Assignment 1

Exercises on lecture 1/chapter 1

1 September 2026

We will work on the following exercises during the tutorial session. Make sure that you understand the solution that we work out together and to solve the remaining exercises yourself.

Exercise 1.1 — Check which of the following three expressions e is typable with $\vdash e : \mathrm{N}_+$ and $\vdash e : \mathrm{N}$, respectively, and provide the derivation tree when possible: $\underline{2} \oplus \underline{3}$, $\underline{2} \oplus (\underline{0} \otimes \underline{3})$ and $\underline{2} \oslash (\underline{0} \otimes \underline{3})$.

Exercise 1.2 — Complete the proof of theorem 1.4 by proving that $\vdash \underline{0} : \mathrm{N}_+$ is not derivable. (Hint: Prove first that if $\vdash \underline{a} : \mathrm{N}_+$ for some numeral $\underline{a}$, then $a > 0$. Use induction on proof trees.)

Exercise 1.3 — Compute all possible steps of the small-step semantics for the following three `Arith`-expressions: $\underline{2} \oplus \underline{3}$, $\underline{2} \oplus (\underline{0} \otimes \underline{3})$ and $\underline{2} \oslash (\underline{0} \otimes \underline{3})$.

Exercise 1.4 — Prove theorem 1.6. Hint: Prove the result first for the contraction relation by induction on type derivations, and then prove the theorem by induction on derivations for the small-step relation.

Exercise 1.5 — Compute the big-step semantics for the following three `Arith`-expressions: $\underline{2} \oplus \underline{3}$, $\underline{2} \oplus (\underline{0} \otimes \underline{3})$ and $\underline{2} \oslash (\underline{0} \otimes \underline{3})$.

Exercise 1.6 — Prove theorem 1.8 by induction on type derivations.

Exercise 1.7 — Compute the denotational semantics $\mathcal{E}_{\mathrm{N}}[\![e]\!]$ and $\mathcal{E}_{\mathrm{N}_+}[\![e]\!]$ for the following three `Arith`-expressions e, where possible: $\underline{2} \oplus \underline{3}$, $\underline{2} \oplus (\underline{0} \otimes \underline{3})$ and $\underline{2} \oslash (\underline{0} \otimes \underline{3})$.

Exercise 1.8 — Compute the denotational semantics $\mathcal{E}[\![e]\!]$ for the following three `Arith`-expressions e: $\underline{2} \oplus \underline{3}$, $\underline{2} \oplus (\underline{0} \otimes \underline{3})$ and $\underline{2} \oslash (\underline{0} \otimes \underline{3})$.

Exercise 1.9 — Prove theorem 1.10 by induction on type derivations.