

# Introduction to Logic (Spring 2020)

## Assignment 2

Monday 10th February 2020

**Instructions** This sheet contains two kinds of assignments: exercises and homework. The first are not mandatory and are meant for practising during the exercise class or by yourself. Tutors will be available during the exercise class to help with the assignment. Homework assignments are mandatory, and the combined grade of the homework makes 30% of the final grade. The grade for the homework on this sheet corresponds to the number of points obtained + 1. Maximally 9 points can be obtained.

**Handing in your answers** Submit your solution through **Blackboard** as a single PDF file named `hw2sN.pdf`, where N is your student number. The document has to be created using  $\text{\LaTeX}$  (or variants like  $\text{\XeTeX}$ ). A template is available on the website of the course. If you do not have a working  $\text{\LaTeX}$  installation, then you can use Overleaf (<https://www.overleaf.com/>). Make sure that your **name, student number and studies** are **clearly written on the document**. All students have to prepare and submit their own solution. Only submit the 2 exercise(s) marked as **Homework**. Answers have to be provided in **Dutch or English**. Submissions that fail to meet these requirements are not considered.

**Deadline** The homework must be uploaded before **Friday 14th February 2020 2:30pm**.

**Learning Objectives** After completing this assignment, you should be able to **calculate the semantics** of propositional formulas. Furthermore, you should be able to determine whether formulas are **satisfiable, tautologies or semantically equivalent**.

### Exercise 1

Which of the following formulas are satisfiable/tautologies? Give the corresponding truth tables.

a)  $\neg p \rightarrow p$

b)  $\perp \rightarrow p$

c)  $p \wedge \neg(q \rightarrow p)$

### Exercise 2

Let  $\varphi$  be a propositional formula. Show that  $\varphi$  is a tautology if and only if  $\emptyset \models \varphi$ .

### Exercise 3

- a) Let  $a, b, c \in \mathbb{B}$ . Show that  $a \leq b \Rightarrow c$  iff  $\min\{a, b\} \leq c$ .
- b) Let  $\varphi, \psi$  and  $\rho$  be formulas. Show that  $(\varphi \wedge \psi) \rightarrow \rho \equiv \varphi \rightarrow (\psi \rightarrow \rho)$ .

### Exercise 4

Let  $\Gamma = \{p, p \rightarrow q\}$ . Show that  $\Gamma \models q$ .

### Exercise 5

Let  $\varphi$  be a formula and  $v$  be a valuation. Show that  $\llbracket \neg\varphi \rrbracket_v = 1 - \llbracket \varphi \rrbracket_v$ .

### Homework 1

\_\_\_\_\_ /5 p.

Let  $\varphi$  be the formula  $(\neg p \vee q) \rightarrow r$ .

- a) Give the truth table of  $\varphi$ .
- b) Let  $v$  be the valuation given by  $v(p) = v(r) = 1$  and  $v(s) = 0$  for all other  $s \in \text{PVar}$ .  
Compute  $\llbracket \varphi \rrbracket_v$  by using the truth table. Is  $\varphi$  satisfiable?

### Homework 2

\_\_\_\_\_ /4 p.

Show that  $p \rightarrow q \rightarrow p$  is a tautology.