Problem 1. (*Complex numbers*). Consider the following two complex numbers

$$x = 1 + i \ \& \ y = \frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}.$$

1. What are $\mathrm{Re}(x)$, $\mathrm{Re}(y)$, $\mathrm{Im}(x)$ and $\mathrm{Im}(y)$? Draw both x and y in the complex plane.
2. Compute $x + y$ and xy.
3. Compute $|x|$ and $|y|$.
4. Rewrite x, y, $x + y$ and xy in polar coordinates, that is, for each of those four complex numbers z find the $r > 0$ and $\varphi \in [0, 2\pi)$ such that $z = r^{i\varphi}$.

Problem 2. (*Matrices and vectors*). Consider the following two complex vectors

$$v = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \ \& \ w = \frac{1}{\sqrt{2}} \begin{pmatrix} -i \\ 1 \end{pmatrix}.$$

Furthermore, consider the complex matrices

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \ \& \ U = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}.$$

1. Compute $x = v - i\sqrt{2}w$.
2. Compute Hv, Hw and Hx. What is the relationship between these three vectors?
3. Compute Uv, Uv and Ux. What is the relationship between these three vectors?
4. Compute $\langle v, w\rangle$, $\langle v, x\rangle$ and $\langle w, x\rangle$. Which of the vectors v, w and x are orthogonal?

   Remark. *When considering complex vectors the inner product is given by*

   $$\langle v, w\rangle = \sum_{j=0}^{n-1} \overline{v_j} w_j, \ \text{where for } v_j = a + bi \text{ we have } \overline{v_j} = a - bi.$$

5. Compute $||v||, ||w||$ and $||x||$. Which of the vectors v, w and x are orthonormal?
6. Compute HU, UH and $H \otimes U$. On what spaces do they operate, i.e., what kind of vectors can we plug-in to multiply with?
7. Is H and/or U Hermitian? Explain.

Problem 3. (*Complexity theory*). Draw a Venn diagram of how you think the inclusions are among the complexity classes `P`, `EXP`, `NP`, `BPP` and `BQP`. Also, point out where you think the problem of `factoring` lies.

Remark. *Some of the inclusions among the above complexity classes are still open problems. For example, the famous* `P` $\neq$ `NP` *conjecture is still unproven and we also don't know whether* `BPP` $\subsetneq$ `BQP` *or* `BPP` $=$ `BQP`*. See the next page for a picture of the most widely conjectured Venn diagram.*

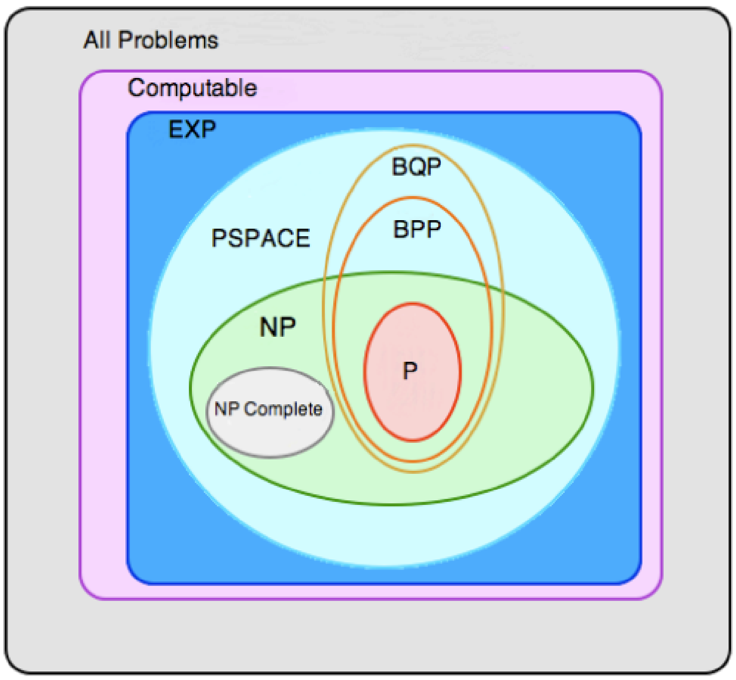

Figure 1: taken from `arXiv:1108.0560`