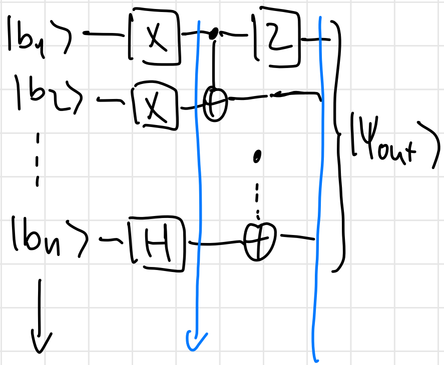
GENERAL CIRCUITS



$|\psi_0^{(n)}\rangle \qquad |\psi_1\rangle \cdots |\psi_\ell\rangle = |\psi_{out}\rangle$

$$\rightsquigarrow \quad |\psi_1^{(n)}\rangle \not- \boxed{U} \not- |\psi_{out}\rangle$$

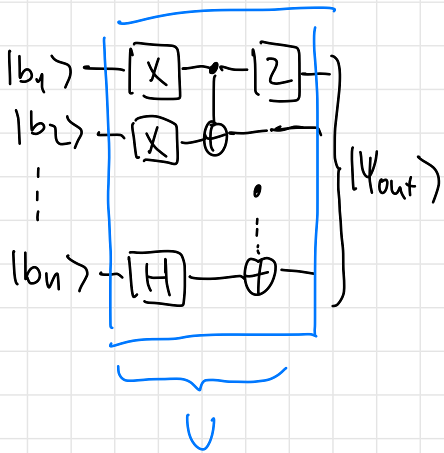
M-M product

$$|\psi\rangle - \boxed{A} - \boxed{B} - \boxed{C} - |\psi'\rangle \quad \Rightarrow \quad |\psi'\rangle = \overbrace{CB}A|\psi\rangle$$

$$\begin{array}{l} |\psi_1\rangle - \boxed{A} - |\psi_1'\rangle \\ |\psi_2\rangle - \boxed{B} - |\psi_2'\rangle \end{array} \quad \Rightarrow \quad |\psi_1'\rangle \otimes |\psi_2'\rangle = \left(A|\psi_1\rangle\right) \otimes \left(B|\psi_2\rangle\right)$$

$$= (A \otimes B)\, |\psi_1\rangle|\psi_2\rangle$$

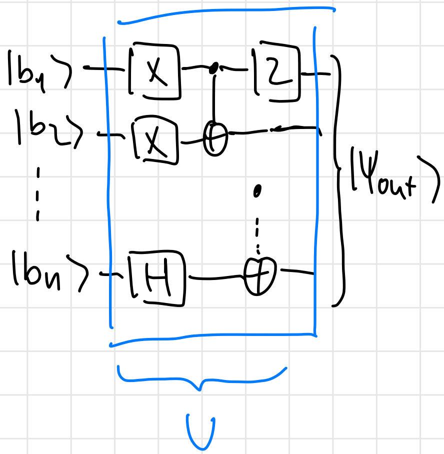
NOTE



$$U \in \mathbb{C}^{2^n \times 2^n}$$

CIRCUIT $\Rightarrow$ EXPONENTIALLY-SIZED MATRIX!

NOTE



$$U \in \mathbb{C}^{2^n \times 2^n}$$

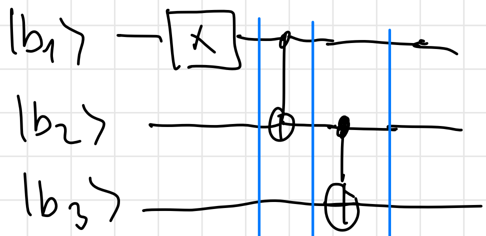
$\boxed{\text{CIRCUIT} \Rightarrow \text{EXPONENTIALLY-SIZED MATRIX!}}$

FOR SPECIAL INPUTS, AND CIRCUITS

CAN EVALUATE OUTPUT WITHOUT

COMPUTING MATRIX ("EFFICIENT SIMULATION")

EXAMPLE: "CLASSICAL CIRCUIT ON CLASSICAL INPUT"



$\neg b_1 , b_2 , b_3$

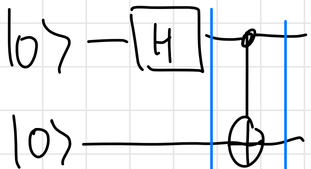
$\neg b_1 , \neg b_1 \oplus b_2 , b_3$

$\neg b_1 , \neg b_1 \oplus b_2 , \neg b_1 \oplus b_2 \oplus b_3$

COMPLETE SPECIFICATION...

LOGICAL OPERATIONS..

From bitstrings to interesting superpositions...



$$|00\rangle \rightarrow \frac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big)|0\rangle = \frac{1}{\sqrt{2}}\big(|00\rangle + |10\rangle\big)$$

$$\rightsquigarrow \frac{1}{\sqrt{2}}\big(\mathrm{CNOT}|00\rangle + \mathrm{CNOT}|10\rangle\big) = \underline{\underline{\frac{1}{\sqrt{2}}\big(|00\rangle + |11\rangle\big)}}$$

MATICES!

QUANTUM V. CLASSICAL UNIVERSALITY

CLASSICAL	QUANTUM
• ALL FUNCTIONS $f:\{0,1\}^n \to \{0,1\}^m$	• ALL "n-qubit unitaries" := unitary $\mathbb{C}^{2^n \times 2^n}$ [$SU(2^n)$]
• AND, OR, NOT, FANOUT ALSO: NAND + FANOUT	• $\boxed{\text{ALL 1-qubit unitaries + CNOT}}$ EXACT UNIVERSALITY
• PROBABILISTIC = DETERMINISTIC + $\underline{\text{RAND}}$	H + $\pi/8$ ARE APPROXIMATELY UNIVERSAL FOR $SU(2)$ CNOT + H + $\pi/8$ ARE APPROXIMATELY UNIVERSAL FOR $SU(2^n)$

BONUS: SOLOVAY-KITAEV TH.

"APPROXIMATION IS EFFICIENT"

$$\text{COST APPROXIMATE} = \text{COST EXACT} \times O(\mathrm{polylog}(1/\varepsilon))$$

NEXT LESSON: Chapter 1.4 OF NIELSEN & Chuang

de Wolf notes: Chapter 2