

Assignment 9

Exercises on lecture 9/chapter 9

28 October 2025

We will work on the following exercises during the tutorial session. Make sure that you understand the solution that we work out together and to solve the remaining exercises yourself.

Exercise 9.1 — Complete the proof of theorem 9.5, by proving the remaining case of the induction that shows that

$$(\chi_1[\chi_2] \triangleleft (f, \sigma)) \Downarrow \sigma' \text{ iff } (\chi_1 \triangleleft (O\chi_2 f, \sigma)) \Downarrow \sigma'$$

Exercise 9.2 — Prove lemma 9.7 by proving that the composition of natural transformations satisfies the unit and associativity laws.

Exercise 9.3 — Let $F, G: \mathcal{C} \rightarrow \mathcal{D}$ be functors and $\alpha: F \rightarrow G$ a natural transformation. Show that a family of morphisms $\beta_X: GX \rightarrow FX$ indexed by objects in \mathcal{C} with $\alpha_X \circ \beta_X = \text{id}_{FX}$ and $\beta_X \circ \alpha_X = \text{id}_{GX}$ for all X , gives a natural transformation $\beta: G \rightarrow F$ that is inverse to α .

Exercise 9.4 — Complete the proof of lemma 9.8 by proving that for all χ, σ and σ' ,

$$(\chi \triangleleft (f, \sigma)) \Downarrow \sigma' \quad \text{implies} \quad (D\chi)(f)(\sigma) = \eta(\sigma')$$

by induction on derivations for the big-step operational semantics.

Exercise 9.5 — Prove corollary 9.10 (full abstraction of Imp) from theorem 9.9 (equivalence of denotational and operational semantics).