Introduction to Logic (Spring 2020)

Assignment 6

Monday 9th March 2020

Instructions This sheet contains two kinds of assignments: exercises and homework. The first are not mandatory and are meant for practising during the exercise class or by yourself. Tutors will be available during the exercise class to help with the assignment. Homework assignments are mandatory, and the combined grade of the homework makes 30% of the final grade. The grade for the homework on this sheet corresponds to the number of points obtained + 1. Maximally 9 points can be obtained.

Handing in your answers Submit your solution through **Blackboard** as a single PDF file named `hw6sN.pdf`, where N is your student number. The document has to be created using LaTeX (or variants like XeLaTeX). A template is available on the website of the course and on Blackboard. Please **use the proper logic connectives and proof rules** as they are shown in the template. If you do not have a working LaTeX installation, then you can use Overleaf (`https://www.overleaf.com/`). Make sure that your **name, student number and studies** are **clearly written on the document**. All students have to prepare and submit their own solution. Only submit the 2 exercise(s) marked as **Homework**. Answers have to be provided in **Dutch or English**. Submissions that fail to meet these requirements are not considered.

Deadline The homework must be uploaded before **Friday 13th March 2020 2:30pm**.

Learning Objectives After completing this assignment, you should be able to **recognise first-order formulas** and use them to **formalise natural language statements**. Moreover, you should be able to tell which variables are **bound and free** in a first-order formula.

Exercise 1

a) Give the smallest first-order signature $\mathcal{L} = (\mathcal{F}, \mathcal{R}, \text{ar})$, such that following is a formula in $\text{Form}(\mathcal{L})$.

$$\exists y.\, \forall x.\, P(f(m), x) \to Q(g(x, y))$$

b) Is there a first-order signature $\mathcal{L}$, such that the following would be a formula in $\text{Form}(\mathcal{L})$? If not, why?

$$\exists y.\, \forall x.\, P(f(m), y) \land P(x)$$

Exercise 2

Suppose we have a unary predicate symbols N, E, O and P, that is, $\text{ar}(N) = \text{ar}(E) = \text{ar}(O) = \text{ar}(P) = 1$. The intended meaning of $N(x)$ is that x is a natural number, of $E(x)$ that x is an even number, of $O(x)$ that x is an odd number, and of $P(x)$ that x is a prime number. Using only the above predicate symbols, translate the following sentences into predicate logic:

a) Not all prime natural numbers are odd.

b) Every natural number is an even or odd number.

c) There is no natural number which is both even and odd.

Exercise 3

Compute $\text{fv}(\varphi)$ and $\text{bv}(\varphi)$ for each of the following formulas φ.

a) $\forall x.\, P(y)$

b) $\forall x.\, P(x)$

c) $\exists y.\, P(x) \land \forall x.\, P(y)$

d) $(\forall x.\, P(y)) \land \exists y.\, Q(y) \lor R(x)$

Homework 1

_____/7 p.

Let $\mathcal{L}$ be the language $(\mathcal{F}, \mathcal{R}, \text{ar})$ with $\mathcal{F} = \{m, f\}$, $\mathcal{R} = \{S, B\}$ and $\text{ar}(m) = 0$, $\text{ar}(f) = 1$, $\text{ar}(S) = \text{ar}(B) = 2$. Which of the following strings are first-order formulas in $\text{Form}(\mathcal{L})$?

a) $\forall x.\, S(m, x)$

b) $B(m, f(m))$

c) $f(m)$

d) $B(B(m, x), y)$

e) $B(m)$

f) $B(x, y) \to \exists z.\, P(z, y)$

g) $S(x, y) \to S(y, f(f(x)))$

Homework 2

_____/2 p.

Calculate $\text{fv}(\varphi)$ and $\text{bv}(\varphi)$ for the following formula φ.

$$\exists x.\, P(y, z) \land \forall y.\, Q(y, x) \lor P(y, z)$$