## Synchronous Sequential Circuits: Design Procedure and Examples

## Overview

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## Sequential Circuit Design

F'In sequential circuit design, we turn some description into a working circuit.

- Start: With a list of specifications (descriptions):
- Behavior description of the circuit
- Type of Flip-Flops to be used (SR or JK or D or T)
- Type of gates to be used
- End: With a logic diagram OR list of Boolean functions.
- NOTE:
- \# Flip-Flops to be used depends on the \# of states. At most $2^{n}$ states can be represented with $\boldsymbol{n}$ Flip-Flops.
- The binary coding of the states and the type of the FlipFlops determine the complexity of the circuit.


## Sequential Circuit Design Procedure

- Step 1: Given the problem statement, derive the state table:
- The table should show inputs, present states, next states and outputs.
- It may be easier to find a state diagram first, and then convert that to a table.
- Step 2 (optional): Apply state-reduction methods to reduce (if possible) the number of states.
- We will not discuss state-reduction methods in this course.
- Step 3: Assign binary codes to the states in the state table, if you haven't already.
- If you have $n$ states, your binary codes will have at least $\left\lceil\log _{2} n\right\rceil$ bits.
- Step 4: Determine the number of Flip-Flops needed and the type of Flip-Flops to be used:
- If you have $\boldsymbol{n}$ states, your circuit will have at least $\left\lceil\log _{2} \boldsymbol{n}\right\rceil$ Flip-Flops.
- The types of Flip-Flops may be given in the initial specification.
- If not, select the type according to some criteria, e.g., to get simpler circuit or to make the design procedure easier.


## Sequential Circuit Design Procedure (cont.)

- Step 5: For each flip-flop and each row of your state table, find the flip-flop input values that are needed to generate the next state from the present state:
- You can use Flip-Flop excitation tables here.
- Step 6: Derive the characteristic (Flip-Flop input) equations from the state table.
- Step 7: Derive the primary output equations from the state table.
- Step 8: Simplify the Flip-Flop input equations and output equations:
- Use K-maps or
- Other simplification methods
- Step 9: Draw the logic diagram of the circuit.


## Sequence Recognizers

- I will explain the Design Procedure in detail by designing a sequence recognizer circuit.
- A sequence recognizer is a special kind of sequential circuit that looks for a special bit pattern in some input.
- The recognizer circuit has only one input, X.
- One bit of input is supplied on every clock cycle. For example, it would take 20 cycles to scan a 20 -bit input.
- This is an easy way to permit arbitrarily long input sequences.
- There is one output, $Z$, which is 1 when the desired pattern is found.
- Our example will detect the bit pattern "1001":
- Input X:
... 00011100110100100110 ...
- Output Z:
... 00000000100000100100 ...
- Here, one input and one output bit appear every clock cycle.
- This requires a sequential circuit because the circuit has to "remember" the inputs from previous clock cycles, in order to determine whether or not a match was found.


## Step 1: Deriving the State Table

- Problem Statement: Design the sequence recognizer circuit described in the previous slide.
- The first thing to figure out is precisely how the use of states will help to solve the given problem.
- Make a state table based on the problem statement. The table should show the present states, inputs, next states and outputs.
- Sometimes it is easier to first find a state diagram and then convert that to a table.
- This is usually the most difficult step. Why?
- There is not a formal procedure how to derive a state table or state diagram from a problem specification such as the one we have here.
- In Step 1 you have to relay on your knowledge and design experience.
- Currently, Step 1 is more an art than a science!
- Once you have the state table, the rest of the design procedure is the same for all sequential circuits.


## Step 1: Deriving the State Table (cont.)

- How many and What states do we need for the sequence recognizer?
- We have to "remember" inputs from previous clock cycles.
- For example, if the previous three inputs were 100 and the current input is 1 , then the output should be 1 .
- In general, we will have to remember occurrences of parts of the desired pattern - in this case, 1, 10, and 100.
- So, we need the following four states:

| State | Meaning |
| :---: | :--- |
| A | None of the desired pattern (1001) has been input yet. |
| B | We've already seen the first bit (1) of the desired pattern. |
| C | We've already seen the first two bits (10) of the desired pattern. |
| D | We've already seen the first three bits (100) of the desired pattern. |

- We will derive a state diagram before deriving the state table.
- First, we draw a part of the state diagram:



## Step 1: Deriving the State Table (cont.)

- What happens if we are in state D (the last three inputs were 100), and the current input is 1 ?
- The output should be a 1, because we've found the desired pattern.
- But this last 1 could also be the start of another occurrence of the pattern! For example, 1001001 contains two occurrences of 1001.
- To detect overlapping occurrences of the pattern, the next state should be B.


| State | Meaning |
| :---: | :--- |
| A | None of the desired pattern (1001) has been input yet. |
| B | We've already seen the first bit (1) of the desired pattern. |
| C | We've already seen the first two bits (10) of the desired pattern. |
| D | We've already seen the first three bits (100) of the desired pattern. |

## Step 1: Deriving the State Table (cont.)

- Remember that we need two outgoing arrows for each node, to account for the possibilities of $X=0$ and $X=1$.
- The remaining arrows we need are shown in blue. They also allow for the correct detection of overlapping occurrences of 1001.

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| State | Meaning |
| :---: | :--- |
| A | None of the desired pattern (1001) has been input yet. |
| B | We've already seen the first bit (1) of the desired pattern. |
| C | We've already seen the first two bits (10) of the desired pattern. |
| D | We've already seen the first three bits (100) of the desired pattern. |

## Step 1: Deriving the State Table (cont.)

- Finally, we have the state diagram and we can derive the state table.

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| Input <br> X | Present <br> State | Next <br> State | Output <br> Z |
| :---: | :---: | :---: | :---: |
| 0 | A | A | 0 |
| 1 | A | B | 0 |
| 0 | B | C | 0 |
| 1 | B | B | 0 |
| 0 | C | D | 0 |
| 1 | C | B | 0 |
| 0 | D | A | 0 |
| 1 | D | B | 1 |

Remember how the state diagram arrows correspond to rows of the state table:


## Step 3: Assigning Binary Codes to States

- We skip Step 2 because in this course we will not discuss state-reduction methods.
- We have four states $A, B, C$, and $D$, so we need at least two bits $Q_{1}$ and $Q_{2}$ to encode the states.
- There are many possible ways to encode the states:

| State | $Q_{1} Q_{2}$ | $Q_{1} Q_{2}$ | $Q_{1} Q_{2}$ | $Q_{1} Q_{2}$ | $\ldots$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | 0 | 0 | 0 | 1 | 1 | 0 | 1 |
| B | 0 | 1 | 0 | 0 | 0 | 0 | 0 |$|$

- The state code can have a big impact on circuit complexity, but we will not study this in this course.
- So, we take an arbitrary code. For example, we represent state $A$ with $Q_{1} Q_{2}=10, B$ with $00, C$ with 01 , and $D$ with 11.


## Step 3: Assigning Binary Codes to States (cont.)

- We fill the state table with the selected state code.
- Recall, we selected to represent state A with 10, B with 00, C with 01, and D with 11.

Encoded State Table

| State Table |  |  |  |  | nput | Present State |  | Next State |  | $\begin{array}{\|c\|} \hline \text { Output } \\ \hline Z(t) \\ \hline \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{array}{\|c\|} \hline \text { Input } \\ \mathrm{X} \end{array}$ | Present State | Next State | $\begin{aligned} & \text { Output } \\ & \text { Z } \end{aligned}$ |  | X(t) | $\mathrm{Q}_{1}(\mathrm{t})$ | $\mathrm{Q}_{2}(\mathrm{t})$ | $\mathrm{Q}_{1}(\mathrm{t}+1)$ | $Q_{2}(t+1)$ |  |
| 0 | A | A | 0 | 2 | 0 | 1 | 0 | 1 | 0 | 0 |
| 1 | A | B | 0 | 6 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | B | C | 0 |  | 0 | 0 | 0 | 0 | 1 | 0 |
| 1 | B | B | 0 |  | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | C | D | 0 |  | 0 | 0 | 1 | 1 | 1 | 0 |
| 1 | C | B | 0 | 5 | 1 | 0 | 1 | 0 | 0 | 0 |
| 0 | D | A | 0 | 3 | 0 | 1 | 1 | 1 | 0 | 0 |
| 1 | D | B | , | 7 | 1 | 1 | 1 | 0 | 0 | 1 |

NOTE: The rows of the Encoded State Table are not ordered as we are used to.
Reorder the rows to make the table ordered. This will make further design steps easier.

## Step 3: Assigning Binary Codes to States (cont.)

- Ordered Encoded State Table:

| Input | Present <br> State |  | Next <br> State |  | Output |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{X}(\mathrm{t})$ | $\mathrm{Q}_{1}(\mathrm{t})$ | $\mathrm{Q}_{2}(\mathrm{t})$ | $\mathrm{Q}_{1}(\mathrm{t}+1)$ | $\mathrm{Q}_{2}(\mathrm{t}+1)$ | $\mathrm{Z}(\mathrm{t})$ |
| 0 | 0 | 0 | 0 | 0 | 1 |
| 2 | 0 | 0 | 1 | 1 | 1 |
|  | 4 | 0 |  |  |  |
| 4 | 0 | 1 | 0 | 1 | 0 |
|  | 0 | 1 | 1 | 1 | 0 |
|  | 1 | 0 | 0 | 0 | 0 |
|  | 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 |

- This table contains the same information as the table from the previous slide. Just, the order of the rows is different.


## Step 4: Determine the number and type of Flip-Flops to be used

- We have 4 states and we have encoded them with 2 bits $Q_{1}$ and $Q_{2}$. Thus, we need 2 Flip-Flops.
- Here, we will use JK Flip-Flops (later you will see the same example using D Flip-Flops).
- Thus, for each Flip-Flip $Q_{i}$, we add two columns (for $\mathrm{J}_{\mathrm{i}}$ and $\mathrm{K}_{\mathrm{i}}$ ) in the state table.

|  | Input | $\begin{aligned} & \text { Present } \\ & \text { State } \end{aligned}$ |  | Next State |  | Flip-Flop Inputs |  |  |  | Output |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | for $Q_{1}$ | for $Q_{2}$ |  |  |
|  | X(t) | $\mathrm{Q}_{1}(\mathrm{t})$ | $\mathrm{Q}_{2}(\mathrm{t})$ |  |  | $Q_{1}(t+1)$ | $\mathrm{Q}_{2}(\mathrm{t}+1)$ | $J_{1}$ | $\mathrm{K}_{1}$ | $\mathrm{J}_{2}$ | $\mathrm{K}_{2}$ | Z(t) |
| 0 | 0 | 0 | 0 | 0 | 1 |  |  |  |  | 0 |
|  | 0 | 0 | 1 | 1 | 1 |  |  |  |  | 0 |
|  | 0 | 1 | 0 | 1 | 0 |  |  |  |  | 0 |
|  | 0 | 1 | 1 | 1 | 0 |  |  |  |  | 0 |
| 4 | 1 | 0 | 0 | 0 | 0 |  |  |  |  | 0 |
|  | 1 | 0 | 1 | 0 | 0 |  |  |  |  | 0 |
| 6 | 1 | 1 | 0 | 0 | 0 |  |  |  |  | 0 |
| 7 | 1 | 1 | 1 | 0 | 0 |  |  |  |  | 1 |

## Step 5: Finding Flip-Flop Input Values

- How to actually make the Flip-Flops change from their present state into the desired next state.
- For each Flip-Flip $Q_{i}$, look at its present and next states, and determine what the inputs $J_{i}$ and $\mathrm{K}_{\mathrm{i}}$

JK Execution Table

| $\mathrm{Q}(\mathrm{t})$ | $\mathrm{Q}(\mathrm{t}+1)$ | J | K |
| :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | x |
| 0 | 1 | 1 | x |
| 1 | 0 | x | 1 |
| 1 | 1 | x | 0 | should be in order to make that state change.

- Use the JK execution table.

| Input | Present State |  | Next State |  | Flip-Flop Inputs |  |  |  | Output |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |
| X(t) | $\mathrm{Q}_{1}(\mathrm{t})$ | $\mathrm{Q}_{2}(\mathrm{t})$ |  |  | $Q_{1}(t+1)$ | $\mathrm{Q}_{2}(t+1)$ | $J_{1}$ | $\mathrm{K}_{1}$ | $\mathrm{J}_{2}$ | $\mathrm{K}_{2}$ | Z(t) |
| 0 | 0 | 0 | 0 | 1 | 0 | x | 1 | X | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | x | x | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | x | 0 | 0 | x | 0 |
| 0 | 1 | 1 | 1 | 0 | x | 0 | x | , | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | x | 0 | x | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 | x | x | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 | x | 1 | 0 | , | 0 |
| 1 | 1 | 1 | 0 | 0 | x | 1 | x | 1 | 1 |

## Step 6 and 8: Deriving Simplified Flip-Flop Input Equations

| Input | Present State |  | Next State |  | Flip-Flop Inputs |  |  |  | Output |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |
| X(t) | $\mathrm{Q}_{1}(\mathrm{t})$ | $\mathrm{Q}_{2}(\mathrm{t})$ |  |  | $\mathrm{Q}_{1}(\mathrm{t}+1)$ | $\mathrm{Q}_{2}(\mathrm{t}+1)$ | $\mathrm{J}_{1}$ | $\mathrm{K}_{1}$ | $\mathrm{J}_{2}$ | $\mathrm{K}_{2}$ | Z(t) |
| 0 | 0 | 0 | 0 | 1 | 0 | x | 1 | x | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | x | x | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | x | 0 | 0 | x | 0 |
| 0 | 1 | 1 | 1 | 0 | x | 0 | x | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | x | 0 | x | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 | X | x | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 | x | 1 | 0 | x | 0 |
| 1 | 1 | 1 | 0 | 0 | x | 1 | x | 1 | 1 |


$J_{1}=X(t) \cdot Q_{2}(t)$

$K_{1}=X(t)$

$K_{2}=X(t)+Q_{1}(t)$

## Step 7 and 8: Deriving Simplified Primary Output Equations

| Input | Present State |  | Next State |  | Flip-Flop Inputs |  |  |  | Output |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | for $\mathrm{Q}_{1}$ | for $Q_{2}$ |  |  |
| X(t) | $\mathrm{Q}_{1}(\mathrm{t})$ | $\mathrm{Q}_{2}(\mathrm{t})$ |  |  | $Q_{1}(t+1)$ | $\mathrm{Q}_{2}(\mathrm{t}+1)$ | $\mathrm{J}_{1}$ | $\mathrm{K}_{1}$ | $\mathrm{J}_{2}$ | $\mathrm{K}_{2}$ | Z(t) |
| 0 | 0 | 0 | 0 | 1 | 0 | x | 1 | X | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | x | x | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | x | 0 | 0 | x | 0 |
| 0 | 1 | 1 | 1 | 0 | x | 0 | x | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | x | 0 | x | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 | x | x | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 | x | 1 | 0 | x | 0 |
| 1 | 1 | 1 | 0 | 0 | x | 1 | x | 1 | 1 |



## Step 9: Drawing The Logic Diagram

$$
\begin{aligned}
& Q_{1}(t+1)\left\{\begin{array}{l}
J_{1}=X(t) \cdot Q_{2}(t) \\
K_{1}=X(t)
\end{array}\right. \\
& Q_{2}(t+1)\left\{\begin{array}{l}
J_{2}=X(t) \cdot Q_{1}(t) \cdot \\
K_{2}=X(t)+Q_{1}(t)
\end{array}\right. \\
& Z(t)=X(t) \cdot Q_{1}(t) \cdot Q_{2}(t)
\end{aligned}
$$



- IMPORTANT: Do not forget to connect a clock signal to the Flip-Flops.
- IMPORTANT: Do not forget to connect a reset signal to the Flip-Flops:
- The reset signal must switch the circuit to the initial state. Every sequential circuit must have one initial state. Always, first switch the circuit to the initial state.
- To switch to the initial state, use the asynchronous signals preset $(S)$ and/or clear ( $R$ ) of the Flip-Flops.
- For our sequence recognizer the initial state is $A$ with $Q_{1} Q_{2}=10$, thus:
- While $\overline{\text { reset }}$ is active ( $\overline{\text { reset }}=0$ ), $Q_{1}$ is set to 1 and $Q_{2}$ to 0 - the circuit stays in the initial state.
- While $\overline{\text { reset }}$ is non-active ( $\overline{\text { reset }}=1$ ), the circuit operates normally.

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## Designing the Same Circuit with D Flip-Flops (Step 4)

- What if we want to design the circuit using D Flip-Flops instead of JK?
- We already have the ordered encoded state table, so we can just start from Step 4, determining the number of Flip-Flops.
- We have 4 states and we have encoded them with 2 bits $Q_{1}$ and $Q_{2}$. Thus, we need 2 Flip-Flops.
- For each Flip-Flip $Q_{i}$, we add one column (for $D_{i}$ ) in the state table.

| Input | Present <br> State |  | Next <br> State |  | Flip-Flop Inputs |  | Output |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | for $Q_{1}$ | for $Q_{2}$ |  |  |  |  |  |
| $\mathrm{X}(\mathrm{t})$ | $\mathrm{Q}_{1}(\mathrm{t})$ | $\mathrm{Q}_{2}(\mathrm{t})$ | $\mathrm{Q}_{1}(\mathrm{t}+1)$ | $\mathrm{Q}_{2}(\mathrm{t}+1)$ | $\mathrm{D}_{1}$ | $\mathrm{D}_{2}$ | $\mathrm{Z}(\mathrm{t})$ |
| 0 | 0 | 0 | 0 | 1 |  |  | 0 |
| 0 | 0 | 1 | 1 | 1 |  |  | 0 |
| 0 | 1 | 0 | 1 | 0 |  |  | 0 |
| 0 | 1 | 1 | 1 | 0 |  |  | 0 |
| 1 | 0 | 0 | 0 | 0 |  |  | 0 |
| 1 | 0 | 1 | 0 | 0 |  |  | 0 |
| 1 | 1 | 0 | 0 | 0 |  |  | 0 |
| 1 | 1 | 1 | 0 | 0 |  |  | 1 |

## Finding Flip-Flop Input Values (Step 5)

- Again, for each Flip-Flip $Q_{i}$, look at its present and next states, and determine what the input $D_{i}$ should be in order to make that state change. Use the $\boldsymbol{D}$ execution table.
- The D excitation table is pretty boring; set the D input to whatever the next state should be.
- You don't even need to show separate columns for $D_{1}$ and $D_{2}$; you can just use the Next State columns.

| Input | Present <br> State |  | Next <br> State |  | Flip-Flop Inputs |  | Output |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | for $Q_{1}$ | for $Q_{2}$ |  |  |  |  |  |
| $X(t)$ | $Q_{1}(t)$ | $Q_{2}(t)$ | $Q_{1}(t+1)$ | $Q_{2}(t+1)$ | $D_{1}$ | $D_{2}$ | $Z(t)$ |
| 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 |

## Deriving Simplified Equations (Steps 6, 7, and 8)

- We can use K-maps again, to simplify:


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## Drawing The Logic Diagram (Step 9)

- Use the equations on the right to draw the logic diagram of the circuit.
- IMPORTANT: Again, do not forget to connect properly the clock and reset signals to the Flip-Flops.

$$
\begin{array}{r}
Q_{1}(t+1)=D 1= \\
X(t)^{\prime} \cdot Q_{2}(t)+ \\
X(t)^{\prime} \cdot Q_{1}(t) \\
Q_{2}(t+1)=D 2=X(t)^{\prime} \cdot Q_{1}(t) \cdot
\end{array}
$$

$$
Z(t)=X(t) \cdot Q_{1}(t) \cdot Q_{2}(t)
$$



## Design Comparison



- D Flip-Flops have the advantage that you do not have to set up Flip-Flop inputs at all, since
 $Q(\mathrm{t}+1)=\mathrm{D}$. However, the D input equations are usually more complex than JK input equations.
- In practice, $\boldsymbol{D}$ Flip-Flops are used more often.
- There is only one input for each Flip-Flop, not two.
- There are no execution tables to worry about.
- D Flip-Flops can be implemented with slightly less hardware than JK Flip-Flops.


## Another Example: A Cyclic Shifter

- Design a circuit which has one input $X$ and three outputs $Z_{1}, Z_{2}$, and $Z_{3}$ and the following behavior:
- Initially, the output values are: $Z_{1} Z_{2} Z_{3}=010$;
- On every clock cycle:
- If $X=1$ then $Z_{1}=Z_{2}, Z_{2}=Z_{3}, Z_{3}=Z_{1}$ (cyclic shift-left)

- If $X=0$ then $Z_{1}=Z_{3}, Z_{2}=Z_{1}, Z_{3}=Z_{2}$ (cyclic shift-right)

- This circuit we will call 3-bit cyclic shifter.


## Deriving the State Table

- How many and What states do we need for our circuit?
- We have initial state where outputs are "010".
- If we make cyclic shift-left we can have the following possible output values: "100" or "001" or "010" (which is the initial output value)
- If we make cyclic shift-right we can have the following possible output values: "001" or " 100 " or " 010 " (which is the initial output value)
- We see that we can use only 3 states, one for each of the three distinct output values "010", "100", "001".
- So, for each possible output value we define a state as follows:

| State | Meaning |
| :---: | :--- |
| $A$ | Initial state where the outputs $Z_{1} Z_{2} Z_{3}=" 010 "$. |
| $B$ | While at this state the circuit has outputs $Z_{1} Z_{2} Z_{3}=" 100 "$. |
| $C$ | While at this state the circuit has outputs $Z_{1} Z_{2} Z_{3}=" 001 "$. |

## Deriving the State Table (cont.)

- First, we will derive the state diagram of the circuit.
- We need two outgoing arrows for each node, to account for the possibilities of $X=0$ (cyclic shift-right) and $X=1$ (cyclic shift-left).


| State | Meaning |
| :---: | :--- |
| $A$ | Initial state where the outputs $Z_{1} Z_{2} Z_{3}=" 010 "$. |
| $B$ | While at this state the circuit has outputs $Z_{1} Z_{2} Z_{3}=" 100 "$. |
| $C$ | While at this state the circuit has outputs $Z_{1} Z_{2} Z_{3}=" 001 "$. |

## Deriving the State Table (cont.)

- We have the state diagram and we can derive the state table.


| Input | Present | Next | Outputs |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| X | State | State | $\mathrm{Z}_{1}$ | $\mathrm{Z}_{2}$ | $\mathrm{Z}_{3}$ |
| 0 | A | C | 0 | 1 | 0 |
| 1 | A | B | 0 | 1 | 0 |
| 0 | B | A | 1 | 0 | 0 |
| 1 | B | C | 1 | 0 | 0 |
| 0 | C | B | 0 | 0 | 1 |
| 1 | C | A | 0 | 0 | 1 |



Remember that we have defined that:

- In state $A$ the circuit outputs $Z_{1} Z_{2} Z_{3}=010$
- In state $B$ the circuit outputs $Z_{1} Z_{2} Z_{3}=100$
- In state $C$ the circuit outputs $Z_{1} Z_{2} Z_{3}=001$


## Assigning Binary Codes to States

## State Table

| Input | Present | Next | Outputs |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | State | State | $\mathrm{Z}_{1}$ | $\mathrm{Z}_{2}$ | $\mathrm{Z}_{3}$ |
| 0 | A | C | 0 | 1 | 0 |
| 1 | A | B | 0 | 1 | 0 |
| 0 | B | A | 1 | 0 | 0 |
| 1 | B | C | 1 | 0 | 0 |
| 0 | C | B | 0 | 0 | 1 |
| 1 | C | A | 0 | 0 | 1 |

- We will represent state A with $00, B$ with $10, C$ with 01.



## Encoded State Table

|  |  | Input | Present State |  | Next State |  | Outputs |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | X(t) | $\mathrm{Q}_{1}(\mathrm{t})$ | $\mathrm{Q}_{2}(\mathrm{t})$ | $\mathrm{Q}_{1}(\mathrm{t}+1)$ | $\mathrm{Q}_{2}(\mathrm{t}+1)$ | $\mathrm{Z}_{1}(\mathrm{t})$ | $\mathrm{Z}_{2}(\mathrm{t})$ | $\mathrm{Z}_{3}(\mathrm{t})$ |
|  | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 |
| NOTE: We have toorder the encodedstate table. | 4 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
|  | 2 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
|  | 6 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
|  | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
|  | 5 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |

## Ordered Encoded State Table

- For this circuit the ordered encoded state table is not complete. Why?
- We have 3 states that we encode with 2 bits, i.e., state $A$ with $Q_{1} Q_{2}=00$, $B$ with $Q_{1} Q_{2}=10$, and $C$ with $Q_{1} Q_{2}=01$.
- Using two bits we can encode 4 states from which we use only 3 states => one state is unused, i.e., $Q_{1} Q_{2}=11$
- Thus, two rows ( 3 and 7 ) in the table are incomplete.
- We have two main options to complete the table (see next slides).

|  | Input | PresentState |  | Next State |  | Outputs |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | X(t) | $\mathrm{Q}_{1}(\mathrm{t})$ | $\mathrm{Q}_{2}(\mathrm{t})$ | $Q_{1}(t+1)$ | $Q_{2}(t+1)$ | $\mathrm{Z}_{1}(\mathrm{t})$ | $\mathrm{Z}_{2}(\mathrm{t})$ | $\mathrm{Z}_{3}(\mathrm{t})$ |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 |
| 2 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 3 | 0 | 1 | 1 |  |  |  |  |  |
| 4 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| 5 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| 6 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 7 | 1 | 1 | 1 |  |  |  |  |  |

## Option 1: Use don't-care Conditions

- We can use don't-care conditions because if the circuit operates correctly it will never enter unused states.

| Input | Present State |  | $\begin{aligned} & \hline \text { Next } \\ & \text { State } \end{aligned}$ |  | Outputs |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| X(t) | $\mathrm{Q}_{1}(\mathrm{t})$ | $\mathrm{Q}_{2}(\mathrm{t})$ | $\mathrm{Q}_{1}(\mathrm{t}+1)$ | $\mathrm{Q}_{2}(t+1)$ | $\mathrm{Z}_{1}(\mathrm{t})$ | $\mathrm{Z}_{2}(\mathrm{t})$ | $\mathrm{Z}_{3}(\mathrm{t})$ |
| 0 | 0 | 0 | , | 1 | 0 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | X | X | X | X | $\times$ |
| 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | x | X | X | X | X |

- Now, you can apply Steps 4 to 9 of the design procedure to get the complete circuit.
- Do this at home as an exercise!
- Similar example will be given at the tutorials!


## Option 2: Explicitly Specify Unused States

- It is possible that outside interference or a malfunction will cause the circuit to enter one of the unused states causing temporary or permanent incorrect behavior of the circuit.
- Temporary or permanent incorrect behavior may be harmful.
- Thus, in such case it is necessary to explicitly specify, fully or at least partially, the next state values or the output values for the unused states.
- This will make the behavior of the circuit predictable.
- Undesired harmful behavior can be avoided if the circuit enters an unused state.


## Option 2: Explicitly Specify Unused States (cont.)

- Depending on the function and application of the circuit, a number of ideas may be applied:
- First, the outputs for the unused states are specified such that any action that results from entry into and transitions between unused states are not harmful.
- Second, an unused output combination may be employed which indicates that the circuit has entered an unused (incorrect) state.
- Third, Next-State for each unused state is selected such that one of the normal occurring states is reached within a few clock cycles, regardless of the input values.
- Typically, the next state for an unused state is selected to be the initial state.
- The ideas above may be applied in combination.


## Option 2: An Example

- Example: Assume that our 3-bit cyclic shifter circuit is used within a system with three other devices:
- Each output of our circuit controls only one device.
- Logic '1' on an output enables the corresponding device.
- A harmful situation for the system will occur if more than one device is enabled.
- So, we can avoid harmful situations by specifying the unused states as shown below (see rows 3 and 7):

| Input | Present <br> State |  | Next <br> State |  | Outputs |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{X}(\mathrm{t})$ | $\mathrm{Q}_{1}(\mathrm{t})$ | $\mathrm{Q}_{2}(\mathrm{t})$ | $\mathrm{Q}_{1}(\mathrm{t}+1)$ | $\mathrm{Q}_{2}(\mathrm{t}+1)$ | $\mathrm{Z}_{1}(\mathrm{t})$ | $\mathrm{Z}_{2}(\mathrm{t})$ | $\mathrm{Z}_{3}(\mathrm{t})$ |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| 1 |  |  |  |  |  |  |  |
| 2 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| 2 | 0 | 1 |  |  |  |  |  |
| 3 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 4 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 5 |  |  |  |  |  |  |  |
|  | 1 | 0 | 0 | 1 | 0 | 0 | 1 |
| 7 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| 7 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |

The output:
$Z_{1} z_{2} z_{3}=000$ is not harmful and indicates that the circuit has entered unused (incorrect) state. Why?

Next-State of the unused state
$\left(Q_{1} Q_{2}=11\right)$ is the initial state
$\left(\mathrm{Q}_{1} \mathrm{Q}_{2}=00\right.$ )

