# Introduction to Logic (Spring 2020)

**Assignment 8**

Monday 20th April 2020

**Instructions** This sheet contains two kinds of assignments: exercises and homework. The first are not mandatory and are meant for practising during the exercise class or by yourself. Tutors will be available during the exercise class to help with the assignment. Homework assignments are mandatory, and the combined grade of the homework makes 30% of the final grade. The grade for the homework on this sheet corresponds to the number of points obtained + 1. Maximally 9 points can be obtained.

**Handing in your answers** Submit your solution through **Blackboard** as a single PDF file named `hw8sN.pdf`, where N is your student number. The document has to be created using LaTeX (or variants like XeLaTeX). A template is available on the website of the course and on Blackboard. Please **use the proper logic connectives and proof rules** as they are shown in the template. If you do not have a working LaTeX installation, then you can use Overleaf (`https://www.overleaf.com/`). Make sure that your **name, student number and studies** are **clearly written on the document**. All students have to prepare and submit their own solution. Only submit the 2 exercise(s) marked as **Homework**. Answers have to be provided in **Dutch or English**. Submissions that fail to meet these requirements are not considered.

**Deadline** The homework must be uploaded before **Friday 24th April 2020 2:30pm**.

**Learning Objectives** After completing this assignment, you should be able to **provide models and valuations** for first-order signatures use them to **evaluate the semantics** of first-order formulas. Furthermore, you should be able to determine whether first-order formulas are **tautologies or satisfiable**.

## Exercise 1

Let $\mathcal{L}$ be the signature with $\mathcal{F} = \{\underline{0}, s\}$, $\mathcal{R} = \{I\}$, and the following arities.

$$\mathrm{ar}(\underline{0}) = 0 \qquad \mathrm{ar}(s) = 1 \qquad \mathrm{ar}(I) = 2$$

Moreover, let $\varphi_1$, $\varphi_2$ and $\varphi_3$ be formulas given by

$$\begin{aligned}
\varphi_1 &= \forall x.\, \neg I(s(x), x) \\
\varphi_2 &= \forall x.\, \neg I(s(x), \underline{0}) \\
\varphi_3 &= \forall x.\, \forall y.\, I(s(x), s(y)) \to I(x, y).
\end{aligned}$$

Determine for every formula which of the following models $\mathcal{M}$ satisfies them.

a)

$$|\mathcal{M}| = \mathbb{N}$$
$$\underline{0}^{\mathcal{M}}(*) = 0 \qquad s^{\mathcal{M}}(n) = n + 1$$
$$I^{\mathcal{M}} = \{(n, m) \mid n = m\}$$

b)

$$|\mathcal{M}| = \{0, 1, 2\}$$
$$\underline{0}^{\mathcal{M}}(*) = 0 \qquad s^{\mathcal{M}}(n) = \begin{cases} n+1, & n < 2 \\ 0, & n = 2 \end{cases}$$
$$I^{\mathcal{M}} = \{(n, m) \mid n = m\}$$

c)

$$|\mathcal{M}| = \{a, 0, 1, 2\}$$
$$\underline{0}^{\mathcal{M}}(*) = a \qquad s^{\mathcal{M}}(n) = \begin{cases} n+1, & n < 2 \\ 0, & n = a \text{ or } n = 2 \end{cases}$$
$$I^{\mathcal{M}} = \{(n, m) \mid n = m\}$$

## Exercise 2

Let $P$ be a binary predicate. Find a model which satisfies the formula $\forall x.\, \neg P(x, x)$ and one that does not.

## Exercise 3

Show that $(\neg \forall x.\, \neg P(x)) \to \exists y.\, P(y)$ is a tautology.

## Homework 1

______/5 p.

Show that $\forall x.\, P(x) \land Q(x) \vDash (\forall y.\, P(y)) \land (\forall z.\, Q(z))$.

## Homework 2

______/4 p.

Is $\exists x.\, P(x, x)$ satisfiable? If yes, provide a model.