 1$ . Expected  $m = p \cdot \frac{1}{2}n(n-1)$  edges
- **Erdős-Rényi**: a random graph  $G(n, m)$  has  $n$  nodes and  $m$  edges, and this graph is chosen uniformly random from all possible graphs with  $n$  nodes and  $m$  edges
- Result does not really resemble real-world graphs

# Erdős-Rényi



<http://barabasi.com/networksciencebook/chapter/3>

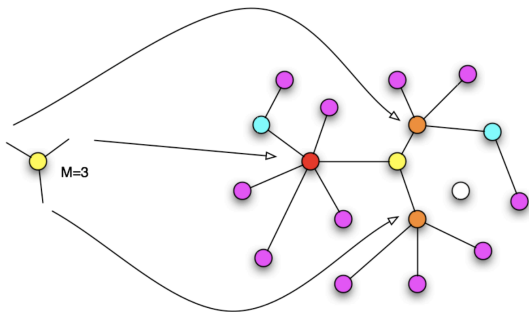
## Barabási-Albert model (1999)

- “Rich get richer”
- **Preferential attachment:** nodes with a high degree more strongly attract new links
- An edge  $(u, v)$  is added between a new node  $u$  and a non-random node  $v$  with probability:

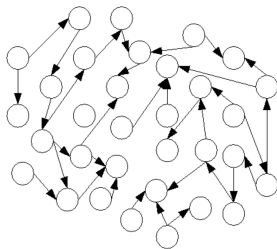
$$p(v) = \frac{\deg(v)}{\sum_{w \in V} \deg(w)}$$

- (Plus some dampening based on the age of the node and correction for links between high-degree nodes)
- Result: giant component and power-law degree distribution: the **scale-free** property

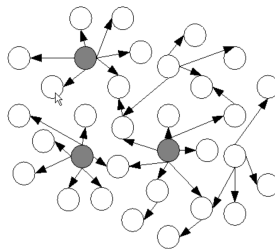
# Barabási-Albert model (1999)



## Random vs. scale-free



(a) Random network



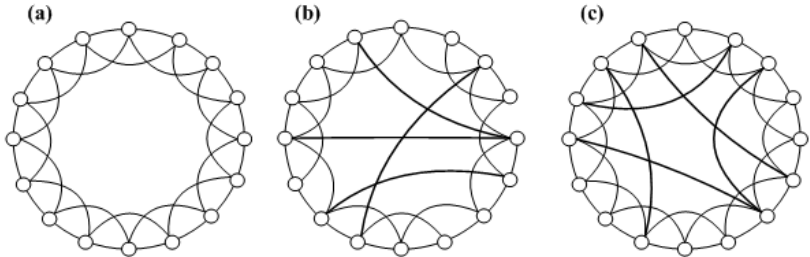
(b) Scale-free network

B. Svenson, Complex networks and social network analysis in information fusion

## Watts-Strogatz model (1998)

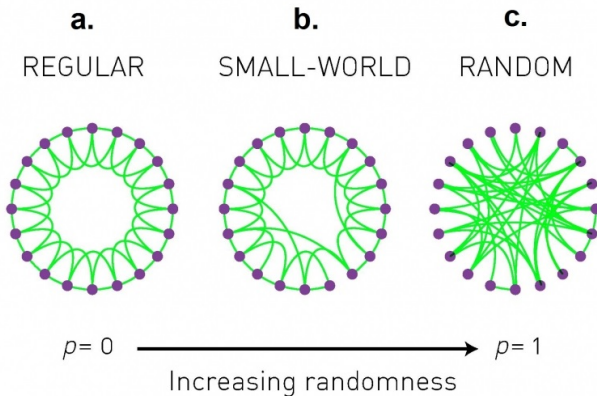
- Input number of nodes  $n$ , average degree  $k$  and parameter  $p$
- Constructs undirected graph with  $n$  nodes and  $\frac{1}{2} \cdot n \cdot k$  edges
- Start with “circle-shaped” graph connecting each node to its  $k$  nearest neighbors
- Until each edge has been considered, in clock-wise order, **Rewire** each node's edge to a closest neighbor, to a random node with probability  $p$
- Result: low distances, giant component, high clustering

# Watts-Strogatz



Watts, D. J., & Strogatz, S. H. (1998). Collective dynamics of 'small-world' networks. *Nature* 393(6684), 440-442.

## Discussion of models



<http://www.cis.upenn.edu/~mkearns/teaching/NetworkedLife/bgc-sci.jpg>

## Discussion of models

- Many generative more models exist: configuration model, stub-matching model, ...
- ERGM, SAOM, REM, stochastic block models, ...

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- Many generative models exist: configuration model, stub-matching model, ...
- ERGM, SAOM, REM, stochastic block models, ...
- Better understanding of system's evolution
- Compare real-world structure with model structure
- Investigate system's complexity

## Discussion of models

- Many generative models exist: configuration model, stub-matching model, ...
- ERGM, SAOM, REM, stochastic block models, ...
- Better understanding of system's evolution
- Compare real-world structure with model structure
- Investigate system's complexity
- Model is never perfect
- Not all small-world properties are captured

# Network evolution

# Levels of evolution

- **Microscopic (local)**
- Macroscopic (global)

# Microscopic evolution

- Node-based investigation of evolution
- Analysis of four online social networks: DELICIOUS, FLICKR, LINKEDIN and YAHOO! ANSWERS
- Other than degree, preferential attachment (assortativity) can also be based on node **age** and the number of **hops** (distance before link is created)
- Derive model based on these properties

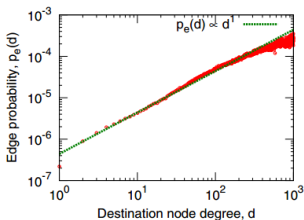
Leskovec et al., Microscopic Evolution of Social Networks, in Proceedings of KDD, pp. 462-470, 2008.

# Datasets

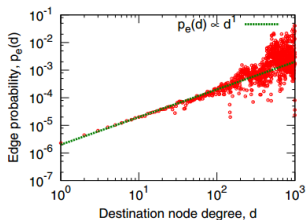
Network	$T$	$N$	$E$	$E_b$	$E_u$	$E_\Delta$	%	$\rho$	$\kappa$
FLICKR (03/2003–09/2005)	621	584,207	3,554,130	2,594,078	2,257,211	1,475,345	65.63	1.32	1.44
DELICIOUS (05/2006–02/2007)	292	203,234	430,707	348,437	348,437	96,387	27.66	1.15	0.81
ANSWERS (03/2007–06/2007)	121	598,314	1,834,217	1,067,021	1,300,698	303,858	23.36	1.25	0.92
LINKEDIN (05/2003–10/2006)	1294	7,550,955	30,682,028	30,682,028	30,682,028	15,201,596	49.55	1.14	1.04

**Table 1: Network dataset statistics.**  $E_b$  is the number of bidirectional edges,  $E_u$  is the number of edges in undirected network,  $E_\Delta$  is the number of edges that close triangles, % is the fraction of triangle-closing edges,  $\rho$  is the densification exponent ( $E(t) \propto N(t)^\rho$ ), and  $\kappa$  is the decay exponent ( $E_h \propto \exp(-\kappa h)$ ) of the number of edges  $E_h$  closing  $h$  hop paths

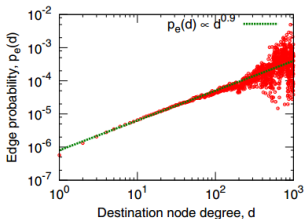
# Preferential attachment: degree



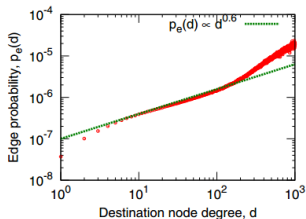
(c) FLICKR



(d) DELICIOUS

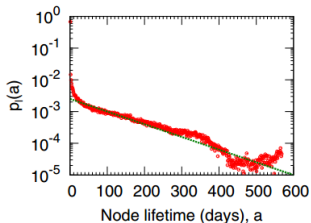


(e) ANSWERS

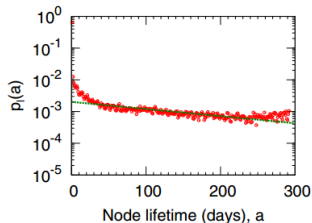


(f) LINKEDIN

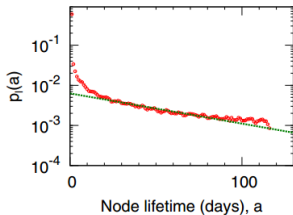
## Preferential attachment: age



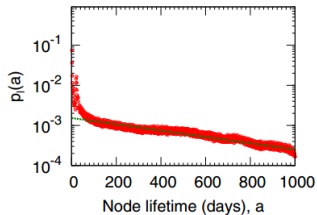
(a) FLICKR



(b) DELICIOUS

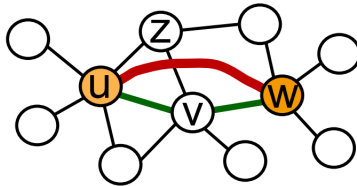


(c) ANSWERS

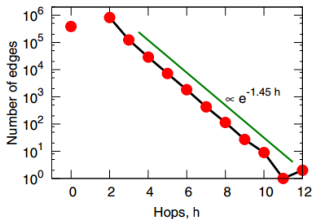


(d) LINKEDIN

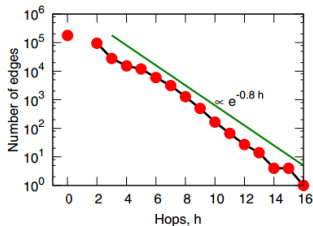
## Triadic closure



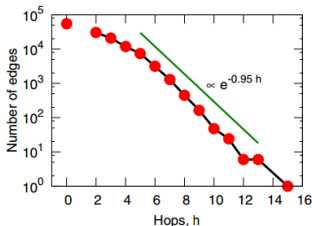
# Preferential attachment: hops



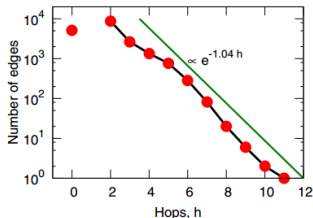
(c) FLICKR



(d) DELICIOUS



(e) ANSWERS



(f) LINKEDIN

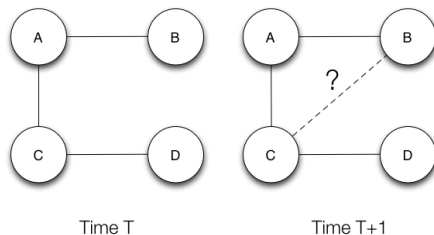
# Microscopic evolution model

- Node arrival and lifetime determined using function (based on derived exponential distribution)
- Node goes to sleep for a time gap, length again sampled from a derived distribution
- Node wakes up to create an edge using (adjusted) triangle closing model and goes to sleep
- Sleep time gets shorter as the degree of a node increases
- Node dies after lifetime is reached

Leskovec et al., Microscopic Evolution of Social Networks, in Proceedings of KDD, pp. 462-470, 2008.

# Link prediction

- Predict “next friendship” to be formed



Liben-Nowell et al., The Link Prediction Problem for Social Networks, in Proceedings of CIKM, pp. 556-559, 2003.

# Levels of evolution

- Microscopic (local)
- **Macroscopic (global)**

# Macroscopic evolution

- Look at evolution of network as a whole
- Observe different characteristic graph properties
- Devise model that incorporates these properties

Dataset	Nodes	Edges	Time	DPL exponent
Arxiv HEP-PH	30,501	347,268	124 months	1.56
Arxiv HEP-TH	29,555	352,807	124 months	1.68
Patents	3,923,922	16,522,438	37 years	1.66
AS	6,474	26,467	785 days	1.18
Affiliation ASTRO-PH	57,381	133,179	10 years	1.15
Affiliation COND-MAT	62,085	108,182	10 years	1.10
Affiliation GR-QC	19,309	26,169	10 years	1.08
Affiliation HEP-PH	51,037	89,163	10 years	1.08
Affiliation HEP-TH	45,280	68,695	10 years	1.08
Email	35,756	123,254	18 months	1.12
IMDB	1,230,276	3,790,667	114 years	1.11
Recommendations	3,943,084	15,656,121	710 days	1.26

Leskovec et al., Graph Evolution: Densification and Shrinking Diameters, in TKDD 1(1): 2, 2007

# Enron

Mid 1980s: Enron business entirely in the USA, focused on gas pipelines and power



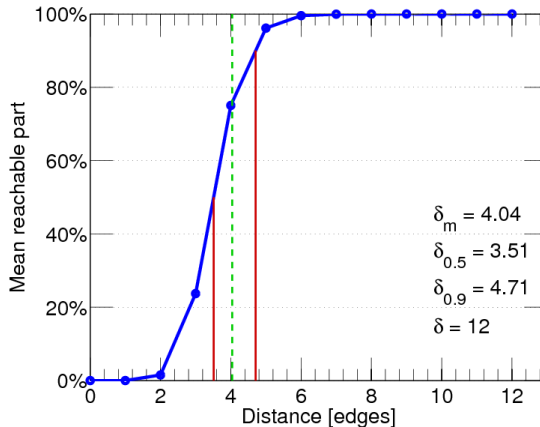
2001: Enron trading in hundreds of commodities  
Interests in: USA, South America, Europe, Asia and Australia



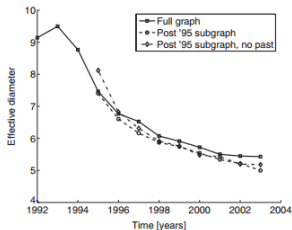
# Macroscopic patterns

- **Densification:** density increases over time
- **Giant component** grows asymptotically
- **Shrinking average distance:**  $d \sim \log(n)$  does not hold over time
- **Shrinking effective diameter**
  - **Effective diameter**  $\delta_{0.9}$ : path length such that 90% of all node pairs are at distance  $\delta_{0.9}$  or less
  - **Diameter:** longest shortest path length

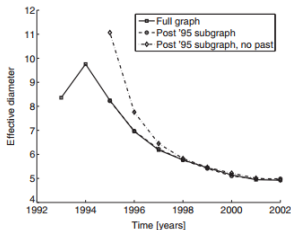
# Effective diameter



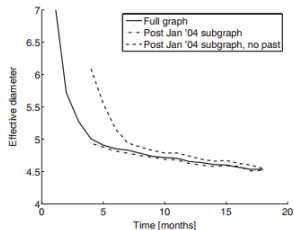
# Effective diameter



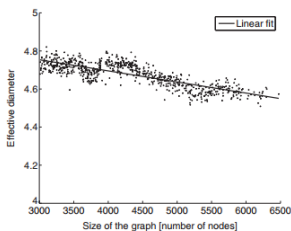
(a) arXiv citation graph



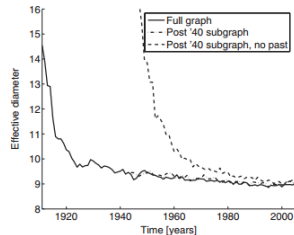
(b) Affiliation network



(c) Patents citation graph

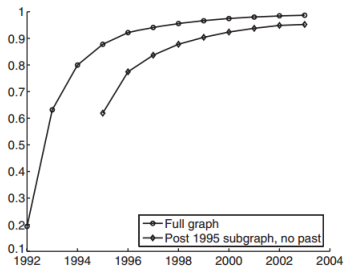


(d) Autonomous Systems

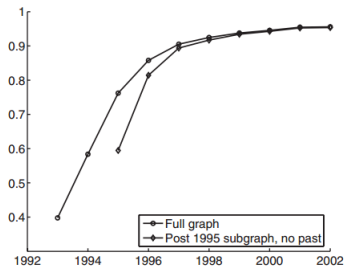


(f) IMDB actors to movies network

# Giant component

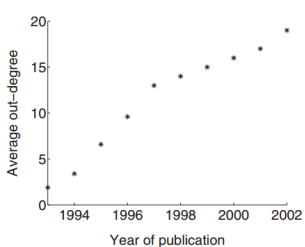


(a) arXiv citation graph

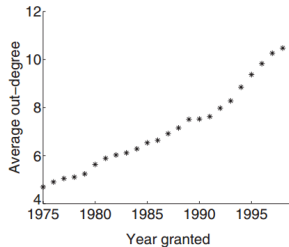


(b) Affiliation network

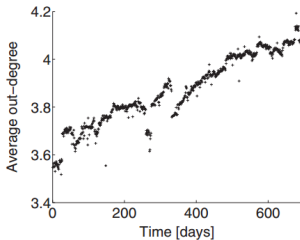
# Densification



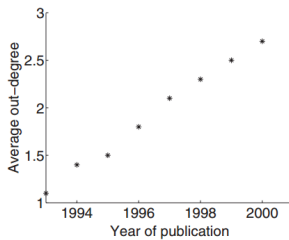
(a) arXiv



(b) Patents



(c) Autonomous Systems



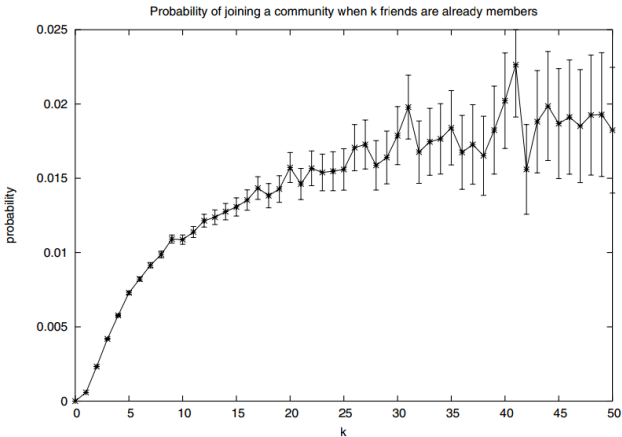
(d) Affiliation network

# Community evolution

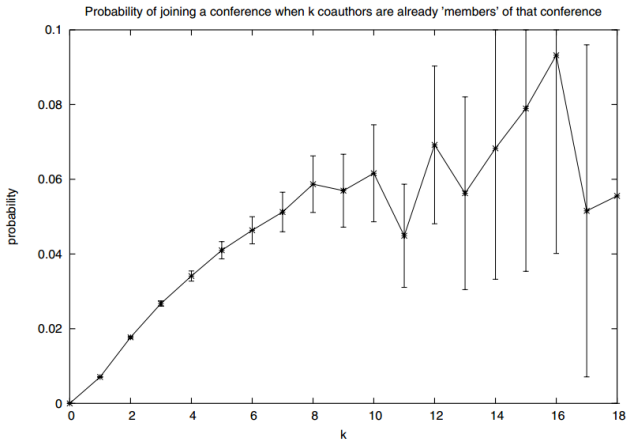
- Slightly different: user-defined communities
- DBLP: scientific collaboration network where communities are conferences that authors visit
- LIVEJOURNAL: online social network with explicit groups based on common interest
- What motivates nodes to join a community?
- What causes nodes to switch between communities?
- When do communities grow?

Backstrom et al., "Group formation in large social networks: membership, growth, and evolution",  
in Proceedings of KDD, pp. 44–54, 2006.

# Community evolution (LIVEJOURNAL)



# Community evolution (DBLP)

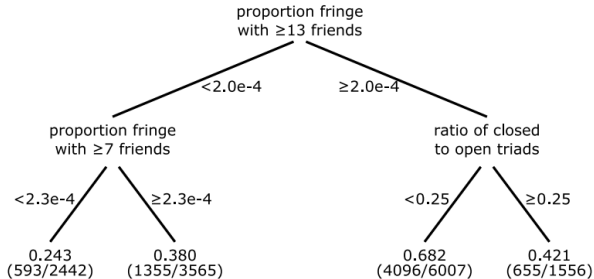


# Features

**Table 1: Features.**

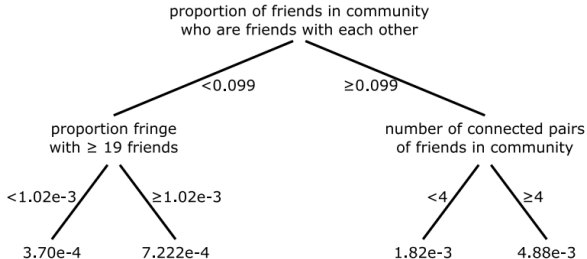
Feature Set	Feature
Features related to the community, $C$ . (Edges between only members of the community are $E_C \subseteq E$ .)	<p>Number of members (<math> C </math>).</p> <p>Number of individuals with a friend in <math>C</math> (the <i>fringe</i> of <math>C</math>).</p> <p>Number of edges with one end in the community and the other in the fringe.</p> <p>Number of edges with both ends in the community, <math> E_C </math>.</p> <p>The number of open triads: <math> \{(u, v, w)   (u, v) \in E_C \wedge (v, w) \in E_C \wedge (u, w) \notin E_C \wedge u \neq w\} </math>.</p> <p>The number of closed triads: <math> \{(u, v, w)   (u, v) \in E_C \wedge (v, w) \in E_C \wedge (u, w) \in E_C\} </math>.</p> <p>The ratio of closed to open triads.</p> <p>The fraction of individuals in the fringe with at least <math>k</math> friends in the community for <math>2 \leq k \leq 19</math>.</p> <p>The number of posts and responses made by members of the community.</p> <p>The number of members of the community with at least one post or response.</p> <p>The number of responses per post.</p>
Features related to an individual $u$ and her set $S$ of friends in community $C$ .	<p>Number of friends in community (<math> S </math>).</p> <p>Number of adjacent pairs in <math>S</math> (<math> \{(u, v)   u, v \in S \wedge (u, v) \in E_C\} </math>).</p> <p>Number of pairs in <math>S</math> connected via a path in <math>E_C</math>.</p> <p>Average distance between friends connected via a path in <math>E_C</math>.</p> <p>Number of community members reachable from <math>S</math> using edges in <math>E_C</math>.</p> <p>Average distance from <math>S</math> to reachable community members using edges in <math>E_C</math>.</p> <p>The number of posts and response made by individuals in <math>S</math>.</p> <p>The number of individuals in <math>S</math> with at least 1 post or response.</p>

# Decision tree (LIVEJOURNAL)



**Figure 5: The top two levels of decision tree splits for predicting community growth in LiveJournal.**

# Decision tree (LIVEJOURNAL)

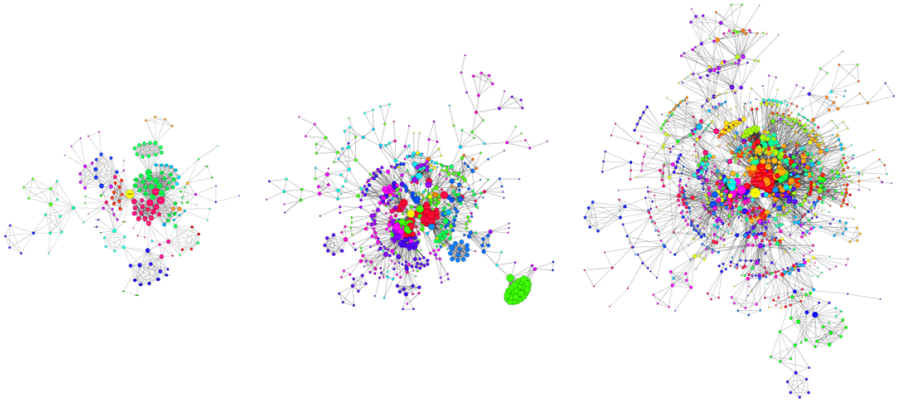


**Figure 3: The top two levels of decision tree splits for predicting single individuals joining communities in LiveJournal. The overall rate of joining is  $8.48e-4$ .**

# Community evolution patterns

- Number of friends already in a community correlates with decision to join a community
- Using various features, decision trees can predict community behavior
- In most models, parameters are specific for considered network
- Challenge: do not flatten data, but use actual network and community structure, perhaps even parameter-free?

# Apple collaboration network



**2007-2008**

**2009-2010**

**2011-2012**

<http://www.kenedict.com/apples-internal-innovation-network-unraveled/>

# Network contraction

- Example: social network losing members to competitor
- Deletion of nodes (and its edges)
- Deletion of edges (and ultimately nodes)
- Merging nodes (a corporate network in which companies merge)
- What happens when you remove a hub?
- How about reversing existing models?

Data Science Lab  
<http://rel.liacs.nl/labs/dslab>

# Structure of the web

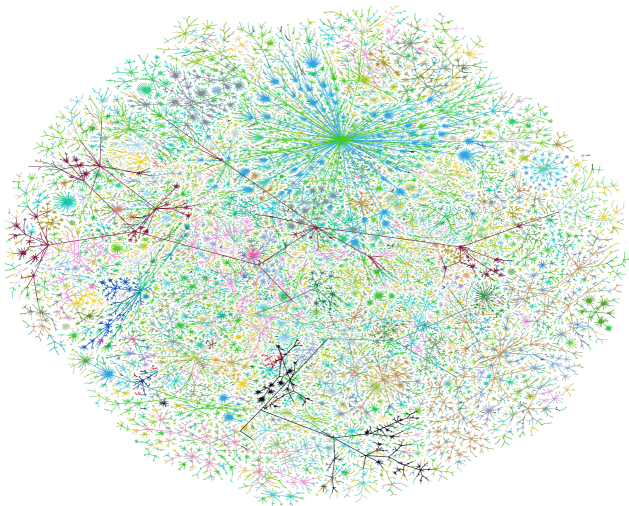
# Webgraph

- **Webgraph:** directed unweighted network  $G = (V, E)$
- Nodes  $V$  are webpages
- Links  $E$  are “hyperlinks” to other pages
- Many dense subgraphs ...

# Webgraph

- **Webgraph**: directed unweighted network  $G = (V, E)$
- Nodes  $V$  are webpages
- Links  $E$  are “hyperlinks” to other pages
- Many dense subgraphs ... because pages (nodes) may belong to the same domain
- Alternative: draw webgraph with only (sub)domains as nodes, referred to as **host graph**
- Idea: search engine ranks webpages using the structure of the webgraph
- Centrality measures

# The web



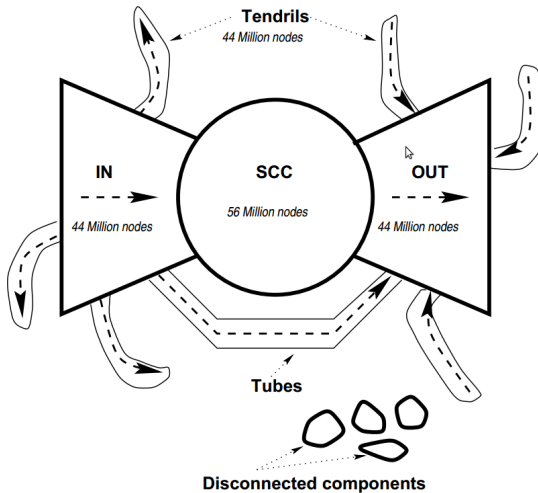
<http://www.cheswick.com/ches/map/gallery/index.html>

# Why study the webgraph?

- Understanding social mechanisms that govern growth
- Designing ranking methods
- Devising better crawling algorithms
- Creating accurate models of the web's structure

Meusel et al., Graph Structure in the Web — Revisited, WWW 2014: 427–431, 2014.

# Webgraph in 1999



Broder et al., Graph structure in the web, Computer Networks 33(1): 309–320, 2000.

# Webgraph in 1999

- Altavista: 200 million nodes
- 186 million nodes in the weakly connected component (90% of the links)
- 56 million nodes in the strongly connected component
- Power law degree distribution
- Average distance of 16 (if there is a path, 25% of the cases)
- Average (undirected) distance of 6.83 (small world!)
- Diameter is 28

Broder et al., Graph structure in the web, Computer Networks 33(1): 309–320, 2000.

# Crawling the webgraph in 2012

- Crawled by Common Crawl Foundation
- First half of 2012
- Breadth-first visiting strategy
- Heuristics to detect spam pages
- Seeded with the list of domains from a previous crawl and a set of URLs from Wikipedia

Meusel et al., Graph Structure in the Web — Revisited, WWW 2014: 427–431, 2014.

# Webgraph in 2012

- Page graph
- Host graph
- PLD graph (Pay-Level Domain (PLD): a subdomain of a public top-level domain, for which users have to pay. PLDs identify a single user or organization)

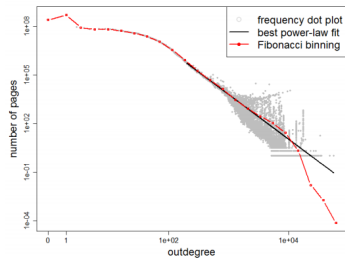
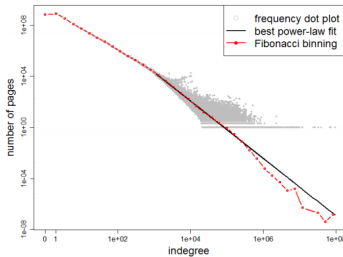
Granularity	# Nodes in millions	# Arcs in millions
Page Graph	3 563	128 736
Host Graph	101	2 043
PLD Graph	43	623

**Table 1: Sizes of the graphs**

Meusel et al., Graph Structure in the Web — Revisited, WWW 2014: 427–431, 2014.

# Degree

- Compared to 1999, average degree increased from 7.5 to 36.8
- Perhaps due to use of content management systems (they tend to create dense websites)



Meusel et al., Graph Structure in the Web — Revisited, WWW 2014: 427–431, 2014.

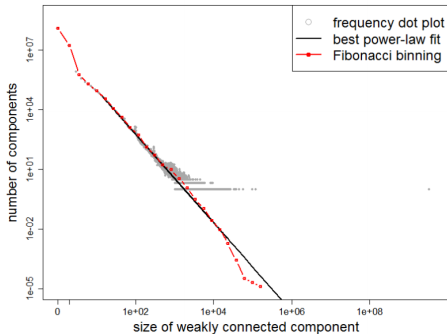
# Centrality in the webgraph

PageRank	Indegree	Harmonic Centrality
gmpg.org	wordpress.org	youtube.com
wordpress.org	youtube.com	en.wikipedia.org
youtube.com	gmpg.org	twitter.com
<b>livejournal.com</b>	en.wikipedia.org	google.com
tumblr.com	tumblr.com	wordpress.org
en.wikipedia.org	twitter.com	flickr.com
twitter.com	google.com	facebook.com
<b>networkadvertising.org</b>	flickr.com	<b>apple.com</b>
<b>promodj.com</b>	<b>rtalabel.org</b>	vimeo.com
<b>skriptmail.de</b>	wordpress.com	creativecommons.org
<b>parallels.com</b>	<b>mp3shake.com</b>	<b>amazon.com</b>
<b>tistory.com</b>	w3schools.com	<b>adobe.com</b>
google.com	domains.lycos.com	<b>myspace.com</b>
miibeian.gov.cn	<b>staff.tumblr.com</b>	<b>w3.org</b>
phpbb.com	<b>club.tripod.com</b>	<b>bbc.co.uk</b>
<b>blog.fc2.com</b>	creativecommons.org	<b>nytimes.com</b>
<b>tw.yahoo.com</b>	vimeo.com	<b>yahoo.com</b>
w3schools.com	miibeian.gov.cn	<b>microsoft.com</b>
wordpress.com	facebook.com	<b>guardian.co.uk</b>
domains.lycos.com	phpbb.com	<b>imdb.com</b>

Meusel et al., Graph Structure in the Web — Revisited, WWW 2014: 427–431, 2014.

# WCC of the webgraph

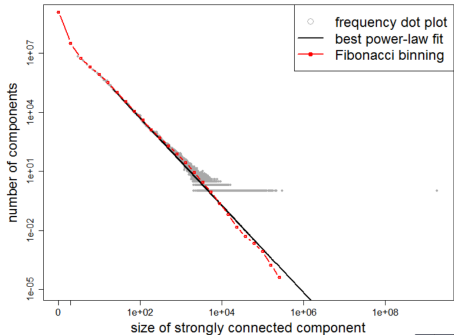
- Weakly Connected Component (WCC)
- 91.8% in 1999, 94% in 2012
- Component size distribution



Meusel et al., Graph Structure in the Web — Revisited, WWW 2014: 427–431, 2014.

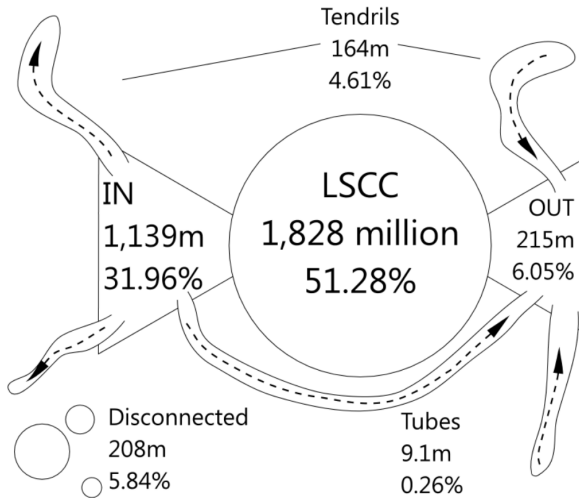
# SCC of the webgraph

- Strongly Connected Component (SCC): 51.3% of the nodes
- Computation required 1TB of RAM
- Graph compression framework WebGraph was used



Meusel et al., Graph Structure in the Web — Revisited, WWW 2014: 427–431, 2014.

## Bow-tie structure in 2012



Meusel et al., Graph Structure in the Web — Revisited, WWW 2014: 427–431, 2014.

## Bow-tie structure in 2012

Component	Common Crawl 2012		Broder <i>et al.</i>	
	# nodes (in thousands)	% nodes (in %)	# nodes (in thousands)	% nodes (in %)
LSCC	1 827 543	51.28	56 464	27.74
IN	1 138 869	31.96	43 343	21.29
OUT	215 409	6.05	43 166	21.21
TENDRILS	164 465	4.61	43 798	21.52
TUBES	9 099	0.26	-	-
DISC.	208 217	5.84	16 778	8.24

**Table 3: Comparison of sizes of bow-tie components**

Meusel et al., Graph Structure in the Web — Revisited, WWW 2014: 427–431, 2014.

# Distances and diameter

- Diameter lower bound: 5,282

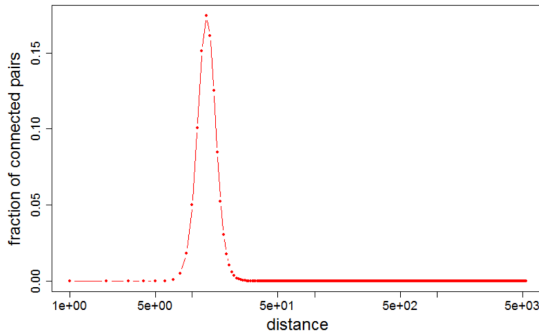


Figure: Distance distribution

Meusel et al., Graph Structure in the Web — Revisited, WWW 2014: 427–431, 2014.

## Webgraph conclusions

- Measurements on the largest webgraph available to the public
- Average degree has significantly increased, almost by a factor of 5
- Connectivity has increased, average distance has decreased
- Structure of the web appears dependent on the specific web crawl
- The distribution of indegrees and outdegrees is extremely different

Meusel et al., Graph Structure in the Web — Revisited, WWW 2014: 427–431, 2014.

## Making a “better” webgraph

- Not just an unweighted unlabeled directed network
- **Resource Description Framework (RDF)**: link is a triple  
[subject] [predicate] [object]
- Link weighting: define a weight for outgoing links (to give hints to PageRank algorithm)
- Link annotation: make more use of the `rel=""` attribute to describe the kind of link: `alternate`, `search`, `next`, etc.
- Requires new algorithms for ranking ...

# Network science challenges

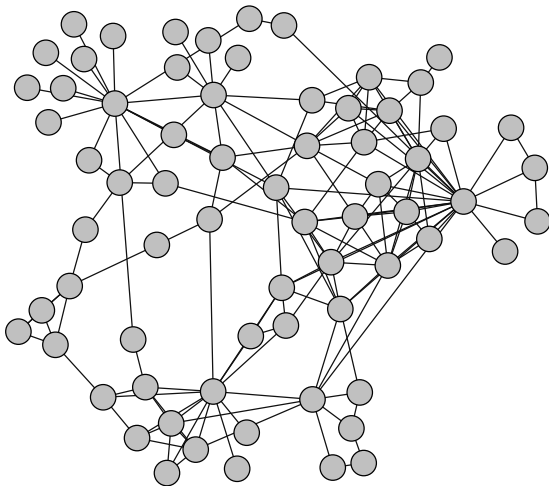
# Network science

- **Network science:** understanding data by investigating interactions and relationships between individual data objects as a network
- **Networks** are the central model of computation

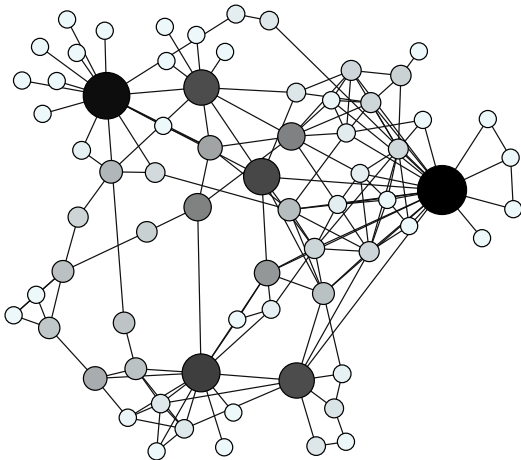
# Network science

- **Network science:** understanding data by investigating interactions and relationships between individual data objects as a network
- **Networks** are the central model of computation
- Branch of data science focusing on network data
- Method in complexity research
- Complex systems approach: the behavior emerging from the network reveals patterns not visible when studying the individuals
- For now assume: network science = social network analysis

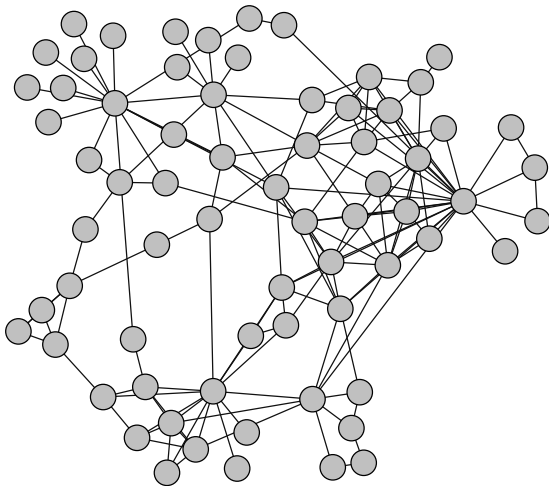
# Network analysis



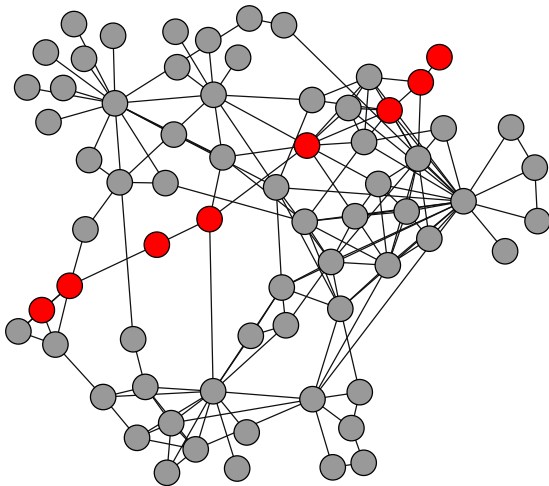
## Micro scale



## Macro scale



## Macro scale



# Network analysis

- **Micro** scale: analyzing the position of individual nodes, based on their structural position in the network (e.g., node centrality, etc.)
- **Macro** scale: analyzing the structure of the network as a whole (e.g., network diameter, small-world effect, etc.)

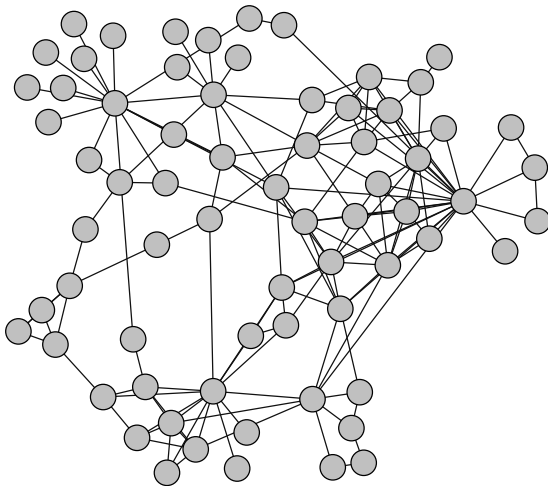
# Network analysis

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- **Meso** scale: analyzing groups of nodes occurring in a particular configuration

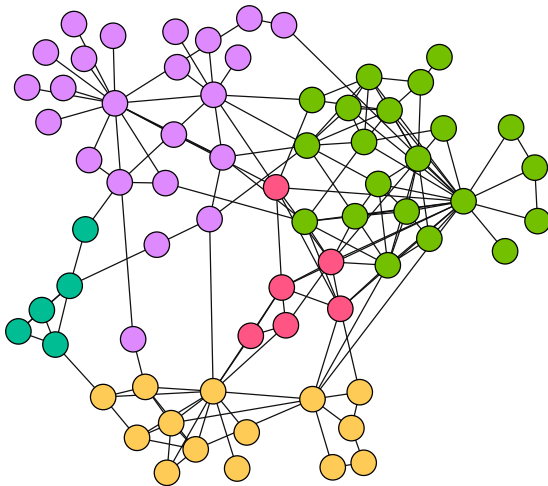
# Network analysis

- **Micro** scale: analyzing the position of individual nodes, based on their structural position in the network (e.g., node centrality, etc.)
- **Macro** scale: analyzing the structure of the network as a whole (e.g., network diameter, small-world effect, etc.)
- **Meso** scale: analyzing groups of nodes occurring in a particular configuration (e.g., communities or networks motifs)

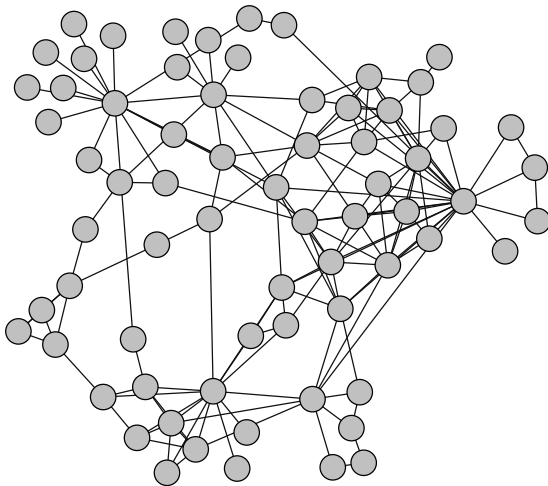
## Meso scale: communities



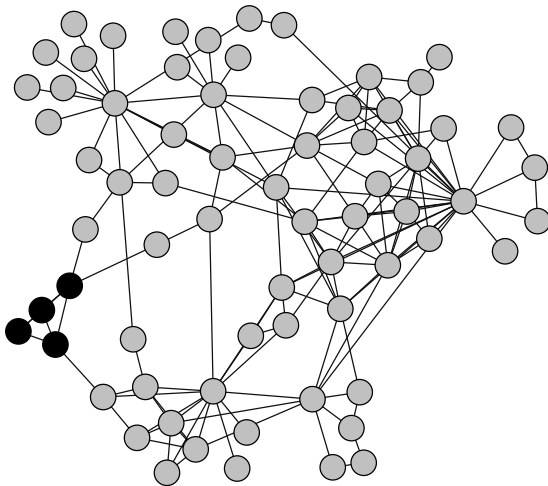
## Meso scale: communities



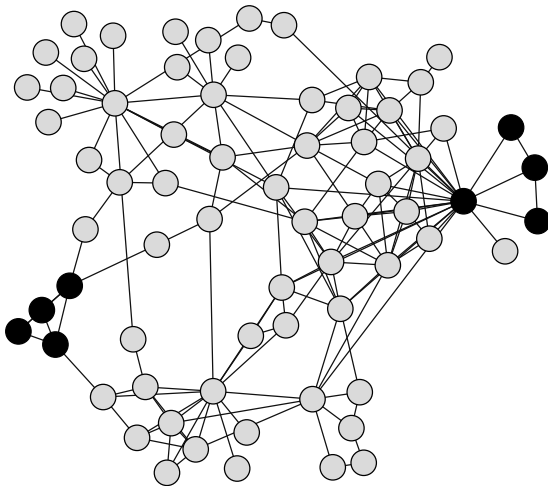
## Meso scale: motifs



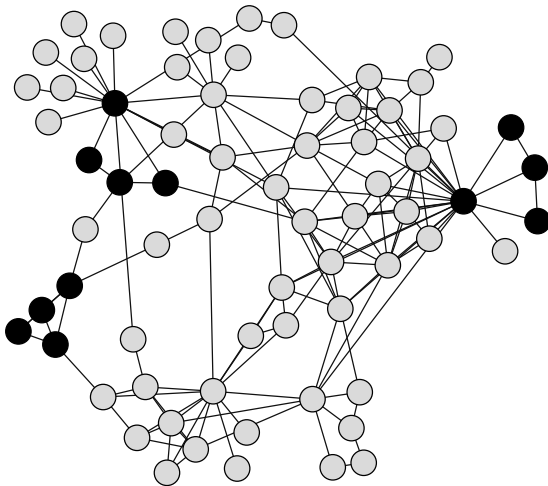
## Meso scale: motifs

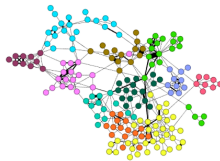


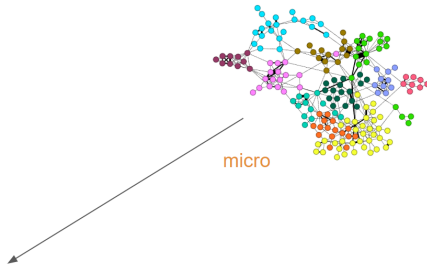
## Meso scale: motifs

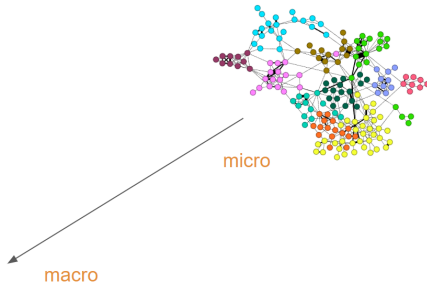


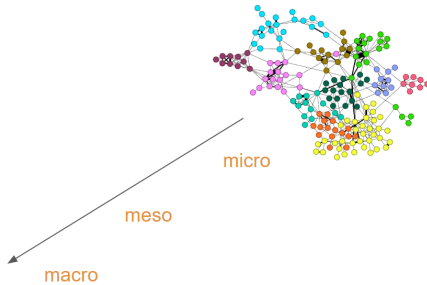
## Meso scale: motifs

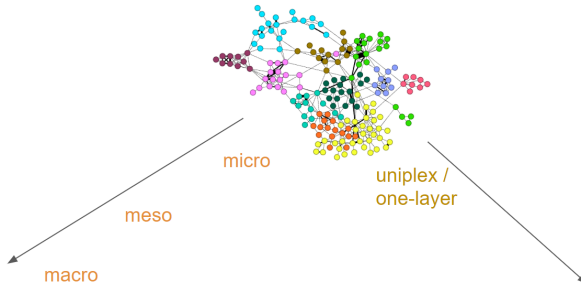


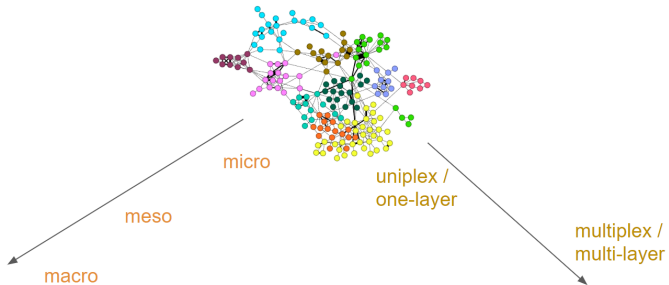


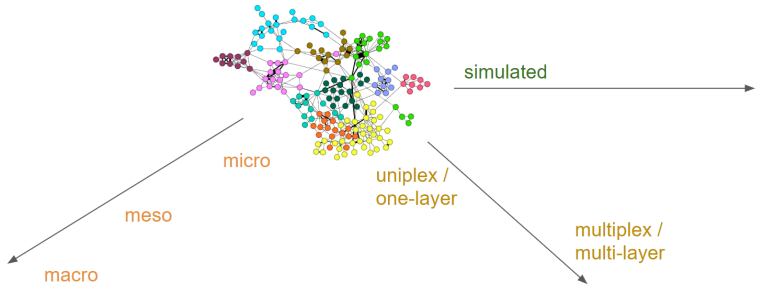


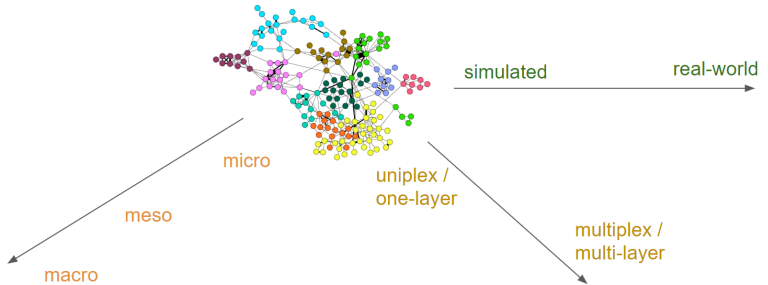


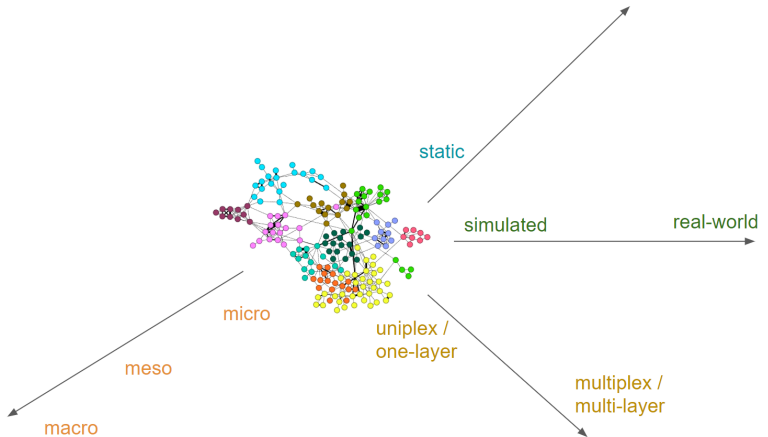


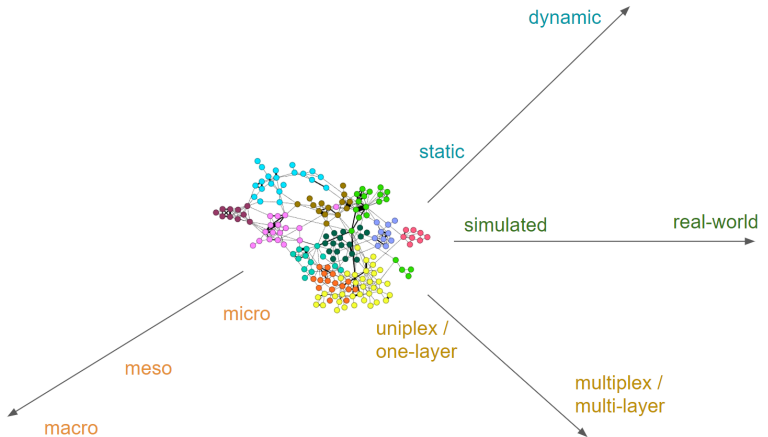


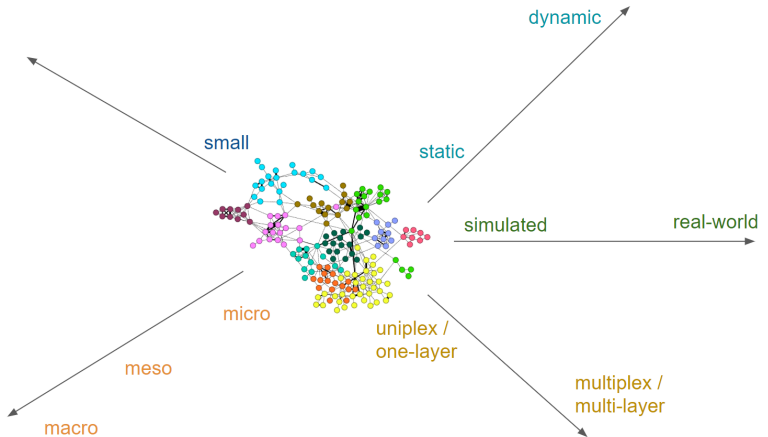


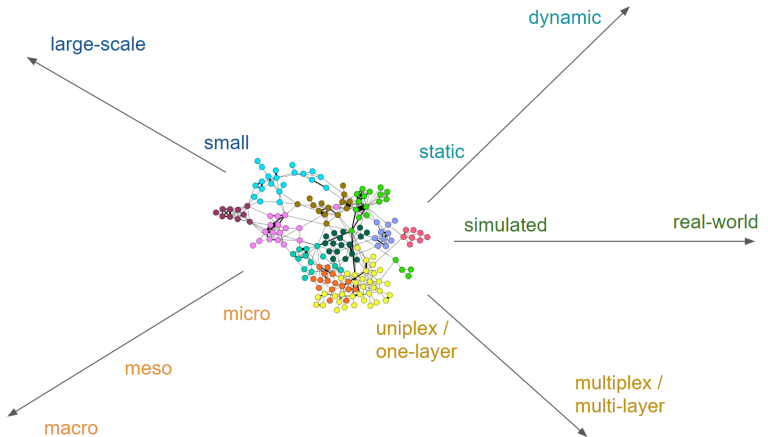


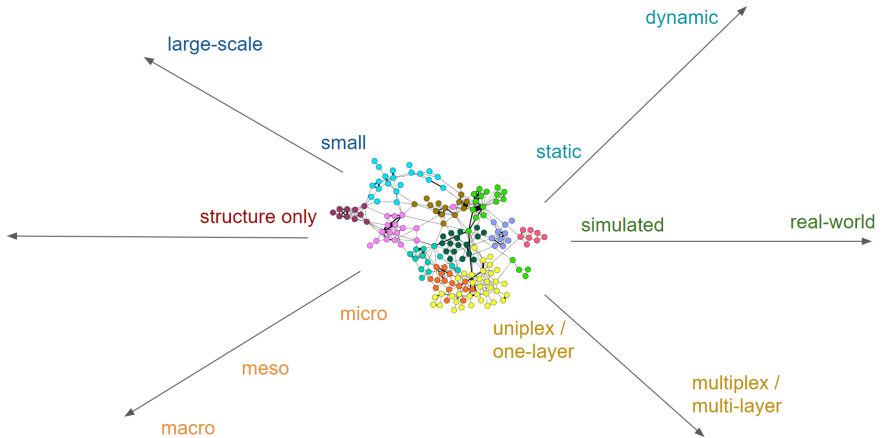


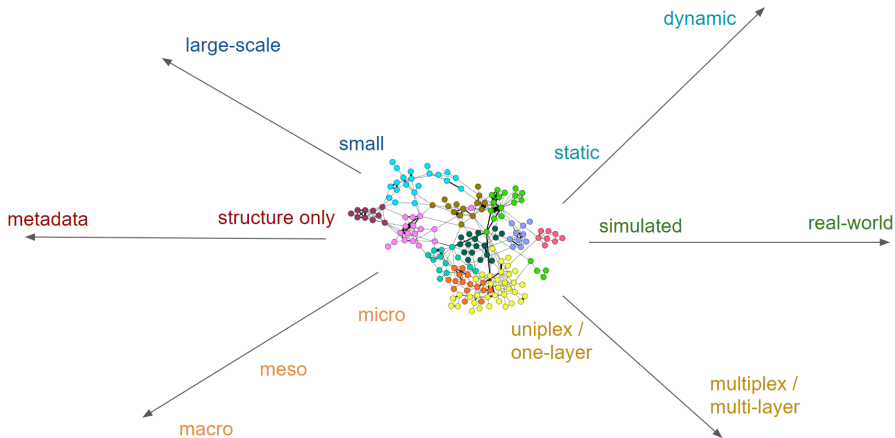




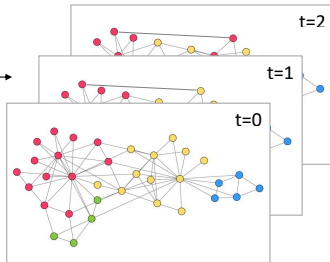
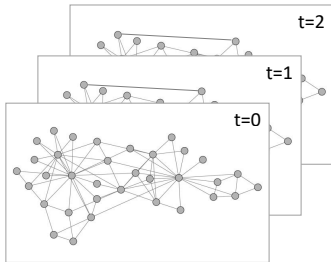
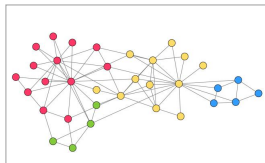
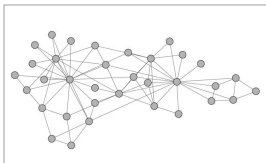




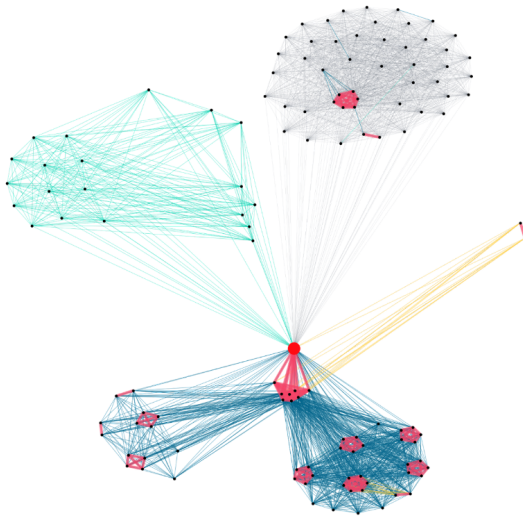




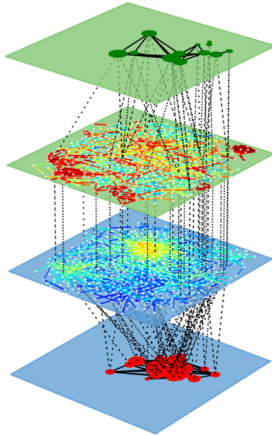
# Network (community) dynamics



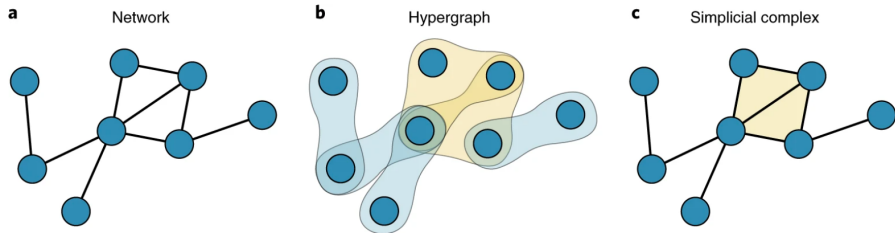
# Multilayer networks



# Multilevel networks



# Higher-order networks / Simplicial complexes



Battiston et al. "The physics of higher-order interactions in complex systems." *Nature Physics* 17 (2021): 1093–1098.

## Upcoming week

- Next week: last lecture; then student presentations start
- Be sure you know the following:
  - Your track letter
    - Track A: room BW 0.08
    - Track B: room BW 0.19
    - Track C: room BW 0.20
- Tracks and rooms are fixed; lecturer differs
- Presenting? On the Tuesday before your Friday presentation, send your slides to `snacs@liacs.leidenuniv.nl` for some feedback, or if you want, make an appointment the week before with the lecturer assigned to your room