Introduction to Logic (Spring 2020)

Assignment 7

Monday 6th April 2020

Instructions This sheet contains two kinds of assignments: exercises and homework. The first are not mandatory and are meant for practising during the exercise class or by yourself. Tutors will be available during the exercise class to help with the assignment. Homework assignments are mandatory, and the combined grade of the homework makes 30% of the final grade. The grade for the homework on this sheet corresponds to the number of points obtained + 1. Maximally 9 points can be obtained.

Handing in your answers Submit your solution through **Blackboard** as a single PDF file named `hw7sN.pdf`, where N is your student number. The document has to be created using LaTeX (or variants like XeLaTeX). A template is available on the website of the course and on Blackboard. Please **use the proper logic connectives and proof rules** as they are shown in the template. If you do not have a working LaTeX installation, then you can use Overleaf (`https://www.overleaf.com/`). Make sure that your **name, student number and studies** are **clearly written on the document**. All students have to prepare and submit their own solution. Only submit the 2 exercise(s) marked as **Homework**. Answers have to be provided in **Dutch or English**. Submissions that fail to meet these requirements are not considered.

Deadline The homework must be uploaded before **Friday 10th April 2020 2:30pm**.

Learning Objectives After completing this assignment, you should be able to **carry out substitution** of terms in first-order formulas and **rename bound variables** in this process. Moreover, you should be able to **prove sequents in first-order logic** in system $\mathbf{ND}_1$ and $\mathbf{cND}_1$ using Fitch-style.

Exercise 1

Do the substitution exercises from the lecture notes on substitution.

Exercise 2

Prove the validity of the following sequents in system $\mathbf{ND}_1$, where P and Q have arity 1, and S has arity 0.

a) $x \mid P(x), \forall y. (P(y) \to Q(y)) \vdash Q(x)$

b) $\exists x. S \to Q(x) \vdash S \to \exists y. Q(y)$

c) $\forall x. P(x) \vdash \neg \exists y. \neg P(y)$

d) $\exists x. P(x) \vdash \neg \forall y. \neg P(y)$

Exercise 3

Prove the validity of the following sequents in system $\mathbf{cND}_1$, where P has arity 1 and $\cdot$ is the empty list. General hint: Use results that have been proved previously in the lecture or in the exercises. Note that exercise b) is very hard!

a) $\neg \forall x. \neg P(x) \vdash \exists y. P(y)$

b) $j, k \mid \cdot \vdash \exists x. P(x) \to P(j) \wedge P(k)$

Homework 1

_____/3 p.

We use the signature $\mathcal{L}$ with a function symbol f of arity one, and two predicate symbols P and Q of arity two. Let σ be the substitution given by

$$\sigma = [y := f(x)]$$

and φ the formula given by

$$\varphi = \exists x. P(y, z) \wedge \forall y. Q(y, x)$$

Carry out the substitution $\varphi[\sigma]$ and annotate each step with the used axiom.

Homework 2

_____/6 p.

We use the signature $\mathcal{L}$ with one predicate symbol P of arity one. Prove the following sequent in $\mathbf{ND}_1$.

$$j \mid \exists x. \neg P(x) \vdash \exists y. (P(y) \to P(j))$$