ENS DE LYON

TD de Sémantique et Vérification

VI– Büchi Automata Regular Properties

Tuesday 5th March 2019

Henning Basold
henning.basold@ens-lyon.fr

In this set of exercises, we will discuss properties of ω-regular expressions and Büchi automata.

Recommendation: The exercises are all purely pen and paper exercises. However, it is quite fun to implement notions from the course and the exercises. At the very end, you may obtain this way your very own model checker. This week, you may implement non-deterministic Büchi automata, the union-, ω-, and concatenation-constructions for NBAs and the check for emptiness of the accepted language of an NBA. Note that your implementation will not be evaluated as part of the course.

ω-Regular Properties

Given $U \subseteq \Sigma^*$ and $A \subseteq \Sigma^\omega$, recall that

- $U \cdot A = \{\hat{\sigma} \cdot \sigma \in \Sigma^\omega \mid \hat{\sigma} \in U \text{ and } \sigma \in A\}$
- $U^\omega = \{\sigma \in \Sigma^\omega \mid \exists (u_k \in U)_{k \in \mathbb{N}}. \sigma = u_0 \cdot u_1 \cdot u_2 \cdots\}$

Exercise 1.

Let $U \subseteq \Sigma^*$ and $A, B \subseteq \Sigma^\omega$.

1. Show that $\text{pref}(A \cup B) = \text{pref}(A) \cup \text{pref}(B)$.
2. Show that $\text{pref}(U \cdot A) = \text{pref}(U) \ \cup \ U \cdot \text{pref}(A)$.
3. Show that $\text{pref}(U^\omega) = \text{pref}(U^*)$.

Exercise 2.

1. Show that if $A \subseteq \Sigma^\omega$ is ω-regular, then $\text{pref}(A) \subseteq \Sigma^*$ is regular. (You may use that $\text{pref}(U)$ is regular if $U \subseteq \Sigma^*$ is regular.)
2. Deduce that if $P \subseteq \mathcal{P}(\text{AP})^\omega$ is an ω-regular safety property, then P is a *regular* safety property.

Büchi Automata

Exercise 3.

1. Let $\text{AP} = \{a, b\}$. Give an non-deterministic Büchi automaton (NBA) that accepts "b holds for a finite time until a holds forever and b never holds again". You may use propositional formulas as labels.
2. Depict an NBA for the language described by the ω-regular expression $(AB+C)^*((AA+B)C)^\omega + (A^*C)^\omega$.

Constructions on Büchi Automata

Exercise 4.

Let $\mathcal{A}_1$ and $\mathcal{A}_2$ be Büchi automata, and $\mathcal{A}$ an NFA.

1. Show that there is a Büchi automaton $\mathcal{A}_1 + \mathcal{A}_2$ with $\mathcal{L}_\omega(\mathcal{A}_1 + \mathcal{A}_2) = \mathcal{L}_\omega(\mathcal{A}_1) \cup \mathcal{L}_\omega(\mathcal{A}_2)$.
2. Show that there is a Büchi automaton $\mathcal{A} \odot \mathcal{A}_2$ with $\mathcal{L}_\omega(\mathcal{A} \odot \mathcal{A}_1) = \mathcal{L}(\mathcal{A}) \cdot \mathcal{L}(\mathcal{A}_1)$.