Introduction to Logic (Spring 2020)
Assignment 3

Monday 17th February 2020

Instructions   This sheet contains two kinds of assignments: exercises and homework. The first are not mandatory and are meant for practising during the exercise class or by yourself. Tutors will be available during the exercise class to help with the assignment. Homework assignments are mandatory, and the combined grade of the homework makes 30% of the final grade. The grade for the homework on this sheet corresponds to the number of points obtained + 1. Maximally 9 points can be obtained.

Handing in your answers   Submit your solution through **Blackboard** as a single PDF file named `hw3sN.pdf`, where N is your student number. The document has to be created using LATEX (or variants like X-LATEX). A template is available on the website of the course. **Please use the proper logic connectives as they are shown in the template.** If you do not have a working LATEX installation, then you can use Overleaf (`https://www.overleaf.com/`). Make sure that your **name, student number and studies** are **clearly written on the document**. All students have to prepare and submit their own solution. Only submit the 2 exercise(s) marked as **Homework**. Answers have to be provided in **Dutch or English**. Submissions that fail to meet these requirements are not considered.

Deadline   The homework must be uploaded before **Friday 21st February 2020 2:30pm**.

Learning Objectives   After completing this assignment, you should have deepened your understanding of **semantic equivalence** and you should be able to construct **syntactic derivations** derive for formula that only involve conjunction and implication.

Exercise 1

Let $\top$ be the formula $\neg\bot$. Show that a formula φ is a tautology if and only if $\varphi \equiv \top$.

Exercise 2

Let φ, ψ, η be propositional formulas. Show that

a) $\varphi \equiv \varphi$ b) $\varphi \wedge \varphi \equiv \varphi$ c) $\varphi \vee (\psi \vee \eta) \equiv (\varphi \vee \psi) \vee \eta$.

Exercise 3

Derive the following sequents.

a) $p \wedge q \vdash q \wedge p$ b) $p \to (p \to q), p \vdash q$ c) $p \to q \vdash p \wedge s \to q \wedge s$

Exercise 4

Show for all formulas φ and ψ that $\varphi \wedge \psi \vdash \varphi$ and $\varphi \wedge \psi \vdash \psi$.

Exercise 5

Derive the sequent $(p \wedge q) \wedge r \vdash (p \wedge q) \wedge r$.

Homework 1

_____/3 p.

Let φ and ψ be the formulas. Show that $\varphi \vee \psi \equiv \psi \vee \varphi$

Homework 2

_____/6 p.

Derive the following sequents.

a) $(p \wedge q) \wedge r, s \wedge t \vdash q \wedge s$ b) $p \vdash q \to (p \wedge q)$