Assignment 1

Exercises on lecture 1/chapter 1

2 September 2025

We will work on the following exercises during the tutorial session. Make sure that you understand the solution that we work out together and to solve the remaining exercises yourself.

Exercise 1.1 — Complete the proof of theorem 1.4 by proving that $\vdash \underline{0} : N_+$ is not derivable. (Hint: Prove first that if $\vdash \underline{a} : N_+$ for some numeral $\underline{a}$, then $a > 0$. Use induction on proof trees.)

Exercise 1.2 — Prove theorem 1.6. Hint: Prove the result first for the contraction relation by induction on type derivations, and then prove the theorem by induction on derivations for the small-step relation.

Exercise 1.3 — Prove theorem 1.8 by induction on type derivations.

Exercise 1.4 — Prove theorem 1.10 by induction on type derivations.