

# Mixed Inductive-Coinductive Reasoning

## Types, Programs and Logic

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PhD Defence  
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Radboud University Nijmegen

# Motivation

**Well-behaved Programs and Proof Methods**

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## Well-behaved Programs and Proof Methods



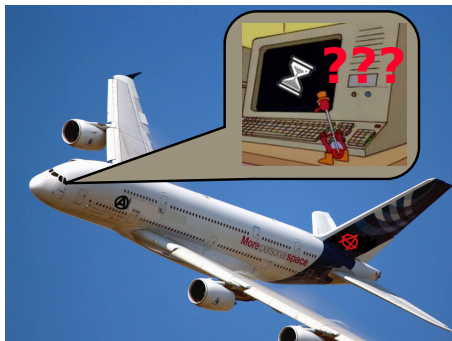
**Crashed Control System**

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## Well-behaved Programs and Proof Methods



**Crashed Control System**



**Non-responsive Control System**

# What are Induction and Coinduction?

Coinduction: Observable systems

**Alive control systems**

Induction: Terminating computations

**Internal computations finish**

Induction-Coinduction: Interleaved control and computations

**Alive and responsive control systems**

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# Aims of the Thesis

Develop **formal languages** for inductive-coinductive programming and reasoning ...

that enable **automatic verification** of proofs, ...

have **formal semantics**, and ...

are easy to understand **for humans**.

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# Approach in the Thesis

## **Category Theory**

Abstraction of Mathematical Theories

## **Type Theory**

Typed Computations and  
Constructive Logic

## **Coalgebraic Methods**

Theory of Systems

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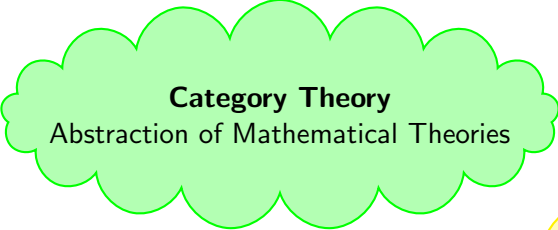
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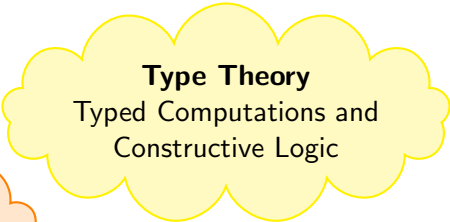
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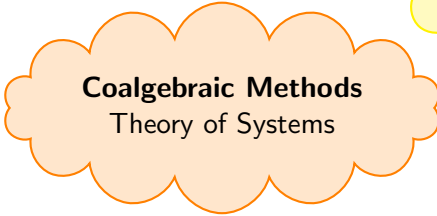
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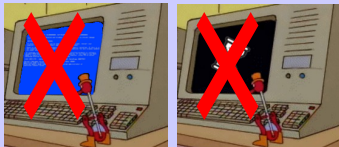


## **Coalgebraic Methods**

Theory of Systems

# Results of the Thesis

## Well-behaved inductive-coinductive programs



## Equivalence of inductive-coinductive programs

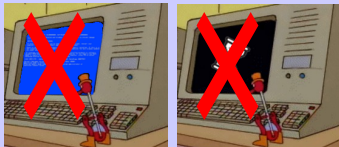
- Coalgebraic description of the equivalence
- Coalgebraic up-to techniques to simplify proofs
- Recursive logic to enable computer-verifiable proofs

## Constructive logic based on induction and coinduction

- Inspired by category theoretical principles
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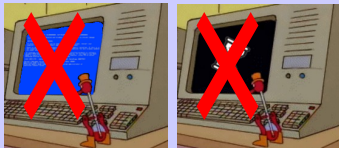
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Thank you and back to the committee.