Introduction to Logic (Spring 2020)

Assignment 10

Monday 25th May 2020

Instructions This sheet contains two kinds of assignments: exercises and homework. The first are not mandatory and are meant for practising during the exercise class or by yourself. Tutors will be available during the exercise class to help with the assignment. Homework assignments are mandatory, and the combined grade of the homework makes 30% of the final grade. The grade for the homework on this sheet corresponds to the number of points obtained + 1. Maximally 9 points can be obtained.

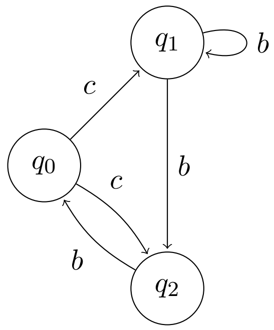
Handing in your answers Submit your solution through **Blackboard** as a single PDF file named `hw10sN.pdf`, where N is your student number. The document has to be created using LaTeX (or variants like XeLaTeX). A template is available on the website of the course and on Blackboard. Please **use the proper logic connectives and proof rules** as they are shown in the template. If you do not have a working LaTeX installation, then you can use Overleaf (`https://www.overleaf.com/`). Make sure that your **name, student number and studies** are **clearly written on the document**. All students have to prepare and submit their own solution. Only submit the 2 exercise(s) marked as **Homework**. Answers have to be provided in **Dutch or English**. Submissions that fail to meet these requirements are not considered.

Deadline The homework must be uploaded before **Friday 29th May 2020 2:30pm**.

Learning Objectives After completing this assignment, you should be able to describe problems using **Horn clause theories** and use **uniform proofs** to query Horn clause theories. The homework on this assignment is a **retake of homework 7**.

Exercise 1

A *labelled transition system* (LTS) is a graph with edges labelled in some alphabet. Alternatively, an LTS can be seen as a non-deterministic automaton without initial and accepting states. We can use labelled transition systems as models of computation. Suppose, for instance, that we want to model a beverage vending machine that takes coins as input and provides beverages. However, this machine has a bug: It may happen that the user of the machine is lucky and the machine enters a state, after receiving a coin, in which the machine may hand out several beverages at no further cost. We can model this machine as an LTS, as shown in the following diagram, where the labels are c for the coin and b for vending a beverage. The state q_0 takes a coin as input, from where the machine non-deterministically moves to either the faulty state q_1 or q_2.



We are now interested in formally representing the machine and its behaviour by Horn clauses, and then reason about it with uniform proofs. To this end, we will need to talk about *traces*. A trace of an LTS is a sequence of labels that arises from transitions between states. We will write such sequences as labels seperated by a dot and ending in ε, where ε stands for the empty trace. For instance, $b \cdot \varepsilon$ is a trace leading from q_1 to q_2 in our vending machine. However, $c \cdot c \cdot \varepsilon$ is not a trace for any pair of states. Formally, we will use the set of function symbols

$$\mathcal{F} = \{q_0, q_1, q_2, c, b, \varepsilon, \cdot\},$$

where $\cdot$ has arity 2 and all other symbols have arity zero (constants). As indicated above, we write $u \cdot w$ instead of $\cdot(u, w)$ to ease readability. To model the vending machine and its behaviour, we will use two predicate symbols

$$\mathcal{R} = \{R, T\},$$

each with arity 3. The idea is that R represents the transitions of the machine and T the traces. For instance, we want that $R(q_1, b, q_2)$ holds because the above machine has a transition from q_1 to q_2 with label b. Similarly, we want that $T(b \cdot \varepsilon, q_1, q_2)$ holds because $c \cdot \varepsilon$ is a trace leading from q_1 to q_2. Note that the same sequence of labels gives rise to different traces and that $T(b \cdot \varepsilon, q_2, q_0)$ holds as well.

a) Provide a logic program Γ_0 with five Horn clauses that determines the transition relation R.

b) Provide two Horn clauses φ_r and φ_t that determine the trace relation T, one clause for the empty sequence ε and one of the sequential composition of traces.

c) Give a uniform proof that shows that we can start the machine in state q_0, input only one coin but receive two beverages, and that we can continue putting in coins. Formally, prove the following sequent using a uniform proof in Fitch-style.

$$\Gamma_0, \varphi_r, \varphi_t \vdash_u \exists x. T(c \cdot b \cdot b \cdot \varepsilon, q_0, x) \land \exists y. R(x, c, y)$$

Homework 1

______/3 p.

We use the signature $\mathcal{L}$ with a function symbol f of arity two, and two predicate symbols P of arity two and R of arity three. Let σ be the substitution given by

$$\sigma = [y := f(x, z)]$$

and φ the formula given by

$$\varphi = \forall z. R(z, y, x) \lor \exists y. P(x, y)$$

Carry out the substitution $\varphi[\sigma]$ and annotate each step with the used axiom.

Homework 2

______/6 p.

We use the signature $\mathcal{L}$ with one predicate symbol P of arity one. Prove the following sequent in $\mathbf{ND_1}$.

$$j \mid \forall x. \neg P(x) \vdash \forall y. P(y) \to P(j)$$