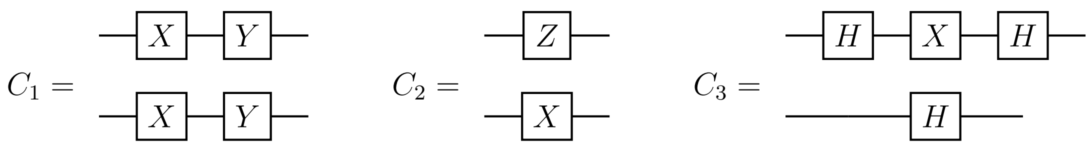
Outline

This assignment consists of 9 questions, each marked out of 1. It constitutes 50% of your total assignment mark (25% of the overall mark for this course).

The deadline the assignment is the beginning of the lecture 20th of March 2020.

1. Consider the following two circuits:



   (a) Express C_1 and C_2 using standard linear-algebraic notation (i.e., using X, Y, Z, H to denote the operators, tensor products $\otimes$ and standard matrix products). (b) Simplify the expression $C_1 - C_2 + \sqrt{2}C_3$ using the rules of tensor algebra, and the relations between Pauli operators and the Hadamard operator– note: it simplifies *a lot*.

2. (a) Draw the circuit diagram for the quantum Fourier transform (QFT) over $n-$qubits. You can use the gate set which includes the controlled-Z rotations by any angle. Don't forget swaps! (b) what is the gate-complexity of QFT with respect to the gate set which includes the Hadamard, swap and controlled-Z rotations. (c) what is the depth complexity of the QFT algorithm (relative to the drawing you provided)?

3. Define the input and output of the (t-ancilla) quantum phase estimation algorithm.

4. (a) Draw the circuit for quantum phase estimation (QPE) for the unitary U with $t-$ancilla qubits (in the eigenvalue-carrying register), assuming access to the unitary ctrl-U. You can draw the quantum Fourier transform as single gate. (b) What is the gate-complexity of the quantum phase estimation algorithm (where you include ctrl-U in the gate set), including the cost of QFT? (c) What is the depth-complexity of this circuit (make explicit which part comes from the QFT)?

5. (a) Draw the circuit for the single-qubit quantum phase estimation (QPE) for the time evolution e^{ikHt}, assuming access to the controlled time evolution e^{ikHt} as a black box. (b) What is the gate-complexity of the quantum phase estimation algorithm? (c) What is the depth complexity of the quantum part of the algorithm?

6. Give the probability of measuring 0 on the ancilla qubit for single ancilla QPE, assuming the system register is prepared in the state $\sum_j a_j|\lambda_j\rangle$ (in the eigenbasis of the Hamiltonian)?

7. Draw a circuit to apply $e^{i\frac{\pi}{4}X_0\otimes X_1}$ to a two-qubit system.

8. (a) Write the first-order Suzuki-Trotter approximation for

$$e^{ih_0(X_0X_1+X_1X_2)+ih_1(Z_0+Z_1+Z_2)}, \tag{1}$$

and (b) calculate the error in the case of the first-order Trotter approximation.

9. Draw the circuit which implements the Trotterized unitary from Question 8.