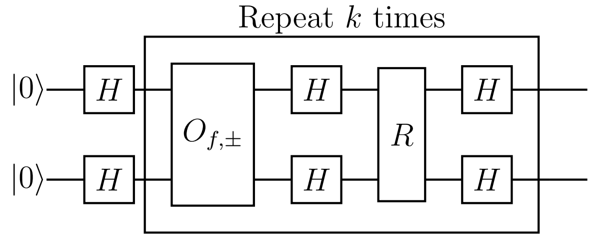
Problem 1. (***Grover with $n = 2$ qubits [Ronald de Wolf's lecture notes, Ch. 7]***). Suppose $f = f(00)f(01)f(10)f(11) = 0001$. Recall that Grover's algorithm is given by



Where R is the unitary that maps $|00\rangle \mapsto |00\rangle$ and $|j\rangle \mapsto -|j\rangle$ for all $j \neq 00$.

1. Give the final state of the above circuit for $k = 1$. What is the success probability?
2. Give the final state of the above circuit for $k = 2$. What is the success probability?

Problem 2. (***Finding all solutions via Grover [Ronald de Wolf's lecture notes, Ch. 7]***). Suppose we have a database with $N = 2^n$ entries with t ones and $N - t$ zeroes. You may assume you know the number t. Show how to use Grover's algorithm to find the positions of *all* t ones, using an expected number of $O(t\sqrt{N})$ queries to the database oracle $O_{f,\pm}$.

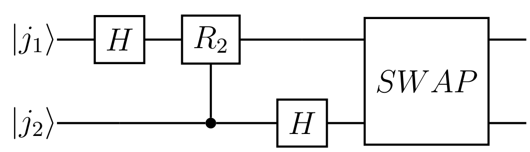
Remark. *You may argue on a high level, no need to draw actual quantum circuits.*

Problem 3. (***The Quantum Fourier transform on 2 qubits***).

1. Confirm the Kronecker product formula for $n = 2$ qubits, i.e., confirm that

$$F_4 |j_1 j_2\rangle = \frac{1}{\sqrt{2}} \left(|0\rangle + e^{2\pi i \cdot \mathrm{dec}(j)/2} |1\rangle\right) \otimes \frac{1}{\sqrt{2}} \left(|0\rangle + e^{2\pi i \cdot \mathrm{dec}(j)/2^2} |1\rangle\right)$$

2. Confirm that the following circuit implements the QFT on $n = 2$ qubits



where $R_2 = \begin{pmatrix} 1 & 0 \\ 0 & e^{2\pi i/4} \end{pmatrix}$ and $SWAP(|\phi\rangle \otimes |\psi\rangle) = |\psi\rangle \otimes |\phi\rangle$.

Problem 4. (***FT of periodic vector [Ronald de Wolf's lecture notes, Ch. 4]***). Suppose $a \in \mathbb{R}^N$ is an r-periodic vector with r dividing N, i.e.,

$$a_k = \begin{cases} 1 \text{ if } k \mod r = 0, \\ 0 \text{ otherwise.} \end{cases}$$

Compute the Fourier transform $F_N a$, i.e., write down a formula for the entries of the vector $F_N a$. Assuming also $r \ll N$, what are the entries with largest magnitude in the vector $F_N a$?