Quantum Approximate Optimization Algorithms

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Expectations Comes with limitations

What we expect from quantum computing :

- Solve intractable problems (e.g. chemistry)
- Having all possible bitstrings using superposition is attractive for Combinatorial Optimization
- Native parallelism as well
- Speed up algorithms (Grover's quadratic speedup)
- Find new ones with advantages
- Maybe more...

But limitations :

- Cannot copy qubits
- Computation must be unitary (hence reversible)
- Instability (noisy computers)
- Hard to design algorithms

But it is still the beginning.

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Hybrid quantum-classical Algorithms

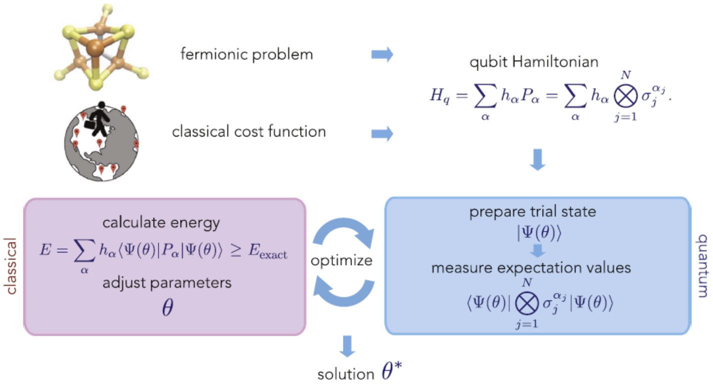

Figure 3. Variational quantum eigensolver method. The trial states, which depend on a few classical parameters $\boldsymbol{\theta}$, are created on the quantum device and used for measuring the expectation values needed. These are combined on a classical computer to calculate the energy $E_q(\boldsymbol{\theta})$, i.e. the cost function, and find new parameters $\boldsymbol{\theta}$ to minimize it. The new $\boldsymbol{\theta}$ parameters are then fed back into the algorithm. The parameters $\boldsymbol{\theta}^*$ of the solution are obtained when the minimal energy is reached.

Source: arXiv:1710.01022

Examples of trial states

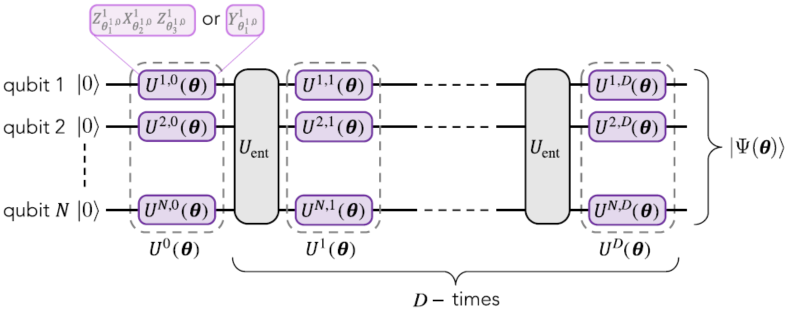

Figure 4. Heuristic preparation of trial states for the variational quantum eigensolver based on single-qubit gates $U(\boldsymbol{\theta})$ interleaved by entangling operations U_{ent} as described in the text.

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Quantum Approximate Optimization Algorithm

QAOA* is a hybrid quantum-classical algorithm introduced in the context of Combinatorial optimization. ϵ-approximation algorithms give lower bound guarantees of ϵ times the optimum on returned solutions. There exist ϵ that are NP-hard to achieve.

- Seek to maximize :

$$C : \{-1;1\}^n \rightarrow \mathbb{R}; C(z) = \sum_{a=1}^{m} C_a(z), C_{max} = \max_{z} C(z)$$

*Farhi et al.: arXiv:1411.4028

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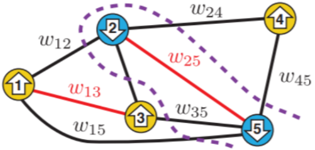
- Example of Maxcut :

$$\sum_{<ij>\in E} w_{ij}(1 - x_i x_j)/2$$

(b)



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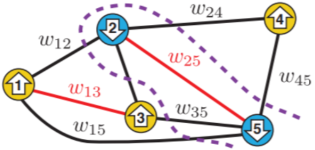

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- Best classical algorithm (Goemans-Williamson semi-definite programming) guarantees an expected ratio of 0.878.

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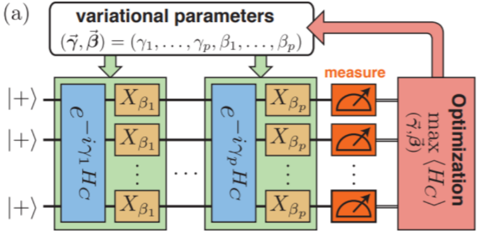
Quantum Approximate Optimization Algorithm on Maxcut

Algorithm :
Requires an integer p, a cost hamiltonian H_C and a so-called mixer hamiltonian H_B:

$$H_C = \sum_{<ij>\in E} w_{ij}(I - \sigma_i^z \sigma_j^z)/2$$

$$H_B = \sum_{i=1}^{n} \sigma_i^X$$



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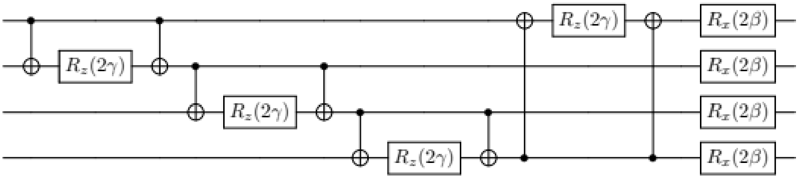
QAOA circuit p=1 and properties

Circuit example p=1 QAOA :



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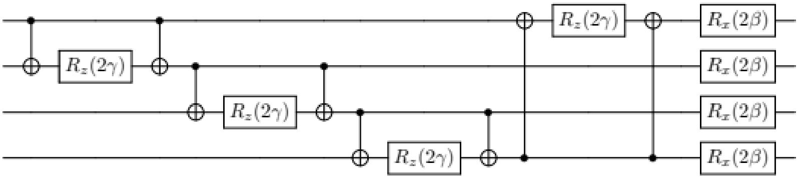

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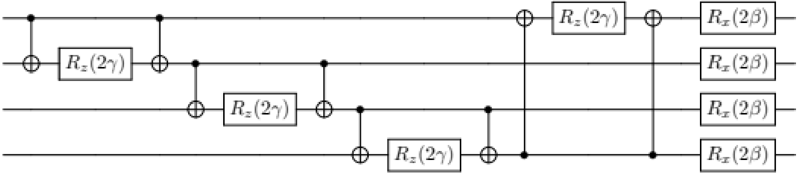

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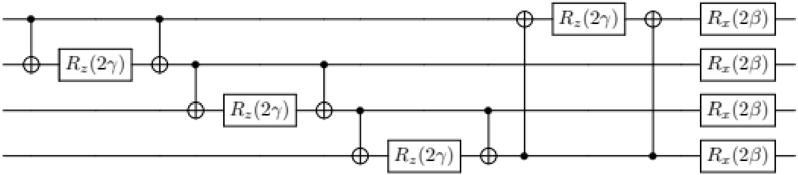

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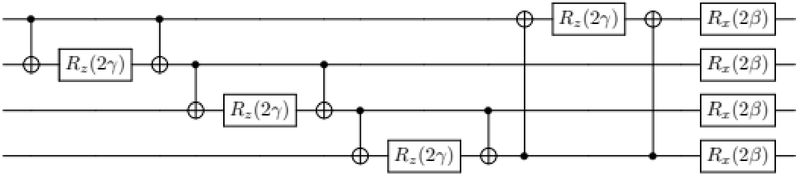

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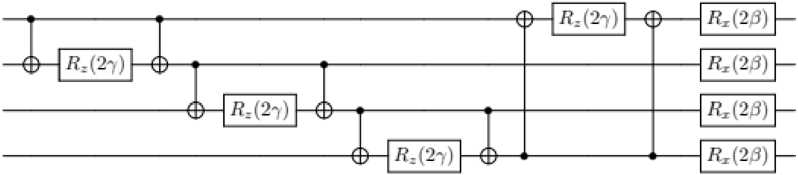

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- Computing optima for the circuit optimization (doubly) exponentially expensive in p
- Efficient classical sampling from arbitrary shallow circuits would cause unlikely collapses in hierarchy of complexity theory
- True for QAOA circuit even at depth 1 (arXiv:1602.07674).

Expectation values on Paulis Hamiltonian

Given a quantum circuit output a quantum state $\psi_\theta\rangle$, evaluate:

$$\langle\psi_\theta|H_C|\psi_\theta\rangle = -\sum w_{ij}\langle\psi_\theta|\sigma_i^z\sigma_j^z|\psi_\theta\rangle$$

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4. Divide expectation by number of samples.
5. If you have σ_i^X, apply before measuring $R_Y(-\pi/2)$ on i-th qubit. For σ_i^Y, $R_X(\pi/2)$.

QAOA problematics

Examples of types of research lines in the QAOA domain:

- Performance of QAOA as an approximate algorithm ?

Challenges given current hardware constraints:

And certainly many more...

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- Hybrid method: connections to divide & quantum methods.

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Different modifications for different use cases

Other versions of QAOA were introduced for :

- **More suitable optimization objective * ,**
  * Li L et al. : "Quantum Optimization with a Novel Gibbs Objective Function and Ansatz Architecture Search", 2019; [http://arxiv.org/abs/1909.07621 arXiv:1909.07621].
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- **Machine Learning** : Clustering, Botzmann machines...

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Current projects

PhD on Quantum Optimization and Quantum Machine Learning for industrial applications

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  Characterizing QAOA advantage against GW by using interpretable Machine Learning

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- **Optimizers for variational quantum eigensolvers** : Determinating power of classical optimizers by benchmarking.

Thank you!