Expressies en Automaten

Opdracht 3

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colleges

- Foundations of Computer Science[1] (eerste jaar, eind)
- Automata Theory (tweede jaar, college 6)

[1] college van afgelopen najaar

Specifying languages

RECOGNIZING, algorithm

$L_2 = \{ x \in \{a, b\}^* \mid n_a(x) > n_b(x) \}$

count *a* and *b*

deterministic [finite] automaton

GENERATING, description

regular expression

$L_1 = (\{ab, bab\}^*\{b\})^*\{ab\} \cup \{b\}\{ba\}^*\{ab\}^*$

recursive definition

$\hookrightarrow$well-formed formulas

grammar

letter, symbool σ $0, 1$ a, b, c
alfabet Σ, A $\{a, b, c\}$
(eindig, niet-leeg)

string, woord over Σ eindig rijtje
$w = a_1 a_2 \ldots a_n$, $a_i \in \Sigma$ $abbabb$
lege string λ Λ, ε, 1
Σ^* alle strings $\{\ \lambda, a, b, c, aa, ab, ac, ba, bb, \ldots, aaa, aab, \ldots\}$

lengte $|w|$ aantal symbolen
$|abbab| = 5$ $|\lambda| = 0$

taal over Σ $L \subseteq \Sigma^*$

wat is een alfabet?

eindige, niet-lege, verzameling

- $\{0, 1\}$ $\{a, b, c\}$ $\{0, 1, \uparrow, \boxplus\}$
- $\{\clubsuit, \spadesuit, \heartsuit, \diamondsuit\} \times \{H, V, B, 10, 9, \ldots, 2, A\}$ $(\spadesuit, V)$
- $\Sigma_G = \{\ (u, v) \mid (u, v) \in E\}$
  taal: paden in graaf
- $\{$ <u>if</u>, <u>then</u>, <u>function</u>, <u>return</u>, $\ldots \}$

taal is verzameling

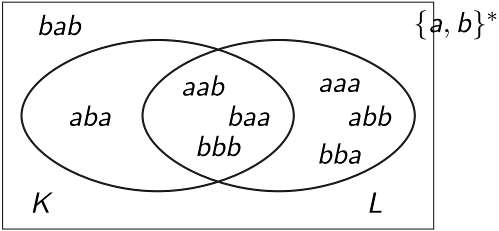
opsommen, eigenschap

- $\varnothing$ nul woorden
- $\{ \lambda \}$ één woord
- $\{ a^m b^n \mid m, n \in \mathbb{N} \} = \{ \lambda, a, b, aa, ab, bb, aaa, aab, abb, bbb, \dots \}$
- $\{ a^n b^n \mid n \in \mathbb{N} \} = \{ \lambda, ab, aabb, aaabbb, a^4 b^4, \dots \}$
- $\{ a^m b a^n \mid m, n \in \mathbb{N} \}$ precies één b
- $L = \{ x \in \{a, b, c\}^* \mid x \text{ heeft subwoord } aab \}$

taal is verzameling

$K = \{ x \in \{a, b\}^* \mid x \text{ heeft een even aantal } a\text{'s} \}$
$= \{ \lambda, b, aa, bb, aab, aba, baa, bbb, \ldots \}$

$L = \{ x \in \{a, b\}^* \mid x \text{ heeft (ergens) twee gelijke letters achter elkaar} \}$
$= \{ aa, bb, aaa, aab, abb, baa, bba, bbb, \ldots \}$



$aab, baa, bbb \in K \cap L \quad aba \in K \setminus L \quad aaa, abb, bba \in L \setminus K \quad bab \in (K \cup L)^c$

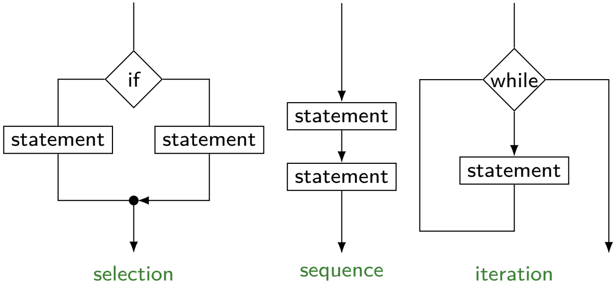
operaties op talen

Boolese operaties

$\cup$	vereniging	$\vee$
$\cap$	doorsnede	$\wedge$
^{c}	complement	$\neg$

reguliere operaties

$\cup$	vereniging
$\cdot$	concatenatie
$*$	ster



concatenatie

$K \cdot L = KL = \{\ x \cdot y \mid x \in K, y \in L\ \}$

$\{\ a, ab\ \} \cdot \{\ a, ba\ \} = \{\ a \cdot a, a \cdot ba, ab \cdot a, ab \cdot ba\ \} = \{\ aa, aba, abba\ \}$

een	$\{\lambda\} \cdot L = L \cdot \{\lambda\} = L$
nul	$\varnothing \cdot L = L \cdot \varnothing = \varnothing$
associatief	$(KL)M = K(LM)$

- $\{\ \lambda, a, b, aa, ab, ba, bb, aaa, aab, \dots \} \cdot \{\ b\ \} =$
  $\{\ \lambda b, ab, bb, aab, abb, bab, bbb, aaab, aabb, \dots \}$
- $\{\ a, ab, abb, abbb, abbbb, \dots \} \cdot \{\ a, ba, aba, baba, ababa, \dots \} =$
  $\{\ aa, aba, aaba, abba, ababa, abbba, aababa, abbaba, abbbba, \dots \}$
  $abb \cdot aba = ab \cdot baba$

macht

$$K^n = \underbrace{K \cdot K \cdot \ldots \cdot K}_{n \text{ keer}}$$

$$K^n = \{\ w_1 w_2 \ldots w_n \mid w_1, w_2, \ldots, w_n \in K\ \}$$ vaste n 'buiten'

- $\{\ a, bb\ \}^3 = \{\ aaa, aabb, abba, ab^4, bbaa, bbabb, b^4 a, b^6\ \}$
- $\{\ \lambda, a, ab\ \}^3 = \{\ \lambda, a, aa, ab, aaa, aab, ab{\cdot}a, aaab, aaba, abaa,$
  $abab, a{\cdot}ab{\cdot}ab, abaab, ababa, ababab\ \}$

- $L = \{\ a^n b a^n \mid n \in \mathbb{N}\} = \{b, aba, aabaa, a^3 b a^3, \ldots\}$
  $L^2 = \{\ a^m b a^{m+n} b a^n \mid m, n \in \mathbb{N}\}$

- $(\ \{a\}^* \{b\} \{a\}^*\ )^3 = \{\ w \in \{a, b\}^* \mid w$ bevat 3 voorkomens van $b\ \}$
- $\{\ \lambda, a, a^4, a^9, a^{16}, \ldots\ \}^4 = \{a\}^*$ Lagrange's four-square theorem☒

(Kleene) ster

$$K^* = \underbrace{K \cdot K \cdot \ldots \cdot K}_{\text{willekeurig}}$$

$$K^* = \{\ w_1 w_2 \ldots w_n \mid w_1, w_2, \ldots, w_n \in K, n \in \mathbb{N}\ \}$$ n varieert 'binnen'

- $\{a, b\}^*$ alle woorden over $\{a, b\}$
  $\{a, b\}^n$ alle woorden over $\{a, b\}$ van lengte n
- $\{a\}^* \cdot \{b\} = \{\lambda, a, aa, aaa, \ldots\} \cdot \{b\} = \{b, ab, aab, aaab, \ldots\}$
- $(\ \{a\}^* \cdot \{b\}\ )^* =$
  $\{b, ab, aab, aaab, \ldots\}^* = \{\lambda, b, ab, bb, aab, abb, bab, bbb, aaab, \ldots\}$
  $= \{a, b\}^* \cdot \{b\} \cup \{\lambda\}$

korte notatie $\{a\} \leftrightarrow a$ a^*b, $(a^*b)^*$ zie ook straks

reguliere talen

alfabet Σ

reguliere talen

- $\varnothing$, $\{\lambda\}$ en $\{a\}$ zijn regulier $\quad a \in \Sigma$
- als K en L regulier, dan ook $K \cup L$, $K \cdot L$ en K^*

- $\{a\}^*\{b\}\{a\}^*$ één b $\quad a^* b a^*$
- $\{b\}^*\{a\}\{b\}^*\{a\}\{b\}^*$ precies twee a's $\quad b^* a b^* a b^*$
- $(\{a\} \cup \{b\})^*\{b\} = \{a, b\}^*\{b\}$ eindigt op b
- $\{a\}\{a\}^*\{b\}\{b\}^*$ één of meer a's gevolgd door één of meer b's
  $aa^* bb^*$
- $(\ \{a\}^*\{b\}\ \{a\}^*\{b\}\ )^*\ \{a\}^*$ even aantal b $\quad (\ a^* b a^* b\ )^*\ a^*$

haakjes weglaten

– associativiteit $+, \cdot$

– voorrangsregels

* voor $\cdot$ voor $+$

$aab + ab^*a + (ab)^*a$

$\{aab\} \cup \{aa, aba, abba, abbba, \ldots\} \cup \{a, aba, ababa, abababa, \ldots\}$

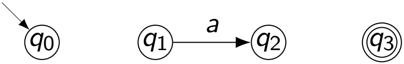
verschil verzamelingsnotatie en reguliere expressie

$\{ab, bab\}^* \; \{\lambda, bb\}$ vs $(ab + bab)^*(\lambda + bb)$

ingredienten

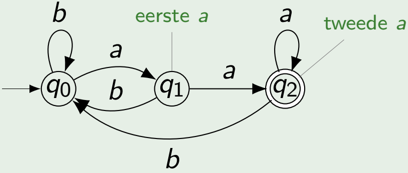
finite control ⇝ gerichte graaf

begin transitie accepterend



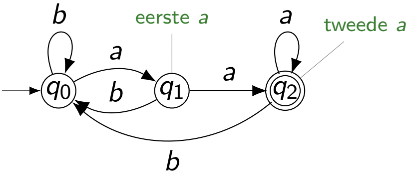
Example

$L_1 = \{ x \in \{a, b\}^* \mid x \text{ ends with } aa \}$



δ	a	b
q_0	q_1	q_0
q_1	q_2	q_0
q_2	q_2	q_0

gerichte graaf, tak-labels, begintoestand, accepterend (eindtoestand)



δ	a	b
q_0	q_1	q_0
q_1	q_2	q_0
q_2	q_2	q_0

accepterend

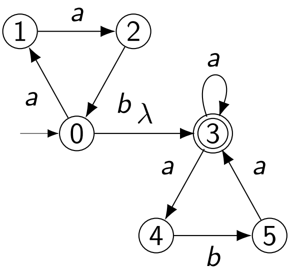
$$\to q_0 \xrightarrow{a} q_1 \xrightarrow{b} q_0 \xrightarrow{b} q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_2 \xrightarrow{b} q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_2$$

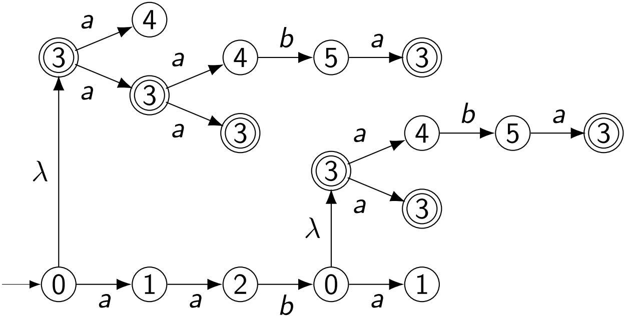
niet accepterend

$$\to q_0 \xrightarrow{a} q_1 \xrightarrow{b} q_0 \xrightarrow{b} q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_2 \xrightarrow{a} q_2 \xrightarrow{b} q_0$$

$L(M)$ *taal van M* labels paden van begin naar accepterende toestand

Computation tree when λ's are around



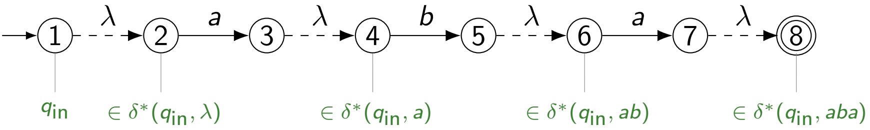


λ-closure

NFA-λ $M = (Q, \Sigma, \delta, q_{\text{in}}, A)$ $\quad S \subseteq Q$

Definition

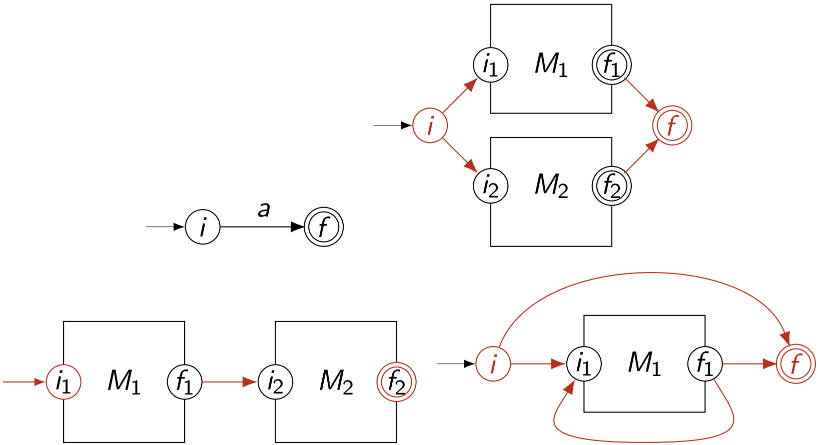
– $S \subseteq \Lambda(S)$
– $p \in \Lambda(S)$ and $(p, \lambda, q) \in \delta$, then $q \in \Lambda(S)$



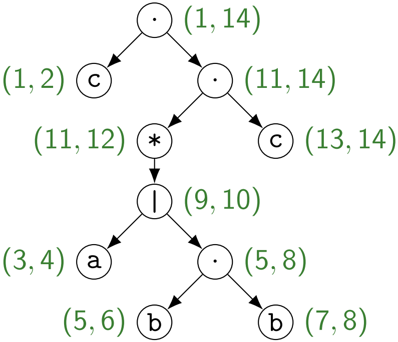
Definition

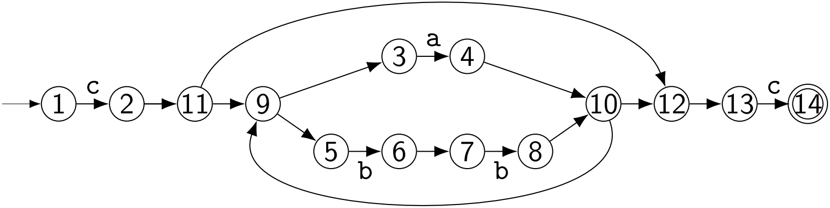
– $\delta^*(q, \lambda) = \Lambda(\ \{q\}\ )$ $\quad q \in Q$
– $\delta^*(q, y\sigma) = \Lambda(\ \delta(\ \delta^*(q, y), \sigma\ )\ )$ $\quad q \in Q,\ y \in \Sigma^*,\ \sigma \in \Sigma$

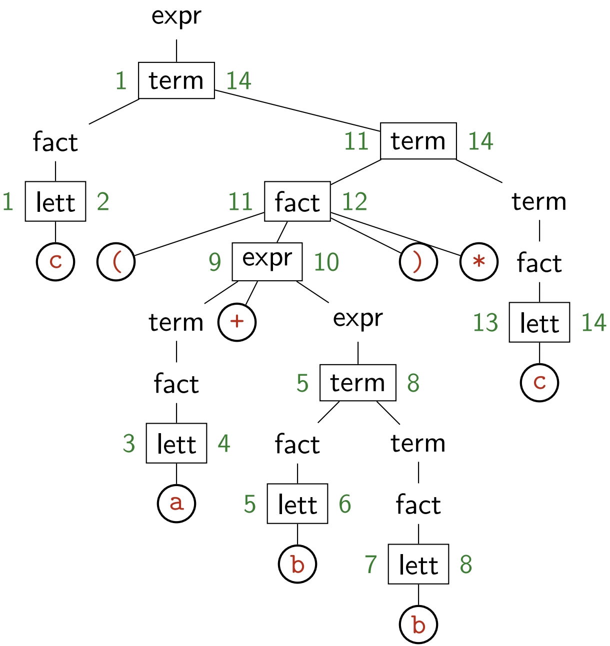
Thompson

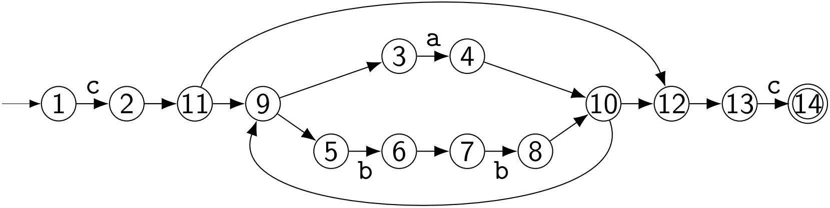


Syntax-boom evaluatie






expr
1 term 14
fact
11 term 14
1 lett 2
11 fact 12
term
c
(
9 expr 10
)
*
fact
term
+
expr
13 lett 14
5 term 8
c
fact
3 lett 4
fact
term
a
5 lett 6
fact
b
7 lett 8
b



1	2	3	4	5	6	7	8	9	10	11	12	13	14
c	-	a	-	b	-	b	-	-	-	-	-	c	-
2	11	4	10	6	7	8	10	3	9	9	13	14	-
-	-	-	-	-	-	-	-	5	12	12	-	-	-

(a*|b)*

(a*|b)* ≡ (a|b)*