# Assignment 13

### Exercises on lecture 13/chapter 13

25 November 2025

We will work on the following exercises during the tutorial session. Make sure that you understand the solution that we work out together and to solve the remaining exercises yourself.

**Exercise 13.1** — Calculate the denotational semantics of the terms $\Omega_{\mathbf{N}}$ from example 11.10, $\lambda x.\, \mathbf{if}_0\ x\ \mathbf{then}\ 0\ \mathbf{else}\ \Omega_{\mathbf{N}}$, and subtraction as given in example 11.7.

**Exercise 13.2** — Let $\mathcal{C}$ be a category with binary products and a terminal object $T$. Show for all objects $A$ that $A \times T \cong A \cong T \times A$. Use this to show that $[T, A] \cong A$ if $\mathcal{C}$ is Cartesian closed.

**Exercise 13.3** — Give an explicit description of $\mathrm{ite}_0$ in each of the characterisation of adjunctions in theorem 10.9.

**Exercise 13.4** — Prove naturality of $\mathrm{fix}_{P,Q}$ in $P$, that is, prove lemma 13.1.

**Exercise 13.5** — Complete the proof of lemma 13.2 by showing that $[\![(\lambda x.\, t)[\sigma]]\!] = [\![\lambda x.\, t]\!] \circ D\sigma$.

**Exercise 13.6** — Prove lemma 13.3 by induction on contexts.

**Exercise 13.7** — Let $x$, $y$ and $z$ be distinct variables. Prove that the contexts $(x : \mathbf{N}, y : \mathbf{N})$ and $(z : \mathbf{N} \times \mathbf{N})$ are isomorphic in $c\mathcal{C}^0_{\lambda\mathrm{Y}}$. Discuss why this does not happen in $\mathcal{C}^0_{\lambda\mathrm{Y}}$.

**Exercise 13.8** — Prove lemma 13.9 by induction on types.