Niching in Evolution Strategies

Ofer M. Shir and Thomas Bäck*

Leiden Institute of Advanced Computer Science, Universiteit Leiden, Niels Bohrweg 1, 2333 CA Leiden, The Netherlands

The Motivation: ES Niching

- EAs, and in particular ES and GAs, have the tendency to converge into a single solution [2].
- Many complex search problems require the location of multiple solutions.
- **Niching methods** are the extension of EAs into the multimodal domain: maintaining the population's diversity and converging in parallel into multiple optima.
- Niching methods have been introduced and studies so far mainly within the GA field [2].
- In our research we extend the ES by a dynamic niching approach to introduce a robust niching method for the optimization of real-valued high-dimensional functions.

ES Dynamic Niching

The strategy of the proposed algorithm can be described as follows:

- **Dynamic Peak Identification**. Once the fitness is evaluated, the various fitness-peaks are identified dynamically using the *dynamic peak identification* (DPI) algorithm [3], with the *euclidean distance* in the decision parameters space as a distance metric (the method is given as algorithm 1). Using an estimated so-called niche radius ρ, the individuals are classified into the peaks and populate those niches.

Algorithm 1 Dynamic Peak Identification (DPI)

```
input:    Pop - array of population members
          N - population size
          q - number of peaks to identify
          ρ - niche radius

Sort Pop in decreasing fitness order
i := 1
NumPeaks := 0
DPS := ∅ (Dynamic Peak Set)
loop until NumPeaks = q or i = N + 1
  if Pop[i] is not within ρ of peak in DPS
     DPS := DPS ∪ {Pop[i]}
     NumPeaks := NumPeaks + 1
  endif
  i := i + 1
endloop
output:   Dynamic Peak Set
```

- **Mating Restriction Scheme**. Competitive mating is allowed only within the niches: every niche can produce a defined number of offspring. A uniform distribution of the resources to q niches is considered: $\tilde{\mu} = \frac{\mu}{q}$ $\tilde{\lambda} = \frac{\lambda}{q}$, i.e. each niche has $\tilde{\mu}$ parents and produces $\tilde{\lambda}$ offspring in every generation.
- **The Evolutionary Mechanism: Standard-ES**. Traditional operators [1] are in use:
  - The *mutation operator*: a single step-size per individual; self-adaptation as in the traditional way [1].
  - First parent is chosen via *tournament selection*, second parent is the best individual of the niche.
  - Recombination: *intermediate* for strategy parameters and *discrete* for decision parameters.
  - The best offspring as well as the best current individuals are *selected*.

Assumptions

- q, the expected/desired number of peaks, is given or can be estimated.
- All peaks are at least in distance 2ρ from each other, where ρ is the fixed radius of every niche.

Calculating the Niche Radius

Given q, consider every niche as surrounded by an n-dimensional hypersphere with radius ρ which occupies $\frac{1}{q}$ of the entire volume of the space. The volume of the hypersphere which contains the entire space is $V = cr^n$, where c is a constant. Given lower and upper boundary values $x_{k,min}$, $x_{k,max}$ of each coordinate in the decision parameters space, r is defined as follows:

$$r = \frac{1}{2}\sqrt{\sum_{k=1}^{n}(x_{k,max} - x_{k,min})^2}$$

If we divide the volume into q parts, we may write

$$c\rho^n = \frac{1}{q}cr^n$$

which yields

$$\rho = \frac{r}{\sqrt[n]{q}}$$

Our algorithm is summarized as algorithm 2.

Algorithm 2 ES Dynamic Niching: A Generation Loop

```
Apply Mutation on the population
Evaluate fitness of population and Sort
Compute the Dynamic Peak Set using the DPI (Algo-1)
for every niche i = 1..q produce the next generation:
  Generate λ̃ offspring as follows:
    Choose 1st parent via Tournament Selection of the niche
    Choose the best individual of that niche as the 2nd parent
    Apply standard recombination
  Select the best η out of the λ̃ offspring and the best μ̃ − η
  individuals of the current niche to form the next generation
endfor
Generate additional ω = μ̃ random individuals
Join all q niches, to yield the new population
```

* *NuTech Solutions*, Martin-Schmeisser-Weg 15, 44227 Dortmund, Germany.

Experimental Results

The Test Functions

1. Himmelblau's function ($minimization$; $x_1, x_2 \in [-6, 6]$):

$$\mathcal{H}(x_1, x_2) = (x_1^2 + x_2 - h_1)^2 + (x_1 + x_2^2 - h_2)^2$$

2. $\mathcal{L}$ ($maximization$; $\vec{x} \in [0, 1]^n$):

$$\mathcal{L}(\vec{x}) = \prod_{i=1}^{n} \sin^k(l_1 \pi x_i + l_2) \cdot \exp\left(-l_3\left(\frac{x_i - l_4}{l_5}\right)^2\right)$$

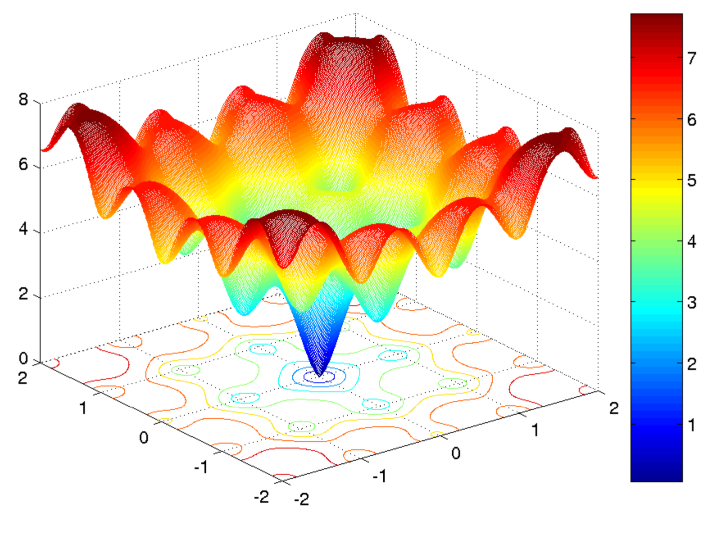
3. Ackley's function ($minimization$; $\vec{x} \in [-10, 10]^n$):

$$\mathcal{A}(\vec{x}) = -c_1 \cdot \exp\left(-c_2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}\right) - \exp\left(\frac{1}{n}\sum_{i=1}^{n} cos(c_3 x_i)\right) + c_1 + e$$

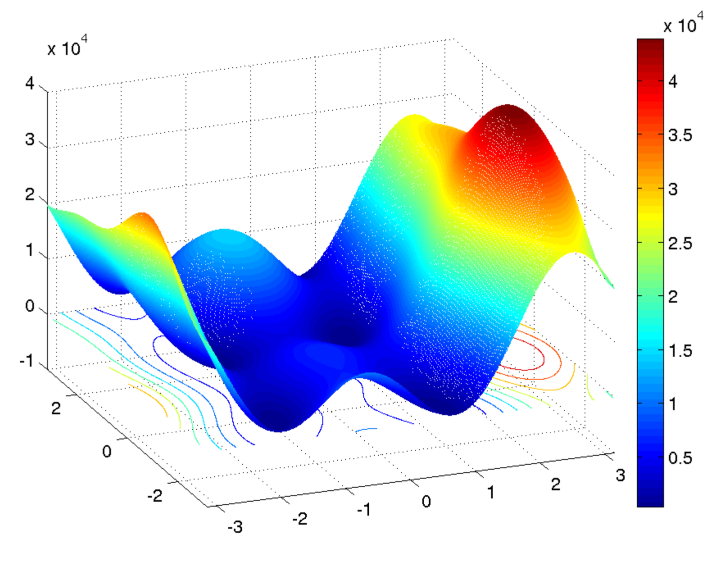
4. The function after Fletcher and Powell (minimization with $\vec{x} \in [-\pi, \pi]^n$):

$$\mathcal{F}(\vec{x}) = \sum_{i=1}^{n}(A_i - B_i)^2$$
$$A_i = \sum_{j=1}^{n}(a_{ij} \cdot sin(\alpha_j) + b_{ij} \cdot cos(\alpha_j))$$
$$B_i = \sum_{j=1}^{n}(a_{ij} \cdot sin(x_j) + b_{ij} \cdot cos(x_j))$$

where a_{ij}, b_{ij}, and α_j are random elements.



Ackley's function $n = 2$



The function after Fletcher and Powell $n = 2$

Performance Criteria

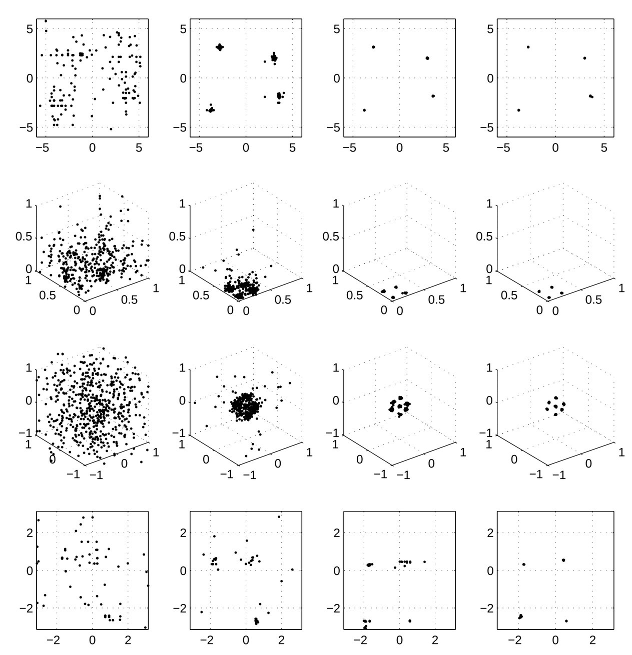
We consider the *maximum peak ratio statistic* [3] as our niching performance criterion. Given the fitness of the optima in the final population $\left\{\tilde{f}_i\right\}_{i=1}^{q}$, and the actual optima of the objective function $\left\{\hat{\mathcal{F}}_i\right\}_{i=1}^{q}$, the *maximum peak ratio* is defined for a *maximization problem* as follows:

$$MPR = \frac{\sum_{i=1}^{q}\tilde{f}_i}{\sum_{i=1}^{q}\hat{\mathcal{F}}_i}$$

Given a minimization problem, the MPR is defined as the actual optima divided by the obtained optima.

Experimental Results

The results refer to an average over 10 runs on each test function. All simulations were run up to an upper bound of 10,000 generations. Three measures are introduced in the table for each test case (mean values): the MPR, the global optimum location percentage, and the number of optima found.



Snapshot gallery with interval of 5 generations: runs for $\mathcal{H}$ (1st), $\mathcal{L}$ (2nd), $\mathcal{A}$ (3rd), and $\mathcal{F}$ (4th)

Function	M.P.R	Global	Optima/q
$\mathcal{H}$	1	100%	4/4
$\mathcal{L}: n = 1$	1	100%	5/5
$\mathcal{L}: n = 2$	1	100%	5/5
$\mathcal{L}: n = 3$	1	100%	7/7
$\mathcal{L}: n = 4$	0.9974	100%	5/5
$\mathcal{L}: n = 10$	0.8612	80%	7.2/11
$\mathcal{A}: n = 2$	1	100%	5/5
$\mathcal{A}: n = 3$	1	100%	7/7
$\mathcal{A}: n = 20$	0.9999	100%	41/41
$\mathcal{A}: n = 30$	0.9681	100%	61/61
$\mathcal{A}: n = 40$	0.9940	100%	81/81
$\mathcal{F}: n = 2$	1	100%	4/4
$\mathcal{F}: n = 4$	0.9321	100%	3.4/4
$\mathcal{F}: n = 10$	0.9141	70%	2.8/4

Final Results

Conclusion

The experimental results show clearly that our method has achieved its goal of locating multiple solutions for the given optimization problems. The algorithm performed well on all test functions, where the excellent results for the high-dimensional Ackley test-cases should be emphasized.

Acknowledgments

This work is part of the research programme of the 'Stichting voor Fundamenteel Onderzoek de Materie (FOM)', financially supported by the 'Nederlandse Organisatie voor Wetenschappelijk Onderzoek (NWO)'.

References

[1] Th. Bäck. *Evolutionary algorithms in theory and practice.* Oxford University Press, New York, 1996.

[2] S. W. Mahfoud. *Niching Methods for Genetic Algorithms.* PhD thesis, University of Illinois at Urbana Champaign, 1995.

[3] B. L. Miller and M. J. Shaw. Genetic algorithms with dynamic niche sharing for multimodal function optimization. In *Proceedings of the 1996 IEEE International Conference on Evolutionary Computation (ICEC'96)*, New York, NY, USA, 1996.