n-Queens

This paper contains references to the well-known n-queens problem. It is available through:

`http://www.liacs.nl/home/kosters/nqueens.html`

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