Introduction to Logic, Spring 2020

Course Technicalities and Introduction

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Part I

Course Technicalities

General Information

- Main source of information is the website

  `https://liacs.leidenuniv.nl/~basoldh/education/logic2020.html`
- Also linked in studiegids. Search for `4031ILOGI`
- If you have questions that are not answered during the lecture, on the website or on the exercise sheet, then you always send an email to

  `itl20@liacs.leidenuniv.nl`

  (itl = introduction to logic)
- We will use Blackboard for assignments, grades and announcements
  - Please enrol during the break
  - Search for course ID `4031ILOGI-1920FWN`

Schedule

- Details on website
- 13 weeks
- 12 lectures with content and exercise classes
- 13th lecture will (most likely) be given by prof. Holger Hoos
  - Monday 25 May, 14:15
  - Content: Implementing and Comparing SAT-solvers
  - Not part of exam, but interesting! 
- 13th exercise class will be last lecture
  - Monday 25 May, 16:00
  - Will be in Sitterzaal! (here)
  - Summary and question session

Lectures

- Monday block 6 and 7 (14:15), but you know that since you're here!
- Topic: Introduction to formal logic and how it fits to computer science
- Goals:
  - Give you an idea of what logic is
  - Enable you to study logic autonomously
- Lecture style
  - On blackboard (yes, old-fashioned!)
  - This means that there are no slides etc.
  - If you miss a lecture, ask your friendly colleagues for their notes
  - I encourage you to participate and ask questions during the lectures!
- A full list of reading material is given on the website
  - Michael R. A. Huth and Mark D. Ryan. `Logic in Computer Science: Modelling and Reasoning about Systems`.
  - Open Logic Project (`openlogicproject.org`)
  - Johan van Benthem et al. `Logica voor informatica`

Exercise Classes and Homework

- Exercise class in Snellius 401, 402, 403, 405 and 412
- Monday blocks after the lecture (16:15)
- I will distribute groups on Blackboard during the break (Please enrol!)
- Weekly assignments
  - Can be worked on during the exercise class
  - Class is supervised
  - Use the chance to work there and ask questions!
  - Each assignment consists of exercises for practising and homework
- Homework
  - The homework is mandatory ...
  - ...in the sense that it makes 30% of the final grade
  - Deadline is always Friday Afternoon
  - Submission via Blackboard
  - Only pdfs generated with LaTeX, XeLaTeXetc. are accepted
  - If you don't have a computer with working LaTeX, use Overleaf (`https://www.overleaf.com/`)
  - Detailed and binding instructions are on exercise sheets and website

Exercise Class Rooms

Instructor	Room
Lau Bannenberg	SN 407-409
Tomke Meyer	SN 402
Tim Poot	SN 403
Nick Radunović	SN 405
Levi Vos	SN 412

Exam and Grade

- Final exam on Monday 15 June, 14:15 retake on Tuesday 07 July, 14:15
- You can bring two hand-written A4 cheat sheets (double-sided)
- I will distribute an sample exam before the question session on 25 May
- Exam should be a walk in the park, if you
  - attend the lectures,
  - do the weekly assignments,
  - hand in weekly the homework, and
  - practise, if necessary, with the sample exam.

Part II

Introduction

What is Logic and Why Should You Care?

A Formal Framework for Reasoning

I would like to argue [...] that without a system of formal constraints there are no creative acts; specifically, in the absence of intrinsic and restrictive properties of mind, there can be only 'shaping of behavior' but no creative acts of self-perfection.
— "Language and Freedom" in "On Anarchism", 2014.

Noam Chomsky [1]

Chomsky Hierarchy

- Modern understanding of formal languages
- Essential to CS (compilers, text search, etc.)

What is Logic?

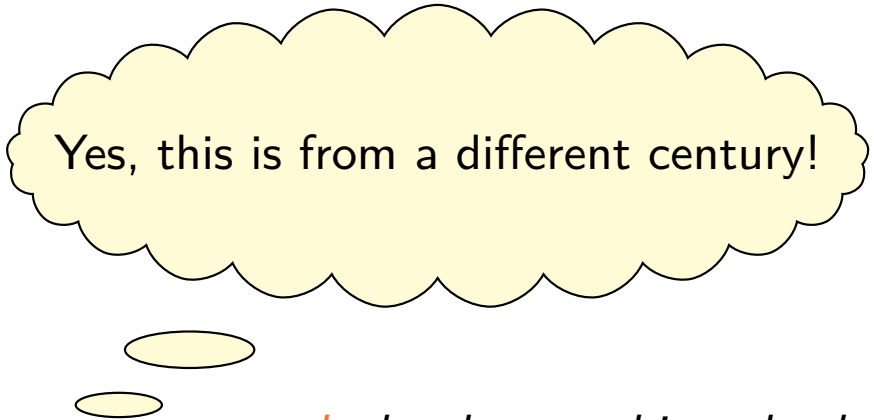
Study of arguments and the form they may take

- Formal languages to express facts and propositions
- Formal proofs to make valid deductions (redeneringen)
- Semantics (betekenis/semantiek) to study where meaning comes from

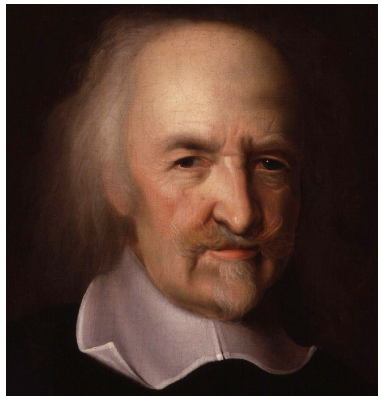
Applications

- Formalise mathematical proofs
- Formalise scientific and philosophical arguments
- Formalise models of computation and correctness of software

Computing would not exist without logic!



When a man reasoneth, he does nothing else but conceive a sum total [...] For as arithmeticians teach to add and subtract in numbers; [...] the logicians teach the same in consequences of words; — Leviathan, *1651.*



Thomas Hobbes [2]

Logic in Computer Science

- Computers do nothing but executing logical operations (very quickly!)
- Semantics of programming languages
- Program correctness
- Automatic deduction
- Computer-verified proofs

Logical Abacus [3]

A Proof Gone Wrong

Ex Nihilo – Creation from Nothing

$$
\begin{aligned}
0 &= 0 + 0 + 0 + \cdots \\
&= (1 - 1) + (1 - 1) + (1 - 1) + \cdots \\
&= (1 + (-1)) + (1 + (-1)) + (1 + (-1)) + \cdots \\
&= 1 + ((-1) + 1) + ((-1) + 1) + \cdots \\
&= 1 + 0 + 0 + \cdots \\
&= 1
\end{aligned}
$$

What Went Wrong?

- We did not adhere to a rigorous definition of convergence
- Without making assumptions and deduction explicit, we can prove anything

Corollary (of Ex Nihilo)

"I will not drink this night!"

Proof.

By induction, using that one beer is no beer 😄 □

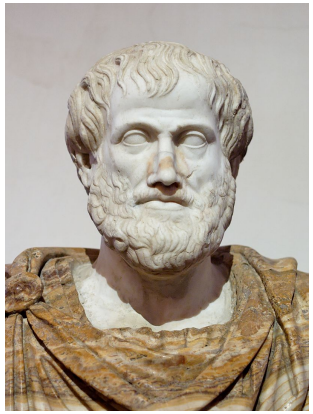
Content of this Course

- Propositional logic — Expressing simple logical relations
- Predicate logic — Expressing facts about data
- Proof systems – Formal deductions
- The essentials of studying logic: syntax, semantics, soundness, completeness, decidability, expressivity
- Focus on computing and constructivism
- Practical relevance of formal logic: SAT/SMT-solving and logic programming
- Limits of logic and reasoning: Gödel's incompleteness theorem

A Bit of Logic History

The Early Days

- Aristotle (384 – 322 BC)
- Euklid (ca. 360 BC)
- Middle ages



Aristotle [4]

Typical Argument

All human are mortal

Socrates is a human

Socrates is mortal

The Birth of Formal Logic

Gottfried Wilhelm Leibniz (1646 – 1716)

- Display relations of objects and concepts in formal language
- Universal language for mathematics (Characteristica universalis)
- Algorithmic deduction (Calculus ratiocinator)

Algebraic Logic

Deductions as calculations.

- George Boole (1815 – 1864)
- Augustus de Morgan (1806 – 1871)
- Charles Sanders Peirce (1839 – 1914)
- Ernst Schröder (1841 – 1902)

Thought Streams

Logicism

All mathematics can be reduced to logic. We only uncover the truth.

- Gottlob Frege (1848 – 1925)
- Betrand Russell (1872 – 1970)
- Alfred North Whitehead (1861 – 1925)
- Guiseppe Peano (1858 – 1932)

Gottlob Frege [5]

Intuitionism

Mathematics only exists as mental activity, not independently.

- Leopold Kronecker (1823 – 1891)
- Luitzen Egbertus Jan Brouwer (1881 – 1966)
- Arend Heyting (1898 – 1980)
- Anne Sjerp Troelstra (1939 – 2019)

Hilbert's Dream

Formalism

All of logic and mathematics is a consequence of string manipulations.

- David Hilbert (1862 – 1943)
- Gerhard Gentzen (1909 – 1945)
- Haskell B. Curry (1900 – 1982)

Hilbert's programme

Axiomatic system to formalise all mathematics and prove this system correct.

- Kurt Gödel (1906 – 1978) showed that this is impossible
- 

Logic and Computing

What does this have to do with computation?

- Ada Lovelace (1815 – 1852): Potential of Babbage's analytical engine
- Thoralf Skolem (1887 – 1963): Primitive recursion
- Alonzo Church (1903 – 1995): λ-calculus
- Alan Turing (1912 – 1954): Turing machine
- Grace Murray Hopper (1906 – 1992): COBOL

Part III

Appendix

References I

References

Σ and Wugapodes. *Noam Chomsky portrait 2017 retouched*. License: CC BY-SA 4.0. 2017. URL: https://commons.wikimedia.org/w/index.php?curid=85616571.

National Portrait Gallery: NPG 106 and Wikimedia (anonymous). *Thomas Hobbes portrait after John Michael Wright*. License: © National Portrait Gallery. 1670. URL: http://www.npg.org.uk/collections/search/portrait/mw03165/Thomas-Hobbes.

Nelson and Shizhao. *William Stanley Jevons Logic Piano*. License: CC BY-SA 2.0. 2006. URL: https://commons.wikimedia.org/w/index.php?curid=3709365.

After Lysippos - Jastrow. *Bust of Aristotle*. License: Public Domain. 2006. URL: https://commons.wikimedia.org/w/index.php?curid=1359807.

References II

Unknown. *Young Gottlob Frege*. License: © Public Domain. 1879. URL: https://commons.wikimedia.org/w/index.php?curid=1932322.