PROBLEM 1. 11 POINTS

(a) $H = \frac{1}{2} |+\rangle\langle+| - \frac{3}{2} |-\rangle\langle-|$

OR : $H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1/2 & 0 \\ 0 & -3/2 \end{pmatrix} \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

(b) let $|\psi_j\rangle$ be λ_i ev. of H...

$$U|\psi_j\rangle = \left(\sum_{k=0}^{\infty} \frac{(2\pi i H)^k}{k!} \right) |\psi\rangle = \left(\sum_k \frac{2\pi i}{k!} (\lambda_j)^k \right) |\psi_j\rangle$$

$$= \exp(2\pi i \lambda_j) \; |\psi_j\rangle$$

$$\Rightarrow \quad \exp\left(\frac{1}{2} 2\pi i\right) = -1$$

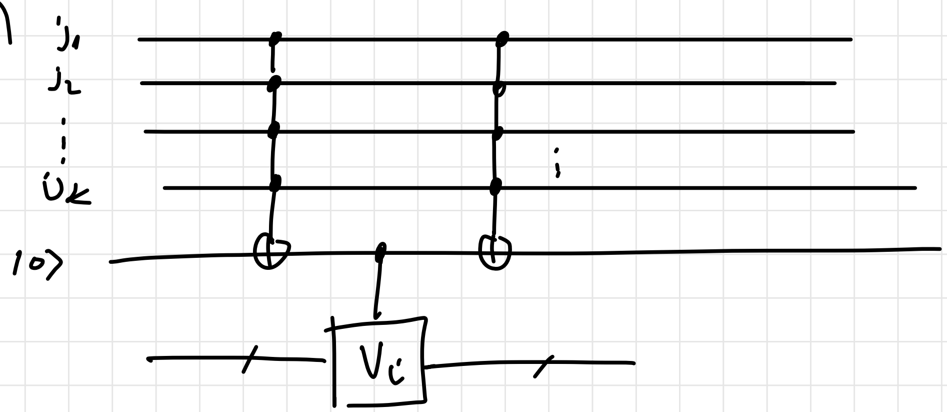
$$\exp\left(-\frac{3}{2} 2\pi i\right) = -1$$

(c) $U = -1 \, |+\rangle\langle+| - 1 \, |-\rangle\langle-| = -1 \, \mathbb{1} = \underline{-\mathbb{1}}$

(d) $UU^\dagger = \mathbb{1}$; $(-\mathbb{1})(-\mathbb{1}) = \underline{\mathbb{1}}$ ✓

PROBLEM 2 12 points

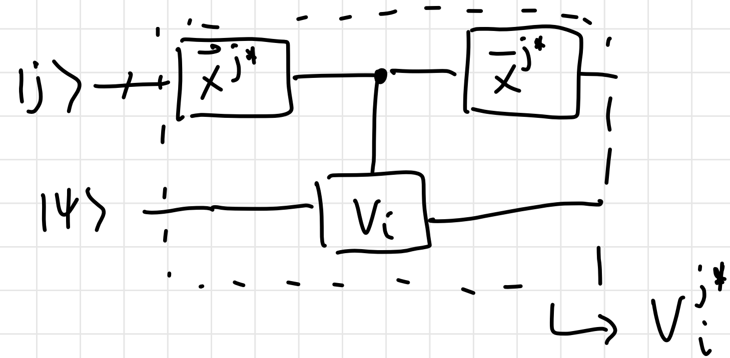
(a)



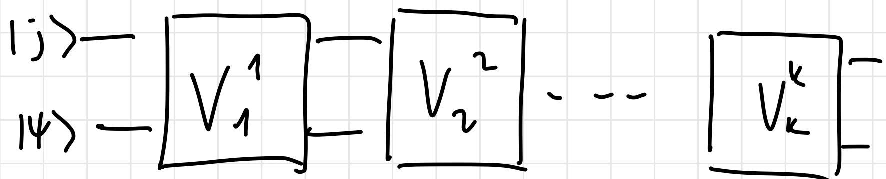
(b) Def. $\vec{X}^{j^*} = \bigotimes_{l=1}^{k} X^{1-j^*_l}$ nb. $X^0 = \mathbb{1}$

$X^1 = X$

$\Rightarrow$



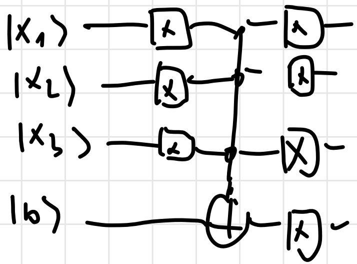
(c)



Note enumeration starts at $\emptyset$ in binary in $\{V_j\}$. or: the set V's is $\{V_0 \dots V_{k-1}\}$.

PROBLEM 3   12 points + 2 bonus

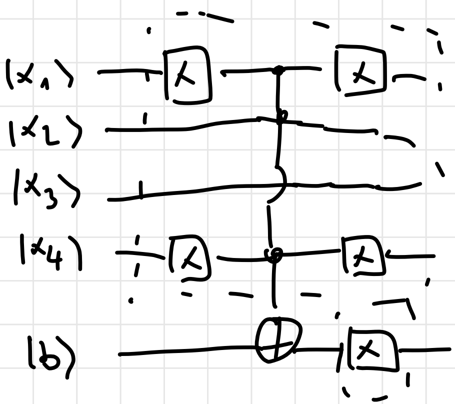
(a)



NB: $(x_1 \vee x_2 \vee x_3)$

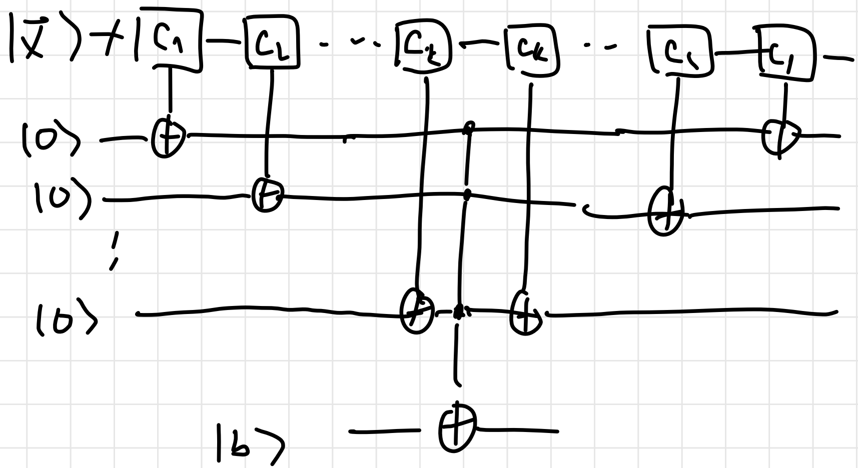
$= \neg(\neg x_1 \wedge \neg x_2 \wedge \neg x_3)$ ..

(b)

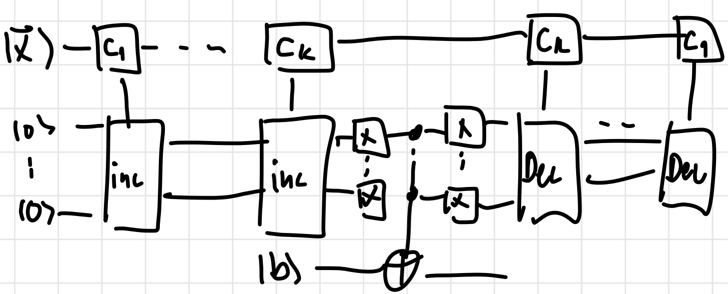


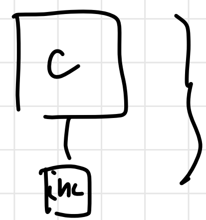
→ Flips $|b\rangle$ if C is satisfied.

(c)

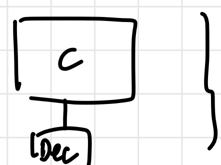


With incrementer





applies incrementer if $C(x) = 0$



applies decrementer $= \boxed{inc^{\dagger}}$ if $C(x) = 0$

PROBLEM 4 12 points

(a)

$$|\varphi_a\rangle = \exp(2\pi \tilde{\theta}_1)|0\rangle = (-1)^{\hat{\theta}_1}|0\rangle$$

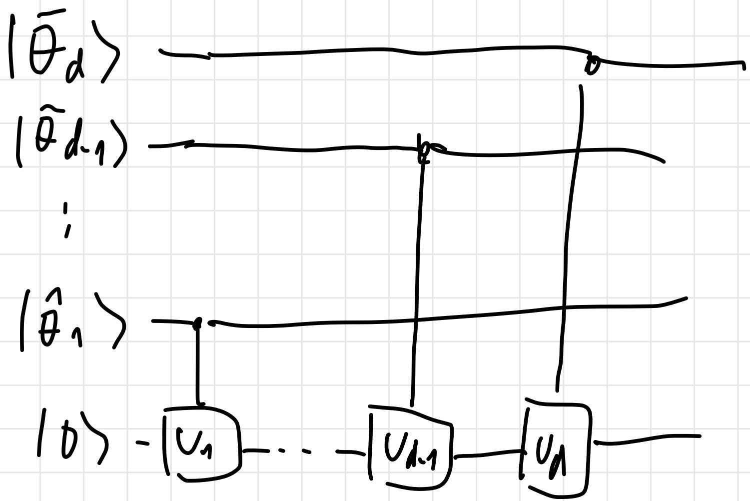
(b) NB:

$$R_x(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

Then $R(\theta)R(\varphi) = R(\theta+\varphi)$.

$$\Rightarrow |\varphi_b\rangle = \cos\left(2\pi\left(\tilde{\theta}_1/2 + \hat{\theta}_2/4\right)\right)|0\rangle + \sin\left(2\pi\left(\frac{\bar{\theta}_1}{2} + \frac{\tilde{\theta}_2}{4}\right)\right)|1\rangle$$

(c)


$|\tilde{\theta}_d\rangle$
$|\tilde{\theta}_{d-1}\rangle$
$|\hat{\theta}_1\rangle$
$|0\rangle$
U_1
U_{d-1}
U_d

PROBLEM 5 13

(a) $\phi_j = \lambda_j$ (note $0 < \underline{\lambda_j < 1}$ $\forall j$)

(b) need to "detect" $\lambda_1 \Rightarrow \log(1/\lambda_1)$ bits
to "hit" first non-trivial most significant
digit].

(c) $|\langle \psi_0 | \varphi \rangle|^2 = |\alpha_0|^2$

Prob = amplitude-norm – square!