ENS DE LYON

TD de Sémantique et Vérification

VIII– LTL, NBAs and Fixed Points

Tuesday 26th March 2019

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In this set of exercises, we will discuss properties of generalised NBAs and of LTL formulas.

Recommendation: The exercises are all purely pen and paper exercises. However, it is quite fun to implement notions from the course and the exercises. At the very end, you may obtain this way your very own model checker. This week, you may implement the translation of LTL formulas into generalised non-deterministic Büchi automata. Note that your implementation will not be evaluated as part of the course.

Notation

We denote the words characterised by an LTL formula φ by $[\![\varphi]\!] = \{\sigma \mid \sigma \models \varphi\}$.

From LTL to NBAs

Exercise 1.

Let φ be a the LTL formula over $AP = \{a, b, c\}$ given by $\varphi = \Box a \wedge (b \,\mathcal{U}\, \neg c)$. Construct an (G)NBA $\mathcal{A}$, such that, $\mathcal{L}_\omega(\mathcal{A}) = [\![\varphi]\!]$.

Order-Theoretic Fixed Points

Exercise 2.

Recall that $(L, \subseteq)$ with $L = \mathcal{P}(\mathcal{P}(AP)^\omega)$ is a complete lattice. We define the derivative of $\sigma \in \mathcal{P}(AP)^\omega$ by $\sigma' = \sigma_1\sigma_2\cdots$, and for $P \in L$ by $N(P) = \{\sigma \mid \sigma' \in P\}$.

1. Let φ and ψ be LTL formulas and define $f : L \to L$ by

$$f(P) = [\![\psi]\!] \cup ([\![\varphi]\!] \cap N(P)).$$

   Show that $[\![\varphi \,\mathcal{W}\, \psi]\!]$ is the greatest fixed point of f.

2. Let φ be an LTL formula and define $f : L \to L$ by

$$f(P) = [\![\varphi]\!] \cap N(P).$$

   Show that $[\![\Box\varphi]\!]$ is the greatest fixed point of f.