Assignment 7

Exercises on lecture 7/chapter 7

14 October 2025

We will work on the following exercises during the tutorial session. Make sure that you understand the solution that we work out together and to solve the remaining exercises yourself.

Exercise 7.1 — Prove lemma 7.2 by induction on contexts.

Exercise 7.2 — Finish the proof of theorem 7.6 and show that eq. (7.4) holds.

Exercise 7.3 — Let P and Q be ωCPOs.

a) Show that that $K(x) : P \to Q$ is monotone and continuous for every $x \in Q$.

b) By **a)**, we can view K as map $Q \to [P, Q]$. Show that this map is also monotone and continuous.

Exercise 7.4 — Prove that for arithmetic a and Boolean expressions b in `Imp` the operational and denotational semantics agree, that is, prove that for all $\sigma \in \Sigma$ and for e being a or b that

$$(e \lhd \sigma) \Downarrow [\![e]\!] .$$

Exercise 7.5 — Let $\mathcal{C}$ be a category, $A_1, A_2, B, C \in |\mathcal{C}|$, $f_k : B \to A_k$ and $g : C \to B$ such that the product of A_1 and A_2 exists in $\mathcal{C}$. Show that $\langle f_1, f_2 \rangle \circ g = \langle f_1 \circ g, f_2 \circ g \rangle$.

Exercise 7.6 — Let $\mathcal{C}$ be a category and A an object for which the product $(A \times A, \pi_1, \pi_2)$ exists. Show that there is a unique $\delta : A \to A \times A$ with $\pi_k \circ \delta = \mathrm{id}_A$. This morphism is called the .

Exercise 7.7 — Continue example 7.9 as follows.

a) Show that in a poset $(P, \leq)$ seen as category, the product of $x, y \in P$ is the meet of $\{x, y\}$ in P, if it exists.

b) Provide an example of a poset that has no binary products for distinct objects.

Exercise 7.9 — Show that both $\boldsymbol{\omega}\mathbf{CPO}_\bot$ and $\boldsymbol{\omega}\mathbf{CPO}_s$ have binary products. (Hint: Use the definition of the product in $\boldsymbol{\omega}\mathbf{CPO}$ as defined in the proof of theorem 7.10.)