Assignment 6

Exercises on lecture 6/chapter 6

7 October 2025

We will work on the following exercises during the tutorial session. Make sure that you understand the solution that we work out together and to solve the remaining exercises yourself.

Exercise 6.1 — Let P and Q be posets. Prove that the pointwise order on maps $P \to Q$ is a partial order.

Exercise 6.2 — Show that the map f_c defined in eq. (6.1) is monotone.

Exercise 6.3 — Prove that any lfp is a fixed point (lemma 6.6).

Exercise 6.4 — Show that the join is monotone and the meet antitone for inclusion: If $U, V \subseteq P$ in a poset $(P, \leq)$ with $U \subseteq V$, then $\sup U \leq \sup V$ and $\inf V \leq \inf U$.

Exercise 6.5 — Prove that $\bigvee \emptyset = \bot_P$ if it exists in P.

Exercise 6.6 — Given a monotone map $f\colon (P, \leq_P) \to (Q, \leq_Q)$ and a set $U \subseteq P$, we write $f^{\to}U$ for the of U under f, given by $f^{\to}U = \{f(x) \mid x \in U\}$. Prove that if the joins $\bigvee U$ and $\bigvee f^{\to}U$ exist, then $\bigvee f^{\to}U \leq_Q f(\bigvee U)$. Formulate and prove the dual results for meets.

Exercise 6.7 — Prove that a map $c\colon \mathbb{N} \to P$ is a chain if and only if $c(n) \leq_P c(n+1)$ for all $n \in \mathbb{N}$.

Exercise 6.8 — Prove lemma 6.2, this is, prove that the pushforward of chains along a monotone map and the derivative map are monotone for the point-wise order.

Exercise 6.9 — A chain c in a poset P is called if there a $k \in \mathbb{N}$, such that $c_n = c_{n+1}$ for all $n \geq k$. Show that any eventually constant chain c has a supremum.

Exercise 6.10 — Complete the proof of lemma 6.11. (Note that monotonicity of sup in this case is is different from exercise 6.4!)

Exercise 6.11 — Prove that the identity map is continuous, and that continuous maps are closed under composition.

Exercise 6.12 — Show that the map f_c defined in eq. (6.1) is continuous. Assume that $[\Sigma, \Sigma_\bot]$ is a based ωCPO, which we will prove in theorem 7.5, and use theorem 6.12 to conclude that $w_\infty = \mu f_c$.