

Introduction to Logic (Spring 2020)

Assignment 5

Monday 2nd March 2020

Instructions This sheet contains two kinds of assignments: exercises and homework. The first are not mandatory and are meant for practising during the exercise class or by yourself. Tutors will be available during the exercise class to help with the assignment. Homework assignments are mandatory, and the combined grade of the homework makes 30% of the final grade. The grade for the homework on this sheet corresponds to the number of points obtained + 1. Maximally 9 points can be obtained.

Handing in your answers Submit your solution through **Blackboard** as a single PDF file named `hw5sN.pdf`, where N is your student number. The document has to be created using \LaTeX (or variants like \XeLaTeX). A template is available on the website of the course and on Blackboard. Please **use the proper logic connectives and proof rules** as they are shown in the template. If you do not have a working \LaTeX installation, then you can use Overleaf (<https://www.overleaf.com/>). Make sure that your **name, student number and studies** are **clearly written on the document**. All students have to prepare and submit their own solution. Only submit the 2 exercise(s) marked as **Homework**. Answers have to be provided in **Dutch or English**. Submissions that fail to meet these requirements are not considered.

Deadline The homework must be uploaded before **Friday 6th March 2020 2:30pm**.

Learning Objectives After completing this assignment, you should be able to **compute conjunctive normal forms (CNF)** of propositional formulas and **decide** for formulas in CNF if they are **tautological** by using the CNF-TAUT algorithm. Moreover, you should be able to use the HORN algorithm to **decide whether a Horn formula is satisfiable**.

Exercise 1

Compute a CNF φ' for each of the following formulas φ . Check whether φ' is a tautology by using the CNF-TAUT algorithm from the lecture.

a) $p \wedge q \rightarrow p \vee q$

b) $p \vee (p \wedge q) \rightarrow p$

c) $p \vee q \rightarrow p \wedge q$

Exercise 2

Suppose we are modelling some facts about children by using the following propositional variables.

Variable	Meaning
FG	First Grade
K	Kindergarten
C	Child
F	Female
M	Male
G	Girl
B	Boy

Let φ be the following Horn formula

$$(FG \rightarrow C) \wedge (K \rightarrow C) \wedge (C \wedge F \rightarrow G) \wedge (C \wedge M \rightarrow B),$$

which expresses some general knowledge about children. Further, let ψ be the following formula

$$FG \wedge F,$$

which expresses some knowledge about a specific person. Use the algorithm HORN to show that the following formula is not satisfiable.

$$\varphi \wedge \psi \wedge \neg G$$

Conclude that the person under consideration is a girl.

Homework 1

Decide which of the following formulas are in CNF? Only answer “yes” or “no”.

a) $p \wedge \neg(q \vee \neg p)$

b) $p \wedge \neg q$

c) $\neg p \wedge (p \vee \neg q)$

d) $\neg p \vee (p \wedge \neg q)$

_____/4 p.

Homework 2

Use the algorithm HORN to decide whether the following Horn formula φ is satisfiable.

$$\neg(p \wedge q \wedge w) \wedge (r \rightarrow p) \wedge r \wedge q$$

_____/5 p.