Introduction to Logic (Spring 2020)
Assignment 4

Monday 24th February 2020

Instructions This sheet contains two kinds of assignments: exercises and homework. The first are not mandatory and are meant for practising during the exercise class or by yourself. Tutors will be available during the exercise class to help with the assignment. Homework assignments are mandatory, and the combined grade of the homework makes 30% of the final grade. The grade for the homework on this sheet corresponds to the number of points obtained + 1. Maximally 9 points can be obtained.

Handing in your answers Submit your solution through **Blackboard** as a single PDF file named `hw4sN.pdf`, where N is your student number. The document has to be created using LaTeX (or variants like XƎLaTeX). A template is available on the website of the course and on Blackboard. Please **use the proper logic connectives and proof rules** as they are shown in the template. If you do not have a working LaTeX installation, then you can use Overleaf (`https://www.overleaf.com/`). Make sure that your **name, student number and studies** are **clearly written on the document**. All students have to prepare and submit their own solution. Only submit the 2 exercise(s) marked as **Homework**. Answers have to be provided in **Dutch or English**. Submissions that fail to meet these requirements are not considered.

Deadline The homework must be uploaded before **Friday 28th February 2020 2:30pm**.

Learning Objectives After completing this assignment, you should be able to construct classical and intuitionistic **natural deduction proofs** for propositional formulas, both in tree- and in Fitch-style. Moreover, you should be able to **use soundness** to prove that formulas are not derivable.

Exercise 1 *Natural Deduction*

Derive the following sequents in the system **ND**, that is, do not use ($\neg\neg$E) or any of its consequences.

    a) $p \vdash \neg\neg p$ b) $p \to q \vdash p \lor r \to q \lor r$

Exercise 2 *Fitch-Style Natural Deduction*

Let φ and ψ be formulas. Derive the following sequents in the system **ND**, that is, do not use $(\neg\neg E)$ or any of its consequences, using Fitch-style.

    a) $\neg\varphi \vee \neg\psi \vdash \neg(\varphi \wedge \psi)$ b) $\neg(\varphi \vee \psi) \vdash \neg\varphi \wedge \neg\psi$

Exercise 3 *Classical Natural Deduction*

Derive the following sequents in the system **cND** (using Fitch-style), that is, you may use $(\neg\neg E)$ or any of its consequences.

    a) $\vdash ((p \rightarrow q) \rightarrow p) \rightarrow p$ b) $\neg(\neg p \wedge \neg q) \vdash p \vee q$

Exercise 4 *Soundness*

Recall that the true proposition $\top$ is defined as $\neg\bot$. Which of the following sequents is derivable in **ND** or **cND**? If the sequent is derivable, then give the derivation, otherwise give an argument (typically using soundness) why it is not derivable.

    a) $p \vee \top \vdash p$ b) $p \vdash p \wedge \top$

    c) $p \rightarrow (q \rightarrow r) \vdash (p \rightarrow q) \rightarrow r$

Homework 1

                                                       _____/4 p.

Derive the following sequent in the system **ND** (Fitch-style), that is, do not use $(\neg\neg E)$ or any of its consequences.
$$p \vee (p \wedge q) \vdash p$$

Homework 2

                                                       _____/5 p.

Derive the following sequent in the system **cND** (Fitch-style).

$$\neg q \rightarrow \neg p \vdash p \rightarrow q$$