# Robotics

Erwin M. Bakker| LIACS Media Lab

13-2 2023

Universiteit Leiden

Bij ons leer je de wereld kennen

## Organization and Overview

**Lecturer:**
Dr Erwin M. Bakker ( erwin@liacs.nl )
Room 126a and LIACS Media Lab (LML)

**Teaching assistant:**
Mor Puigventos ( email )
TBA (email)

**Period:** February 6th - May 22nd 2023
**Time:** Monday 15.15 - 17.00
**Place (Rooms):**
a) Gorlaeus - Lecture Hall C3
b) Sylvius - 15.31
c) Van Steenis - E0.04
d) Oortgebouw - Sitterzaal

**Schedule (tentative, visit regularly):**

| Date | Room | Subject |
|---|---|---|
| 6-2 | a | Introduction and Overview |
| 13-2 | a | Locomotion and Inverse Kinematics |
| 20-2 | b | Robotics Sensors and Image Processing |
| 27-2 | a | SLAM + SLAM Workshop |
| 6-3 | c | Mobile Robot Challenge Introduction |
| 13-3 | a | Project Proposals I (by students) |
| 20-3 | d | Project Proposals II (by students) |
| 27-3 | d | Robotics Vision (Week 13, start 15.30) |
| 3-4 | d | Robotics Reinforcement Learning&Workshop |
| 10-4 | | No Class (Eastern) |
| 17-4 | d | Project Progress I (by students) |
| 24-4 | d | Project Progress II (by students) |
| 1-5 | d | Mobile Robot Challenge I |
| 8-5 | a | Mobile Robot Challenge II |
| 15-5 | d | Project Demos I |
| 22-5 | d | Project Demos II |
| 29-5 | | Whit Monday |
| 5-6 | | Project Deliverables |

Website: http://liacs.leidenuniv.nl/~bakkerem2/robotics/

Grading (6 ECTS):
- Presentations and Robotics Project (60% of grade).
- Class discussions, attendance, assignments (pass/no pass) 2 workshops (0-10) (2 x 20% = 40% of grade).
- It is necessary to be at every class and to complete every workshop and assignment.

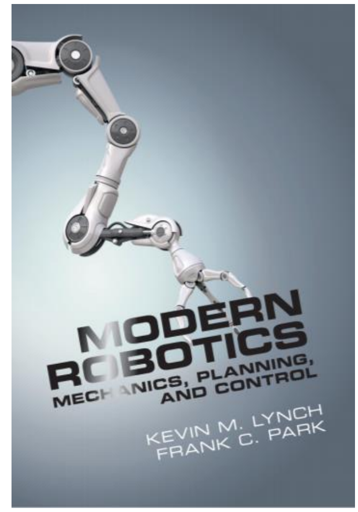
Universiteit Leiden. Bij ons leer je de wereld kennen

# Overview

- Robotic Actuators

- Configuration Space
- Rigid Body Motion
- Forward Kinematics
- Inverse Kinematics

- Link: [http://modernrobotics.org](http://modernrobotics.org)

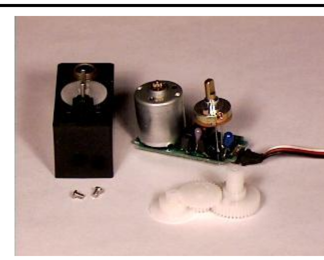


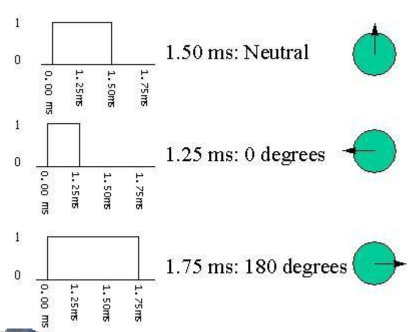

K.M. Lynch, F.C. Park, Modern Robotics: Mechanics, Planning and Control, Cambridge University Press, 2017

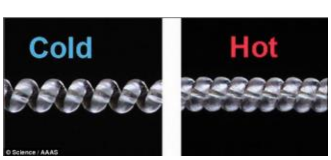


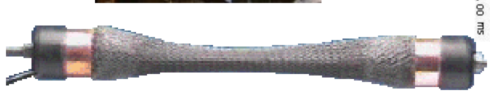
# Robotics Actuators

- **Electro motors**
- **Servo's**
- **Stepper Motors**
- **Brushless motors**
- **Solenoids**
- Hydraulic, **pneumatic actuator's**
- Magnetic actuators
- Artificial Muscles
- Etc.



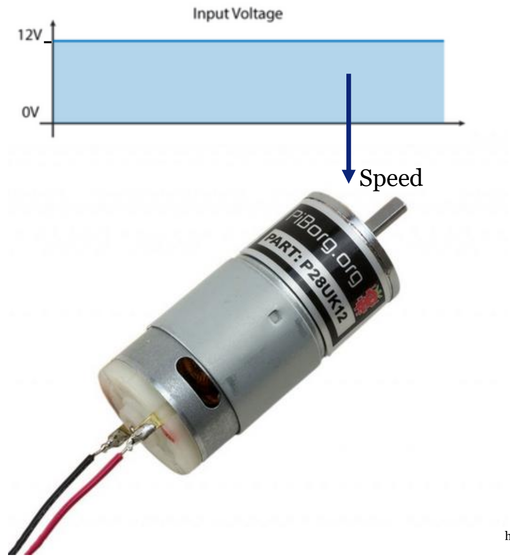


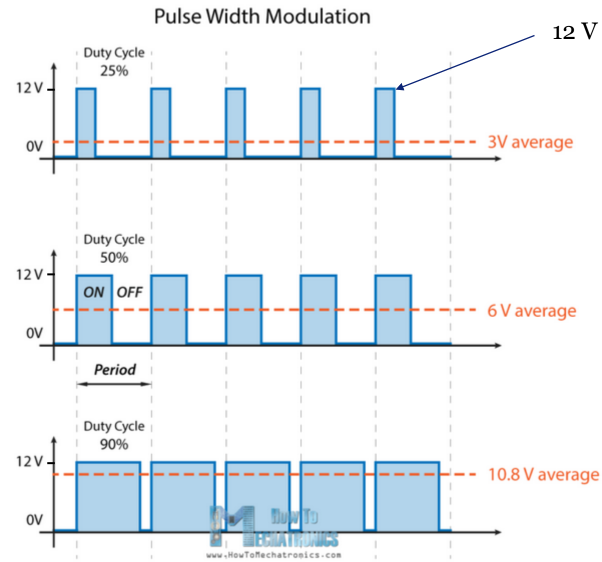



Universiteit Leiden. Bij ons leer je de wereld kennen

# DC Motors



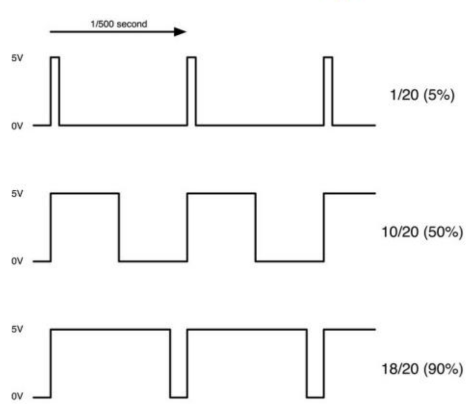

https://howtomechatronics.com/how-it-works/electronics/how-to-make-pwm-dc-motor-speed-controller-using-555-timer-ic/

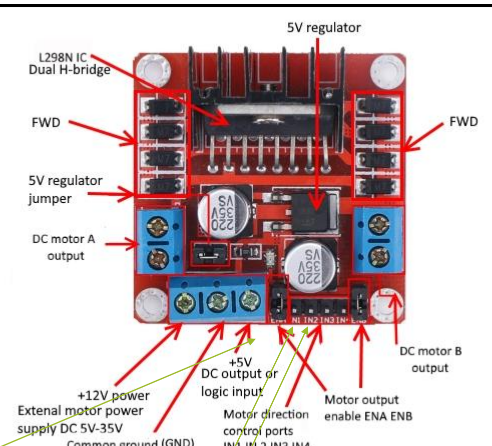


# Direct Current (DC) Electro Motors:

- Duty Cycle

Brushed DC (BDC) motor controller Using L298N



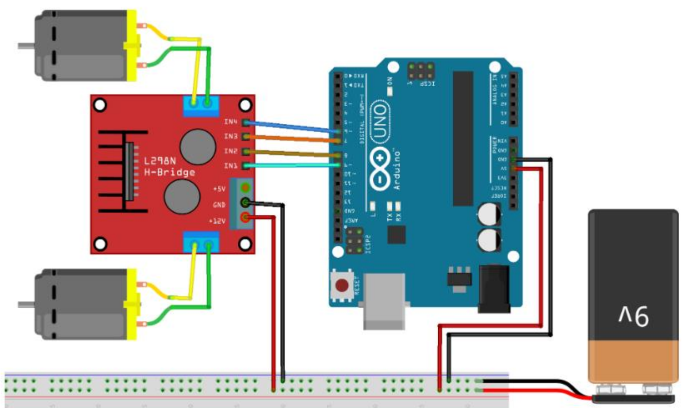




Loop:
PWM(EN**A**,128);
DigitalWrite(IN**1**, HIGH);
DigitalWrite(IN**2**,LOW);
PWM(EN**B**,64);
DigitalWrite(IN**3**, HIGH);
DigitalWrite(IN**4**,LOW);

# Direct Current (DC) Electro Motors:

Brushed DC (BDC)
motor controller
Using L298N

| MotoDriver 2 | Arduino |
|---|---|
| Input 1 | 9 |
| Input 2 | 8 |
| Input 3 | 7 |
| Input 4 | 6 |




```
//Motor 1
const int motorPin1  = 9;
const int motorPin2  = 8;
//Motor 2
const int motorPin3  = 7;
const int motorPin4  = 6;

int speed = 180;

void setup(){

    //Set pins as outputs
    pinMode(motorPin1, OUTPUT);
    pinMode(motorPin2, OUTPUT);
    pinMode(motorPin3, OUTPUT);
    pinMode(motorPin4, OUTPUT);


    //Motor Control A in both directions
    analogWrite(motorPin1, speed);
    delay(2000);
    analogWrite(motorPin1, 0);
    delay(200);
    analogWrite(motorPin2, speed);
    delay(2000);
    analogWrite(motorPin2, 0);

    //Motor Control B in both directions
    analogWrite(motorPin3, speed);
    delay(2000);
    analogWrite(motorPin3, 0);
    delay(200);
    analogWrite(motorPin4, speed);
    delay(2000);
    analogWrite(motorPin4, 0);

}


void loop(){

}
```

Loop:
PWM(EN**A**,128);
DigitalWrite(IN**1**, HIGH);
DigitalWrite(IN**2**,LOW);
PWM(EN**B**,64);
DigitalWrite(IN**3**, HIGH);
DigitalWrite(IN**4**,LOW);



# DC Motor Controllers

Pololu Simple Motor Controllers

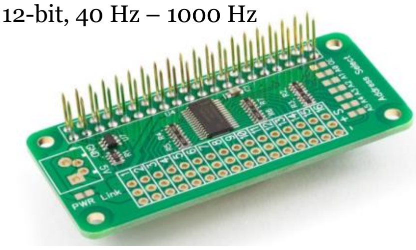
- USB, TTL Serial, Analog, RC Control, I2C

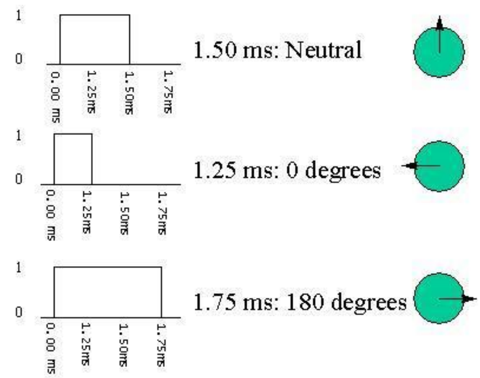
| | Original versions, not recommended for new designs (included for comparison purposes) | | | | | G2 versions, released November 2018 | | | |
|---|---|---|---|---|---|---|---|---|---|
| | SMC 18v7 | SMC 18v15 | SMC 24v12 | SMC 18v25 | SMC 24v23 | SMC G2 18v15 | SMC G2 24v12 | SMC G2 18v25 | SMC G2 24v19 |
| **Minimum operating voltage:** | 5.5 V | 5.5 V | 5.5 V | 5.5 V | 5.5 V | 6.5 V | 6.5 V | 6.5 V | 6.5 V |
| **Recommended max operating voltage:** | 24 V(1) | 24 V(1) | 34 V(2) | 24 V(1) | 34 V(2) | 24 V(1) | 34 V(2) | 24 V(1) | 34 V(2) |
| **Max nominal battery voltage:** | 18 V | 18 V | 28 V | 18 V | 28 V | 18 V | 28 V | 18 V | 28 V |
| **Max continuous current (no additional cooling):** | 7 A | 15 A | 12 A | 25 A | 23 A | 15 A | 12 A | 25 A | 19 A |
| **USB, TTL serial, Analog, RC control:** | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| **I²C control:** | | | | | | ✓ | ✓ | ✓ | ✓ |
| **Hardware current limiting:** | | | | | | ✓ | ✓ | ✓ | ✓ |
| **Reverse voltage protection:** | | | | | | ✓ | ✓ | ✓ | ✓ |

https://www.pololu.com/category/94/pololu-simple-motor-controllers

# Servo's

16-ch, 12-bit, 40 Hz – 1000 Hz

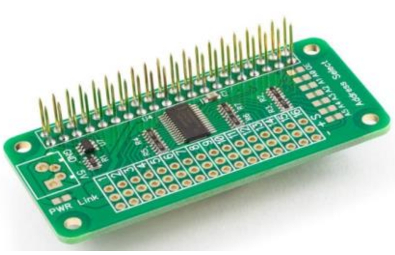




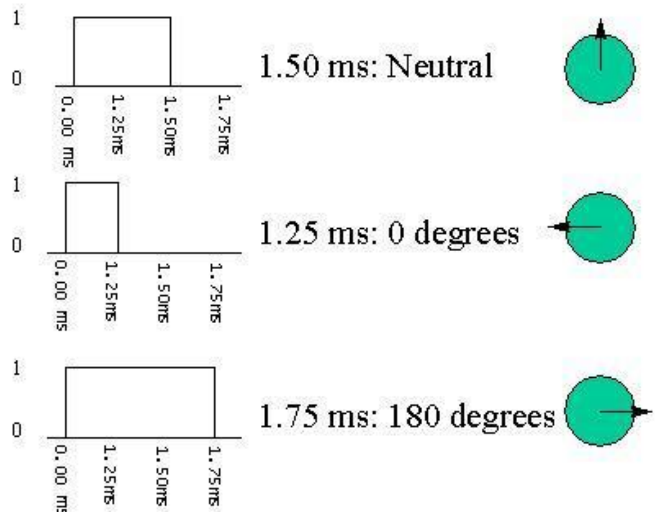


# Servo's

Pulse Width Modulated (PWM) Signal: 50 Hz, i.e. 20ms periods

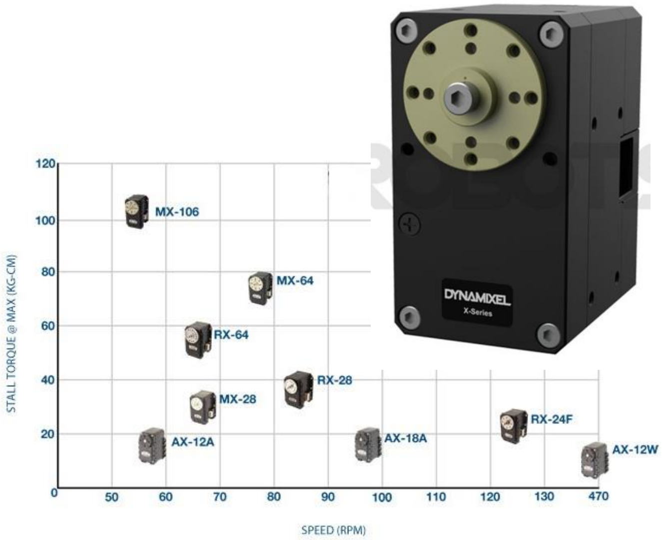



16-ch, 12-bit, 40 Hz – 1000 Hz



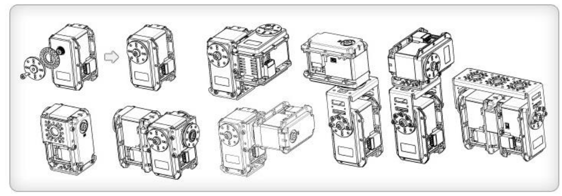

Note: often trimmed Pulse Width

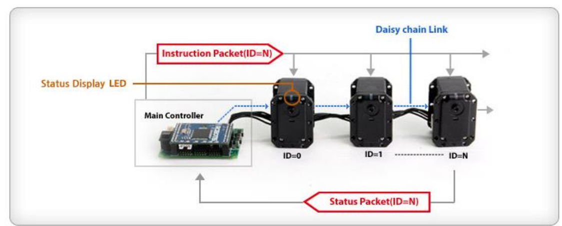
# Dynamixel Servo's





Flexible Construction and Modular Structures

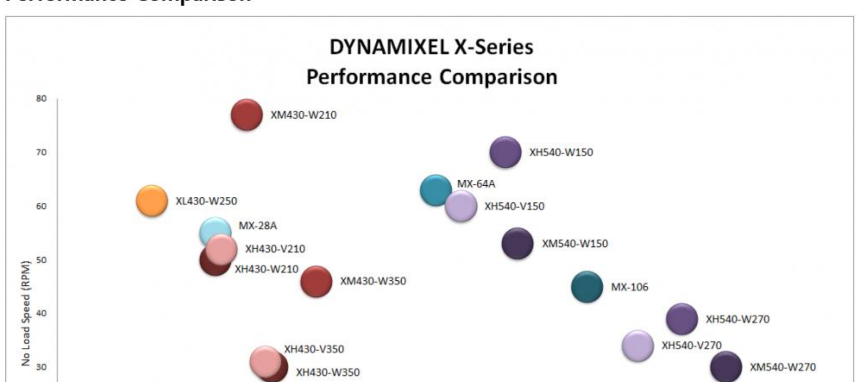







# Servo's

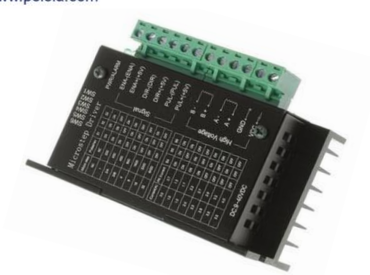

**Performance Comparison**



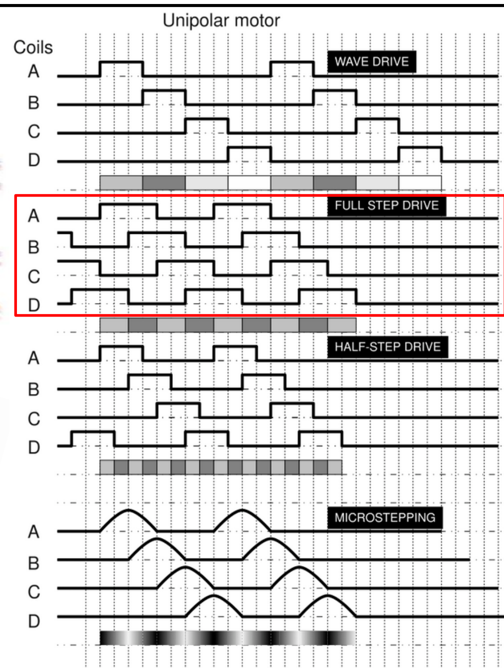

10.6 N.m
30.0 rpm 12.0 V
33.5x58.5x44 (mm³)
165.0 g

9.8N.m ~ 1kgf.m

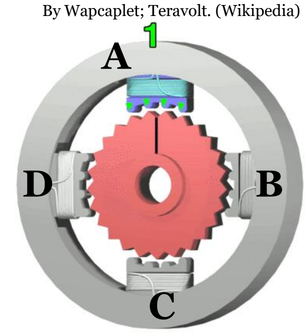
# Stepper Motors

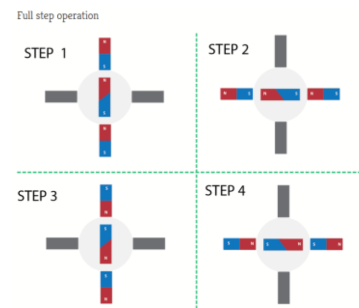



Drivers: low-level, high level



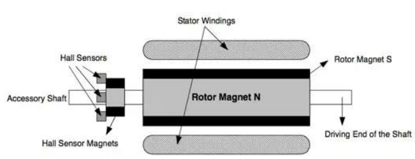

By Wapcaplet; Teravolt. (Wikipedia)

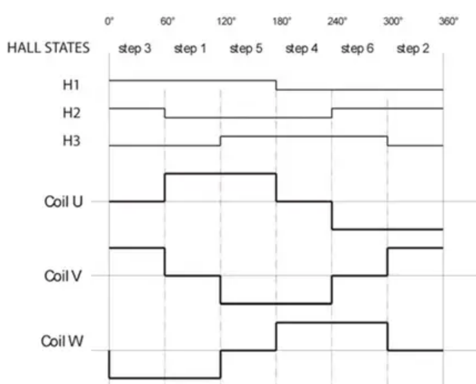


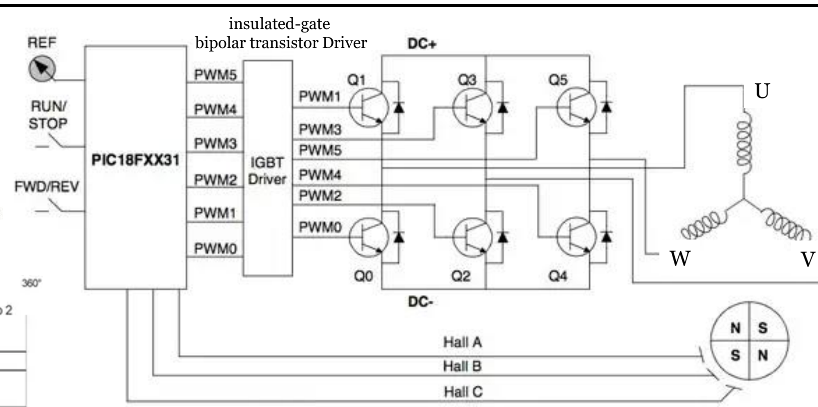



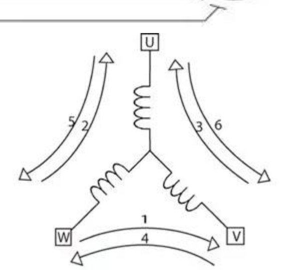



# Brushless Motors



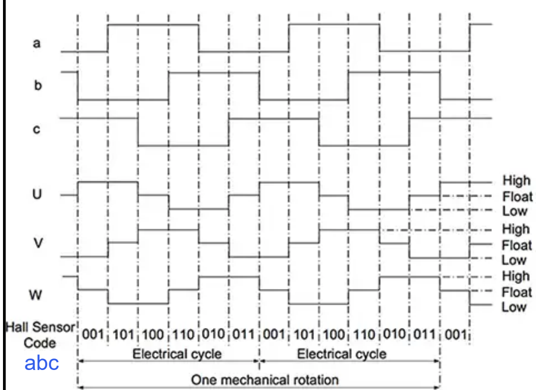



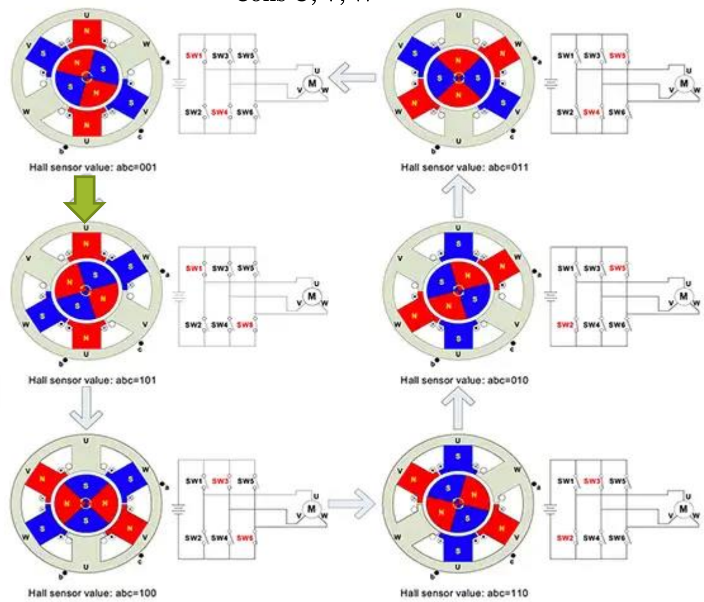




https://www.digikey.com

## Brushless Motors




Coils U, V, W

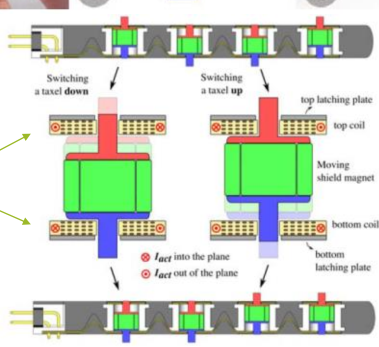



https://www.digikey.com/en/articles/techzone/2016/dec/how-to-power-and-control-brushless-dc-motors



## Flexible Magnetic Actuators


Coils

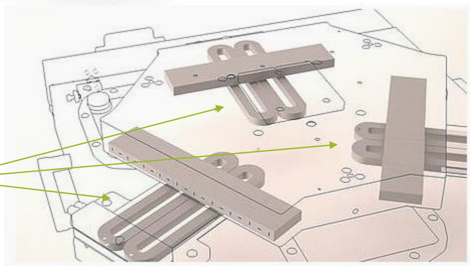



F. Pece et al., MagTics: Flexible and Thin Form Factor Magnetic Actuators for Dynamic and Wearable Haptic Feedback, UIST 2017, Oct. 22–25, 2017, Québec City, Canada

# Robotics Actuators

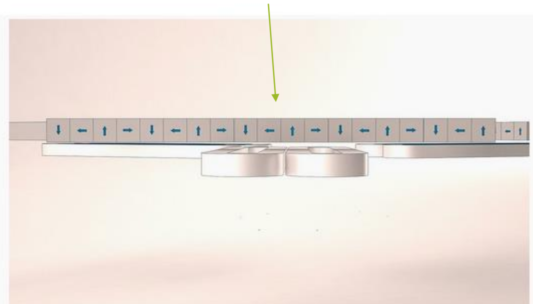
- 6D Magnetic Control
- https://www.pi-usa.us
- pimag-6d-magnetic-levitation

6 Coils



Simple structure: The platform levitates on a magnetic field generated by only six planar coils in the stator

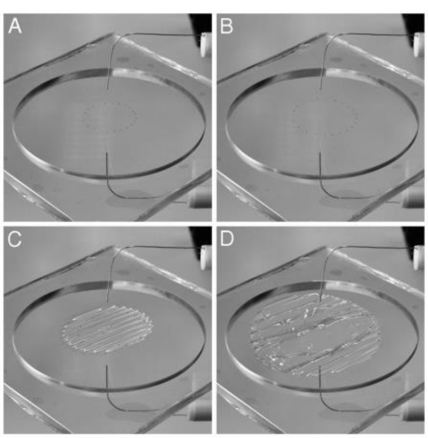
Halbach arrangement of magnets



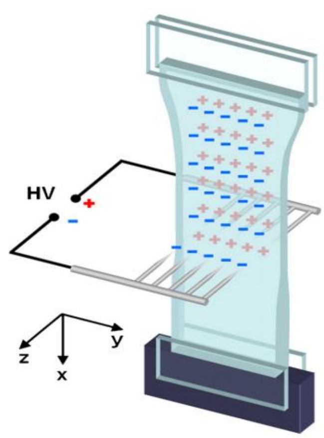
The Halbach arrangement of the magnets makes it possible, to minimize the energy required by the active coils in the stator for carrying the platform, to increase the load carrying capacity and to reduce thermal load



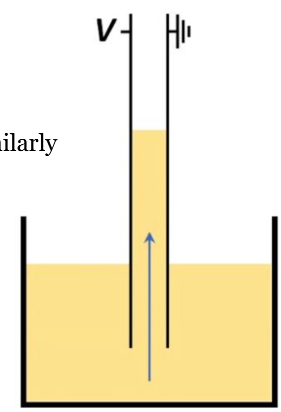
## Artificial Muscles




0 – 25 kV



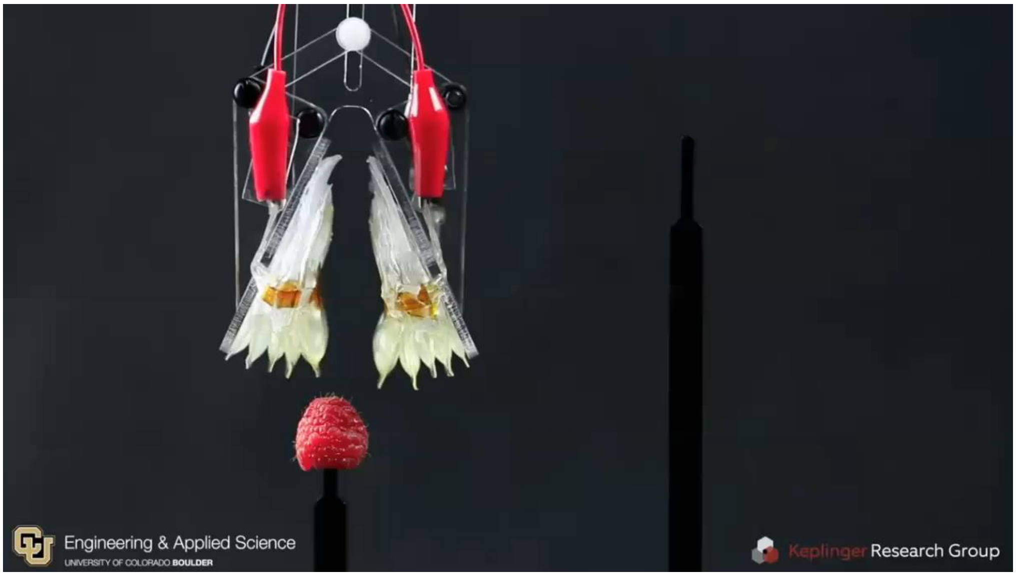

similarly




Maxwell Stress

**Röntgen's electrode-free elastomer actuators without electromechanical pull-in instability by** C. Keplinger, et al. PNAS March 9, 2010 107 (10) 4505-4510; https://doi.org/10.1073/pnas.0913461107

Röntgen WC (1880) Ueber die durch Electricität bewirkten Form—und Volumenänderungen von dielectrischen Körpern. Ann Phys Chem 11:771–786.

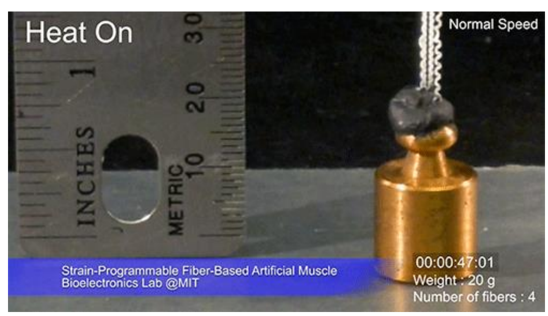


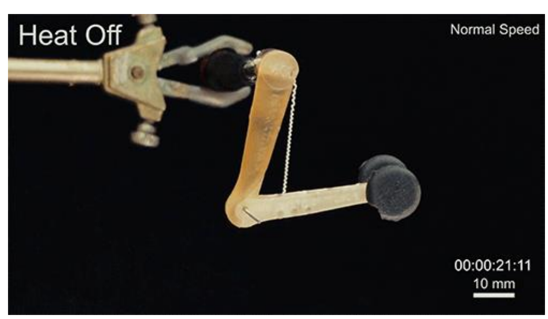

See also TED Talk **The artificial muscles that will power robots of the future by Christoph Keplinger** [https://www.youtube.com/watch?v=ER15KmrB8h8](https://www.youtube.com/watch?v=ER15KmrB8h8)

Universiteit Leiden. Bij ons leer je de wereld kennen

**MIT Artificial Muscles**

- Combination of two dissimilar polymers into a single fiber
- The polymers have very different thermal expansion coefficients (as in bimetals)
- Developed by Mehmet Kanik, Sirma Örgüç, working with Polina Anikeeva, Yoel Fink, Anantha Chandrakasan, and C. Cem Taşan, and five others




http://news.mit.edu/2019/artificial-fiber-muscles-0711

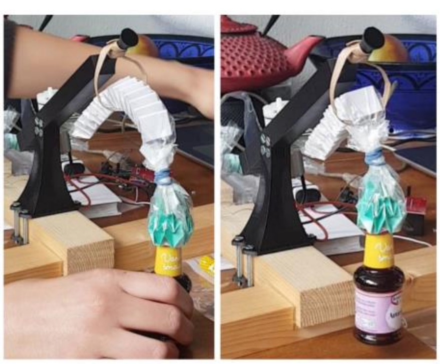
Universiteit Leiden. Bij ons leer je de wereld kennen

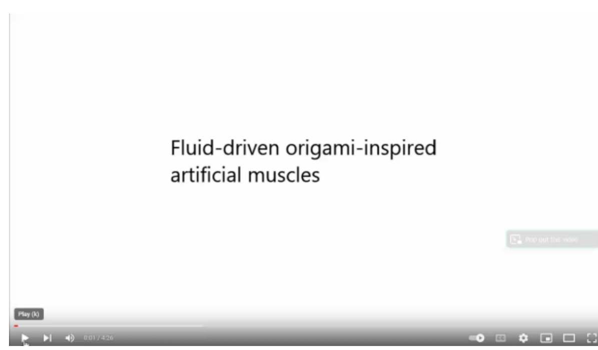
## Spanish Dancer by Micha Heilman and Stella Tsilia



## Artificial Muscle
### S. Raadscheiders, Marton Menyhert, Yven Lommen, Raffi Mirzoyan

Max. Vacuum:
≤-420 mmHg

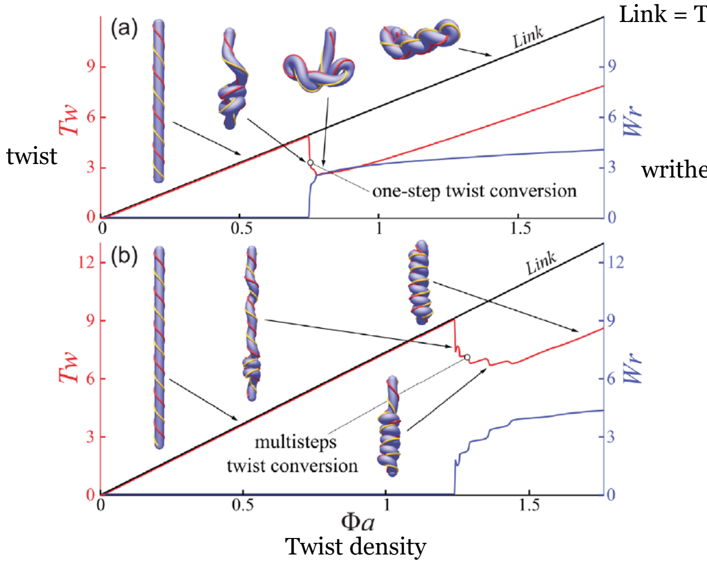





Shuguang Li et al. “Fluid-driven origami-inspired artificial muscles”. In: Proceedings of the National Academy of Sciences 114.50 (2017), pp. 13132–13137.
https://www.youtube.com/watch?v=YK6giJglqjE

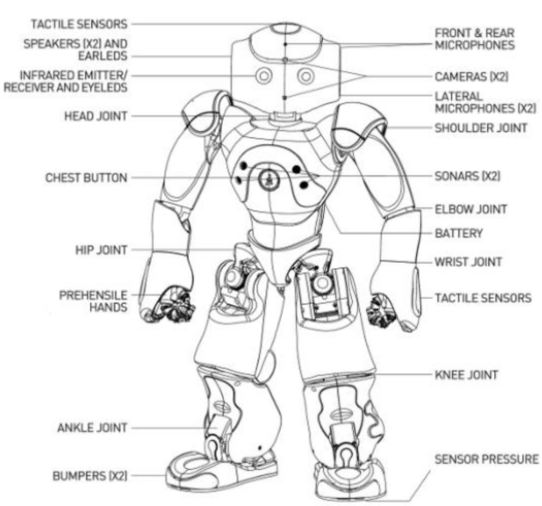
N. Charles, M. Gazzola, and L. Mahadevan, **Topology, Geometry, and Mechanics of Strongly Stretched and Twisted Filaments: Solenoids, Plectonemes, and Artificial Muscle Fibers**
PHYSICAL REVIEW LETTERS 123, 208003 (2019)

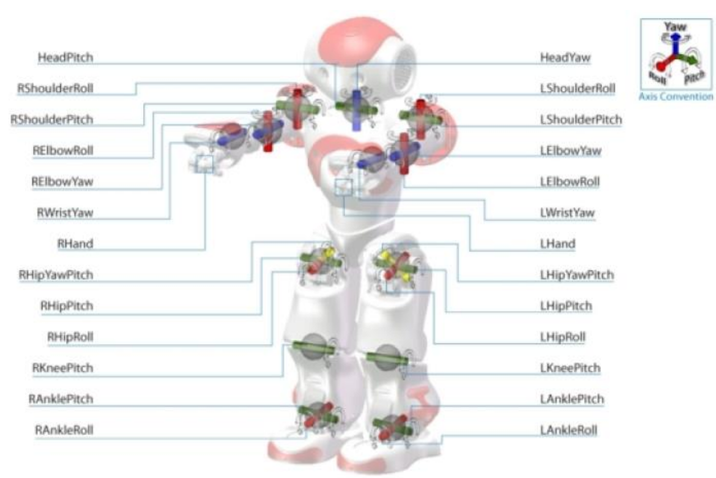
Link = Tw + Wr




Universiteit Leiden. Bij ons leer je de wereld kennen

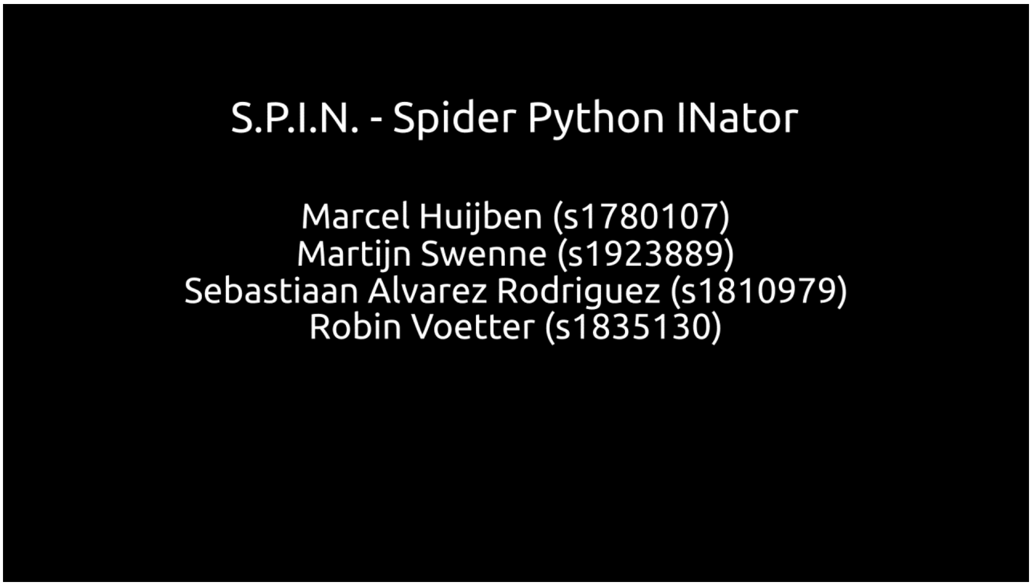
# NAO




http://doc.aldebaran.com/2-1/family/nao_dcm/actuator_sensor_names.html

# Hexapod: S.P.I.N. by M. Huijben, M. Swenne, R. Voeter, S. Alvarez Rodriguez.




2/10/2023

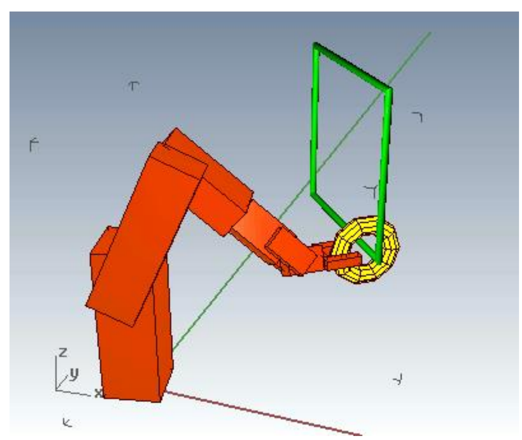
# How to move to a goal?

**Problem: How to move to a goal?**

- Grasp, Walk, Stand, Dance, Follow, etc.

**Solution:**

1. *Program step by step*
   - *Computer Numerical Control (CNC), Automation.*
2. *Inverse kinematics*
   - take end-points and move them to designated points.
3. *Record and Replay movements*
   - by specialist, human, etc.
4. ***Learn the right movements***
   - **Reinforcement Learning**, give a reward when the movement resembles the designated movement.



https://pybullet.org/wordpress/

Universiteit Leiden. Bij ons leer je de wereld kennen

# Configuration Space

Robot Question: Where am I?

Answer:

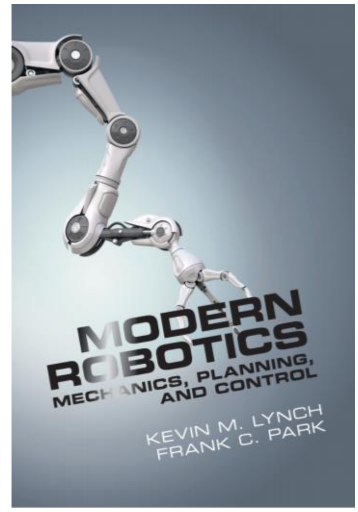
The robot's configuration: a specification of the positions of all points of a robot.

Here we assume:

Robot links and bodies are rigid and of known shape

=>

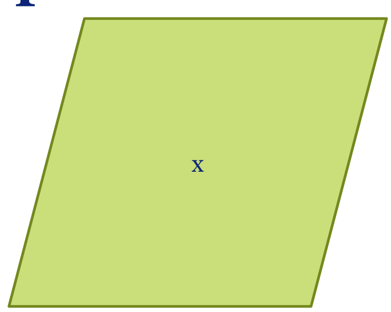
only a few variables needed to describe it's configuration.




K.M. Lynch, F.C. Park, Modern Robotics: Mechanics, Planning and Control, Cambridge University Press, 2017
http://hades.mech.northwestern.edu/index.php/Modern_Robotics



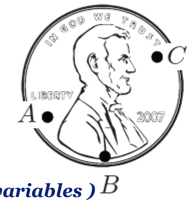
# Configuration Space

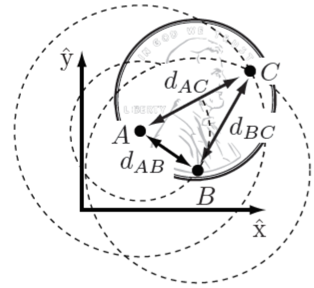



**Degrees of Freedom of a Rigid Body:**

the smallest number of real-valued coordinates needed to represent its configuration

# Configuration Space




**In the plane:**
**Assume a coin (heads) with 3 points A, B, C on it.**

In the plane A,B,C have 6 degrees of freedom: ***($x_A$,$y_A$) , ($x_B$,$y_B$) , ($x_C$,$y_C$)  ( 6 variables )***
A coin is rigid => 3 extra constraints on distances: ***$d_{AB}$, $d_{AC}$, $d_{BC}$*** ***( 3 constraints )***
These are fixed, wherever the location of the coin.

1. The coin and hence **A** can be placed everywhere => ($x_A$,$y_A$) free to choose.
2. B can only be placed under the constraint that its distance to **A** would be equal to **$d_{AB}$**. (1 constraint)

=> freedom to turn the coin around A with angle $\varphi_{AB}$ => ($x_A$, $y_A$, $\varphi_{AB}$ ) are free to choose.

3. **C** should be placed at distance **$d_{AC}$**, **$d_{BC}$** from **A** and **B**, respectively (2 constraints)

=> only 1 possibility, hence no degree of freedom added.

**Degrees of Freedom (DOF) of a Coin**

= sum of freedoms of the points – number of independent constraints
= number of variables – number of independent equations **= 6 – 3 = 3**



# Configuration Space

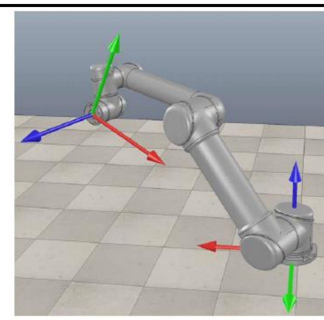
## [1] Definition 2.1.

The **configuration** of a robot is a complete specification of the position of every point of the robot.

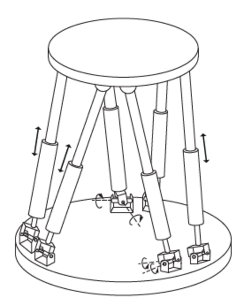
The minimum number *n* of real-valued coordinates needed to represent the configuration is the number of **degrees of freedom (dof)** of the robot.

The *n*-dimensional space containing all possible configurations of the robot is called the **Configuration Space (C-space)**.

The configuration of a robot is represented by a point in its C-space.



Open-chain robot: Manipulator (in V-REP). [1]

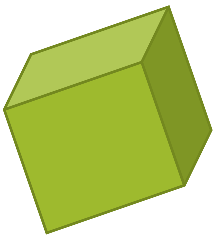


Closed-chain robot: Stewart-Gough platform. [1]

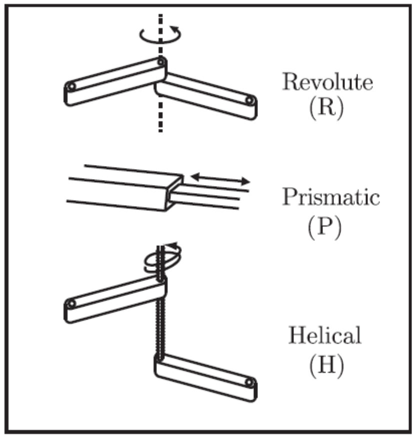
Universiteit Leiden. Bij ons leer je de wereld kennen

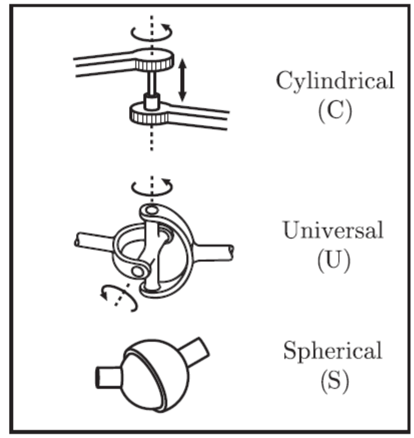
# Degrees of Freedom of a Robot

- A rigid body in 3D Space has **6 DOF**



- A joint can be seen to put constraints on the rigid bodies it connects
- It also allows freedom to move relative to the body it is attached to.

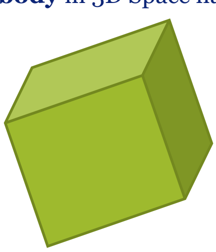


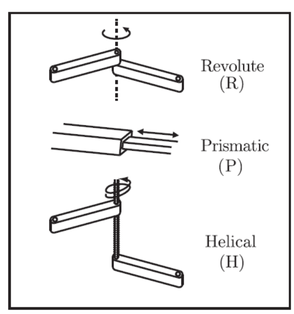

Universiteit Leiden. Bij ons leer je de wereld kennen

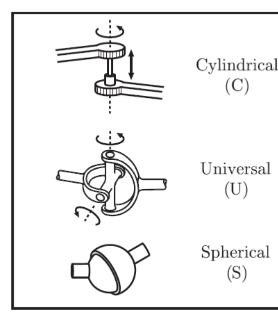
# Degrees of Freedom of a Robot

- A **rigid body** in 3D Space has 6 DOF



- A **joint** can be seen to put constraints on the rigid bodies it connects
- It also allows freedom to move relative to the body it is attached to.



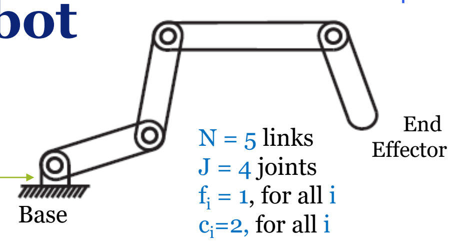




| Joint type | dof $f$ | Constraints $c$ between two planar 2D rigid bodies | Constraints $c$ between two spatial 3D rigid bodies |
|---|---|---|---|
| Revolute (R) | 1 | 2 | 5 |
| Prismatic (P) | 1 | 2 | 5 |
| Helical (H) | 1 | N/A | 5 |
| Cylindrical (C) | 2 | N/A | 4 |
| Universal (U) | 2 | N/A | 4 |
| Spherical (S) | 3 | N/A | 3 |

Universiteit Leiden. Bij ons leer je de wereld kennen

# Degrees of Freedom of a Robot

Planar Mechanism DOF = 4




## Proposition (Grübler's formula)

Consider a mechanism consisting of

- N links, where ground (!) is also regarded as a link
- J number of joints
- m number of degrees of freedom of a rigid body
  (m = 3 for planar (2D) mechanisms and m = 6 for spatial (3D) mechanisms)
- $f_i$ the number of freedoms provided by joint i
- $c_i$ the number of constraints provided by joint i, where $f_i + c_i = m$ for all i.

Then *Grübler's formula* for the number of degrees of freedom of the robot is

$$dof = m(N-1) - \sum_{i=1}^{J} c_i = m(N-1-J) + \sum_{i=1}^{J} f_i$$

This formula holds only if all joint constraints are independent. If they are not independent then the formula provides a lower bound on the number of degrees of freedom.



## Joint reactions in rigid body mechanisms with dependent constraints

Marek Wojtyra *

*Warsaw University of Technology, Institute of Aeronautics and Applied Mechanics, ul. Nowowiejska 24, 00-665 Warsaw, Poland*

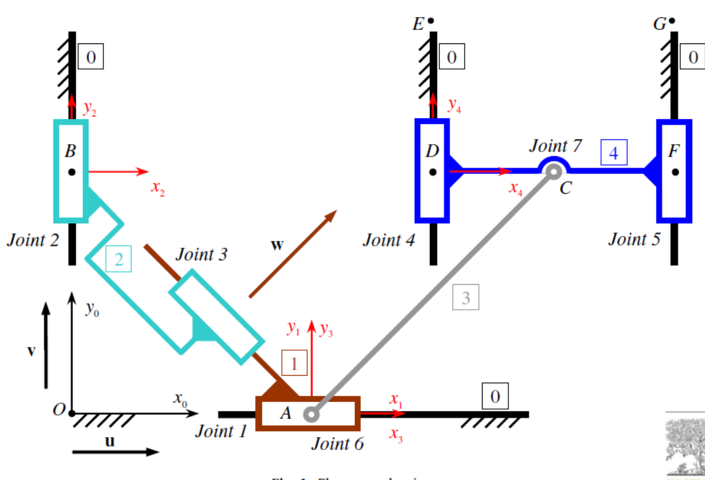



**Fig. 1.** Planar mechanism.

$$dof = m(N-1) - \sum_{i=1}^{J} c_i = m(N-1-J) + \sum_{i=1}^{J} f_i$$

2009

Links
1

3

3

6

3

1

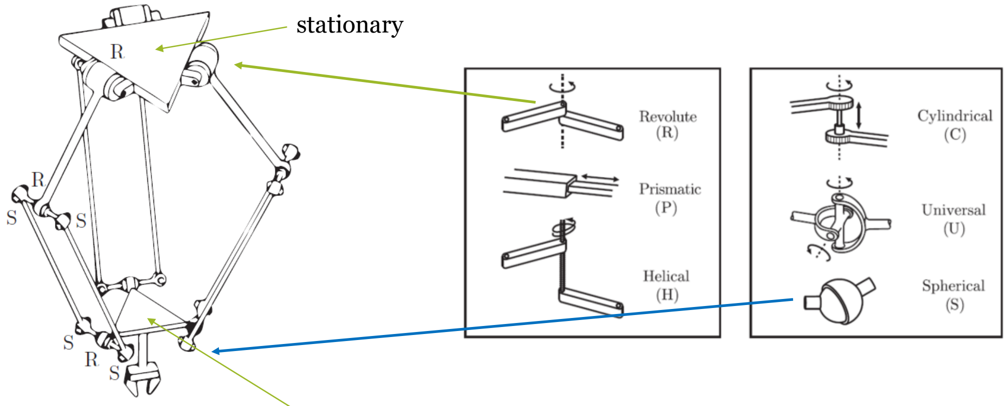



**Figure 2.8:** The Delta robot.

**Example 2.7** (Delta robot). The Delta robot of Figure 2.8 consists of two platforms – the lower one mobile, the upper one stationary – connected by three legs. Each leg contains a parallelogram closed chain and consists of three revolute joints, four spherical joints, and five links. Adding the two platforms, there are $N = 17$ links and $J = 21$ joints (nine revolute and 12 spherical). By Grübler's formula,

$$\boxed{\text{dof} = 6(17 - 1 - 21) + 9(1) + 12(3) = 15.}$$

- Links: 1 + 3 + 3 + 6 + 3 + 1 = 17
- Joints: 21: 9x R(1 dof) and 12 x S(3 dof)
- m= 6

$$dof = m(N-1) - \sum_{i=1}^{J} c_i = m(N-1-J) + \sum_{i=1}^{J} f_i$$



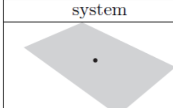
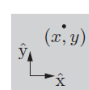
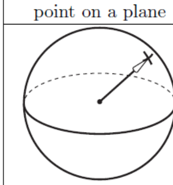
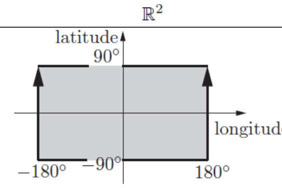
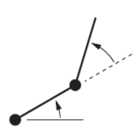
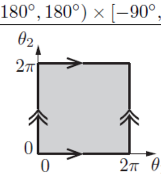
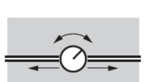
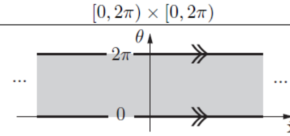
# Systems
## and their
# Topologies

Note: $S^1 \times S^1 = T^2$ (not $S^2$)

## Coordinates can be:

**Explicit Coordinates**
- Euclidean (x,y)
- Polar (r,φ)
- Combined (x,y) x (r, φ)

**Implicit Coordinates**
- { (x,y,z) | x²+y²+z²=1 }

| system | topology | sample representation |
|---|---|---|
|  point on a plane |  $\mathbb{E}^2$ |   $\mathbb{R}^2$ |
|  spherical pendulum |  $S^2$ |   $[-180°, 180°) \times [-90°, 90°]$ |
|  2R robot arm |  $T^2 = S^1 \times S^1$ |   $[0, 2\pi) \times [0, 2\pi)$ |
|  rotating sliding knob |  $\mathbb{E}^1 \times S^1$ |   $\mathbb{R}^1 \times [0, 2\pi)$ |

# C-Space (Configuration Space)

**How to describe a rigid body's position and orientation in C-Space?**

Fixed reference frame {s}

Reference fame attached to body {b}

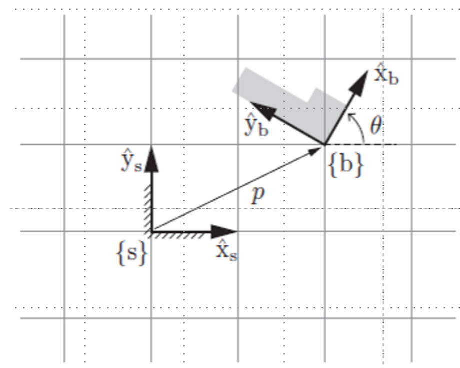
In $\mathbb{R}^3$ described by a 4x4 matrix with 10 constraints

(constraints, e.g.: unit-length, orthogonal)

Note: a point in $\mathbb{R}^3 x S^2 x S^1$

Matrix can be used to:

1. Translate or rotate a vector or a frame
2. Change the representation of a vector or a frame
   - for example from relative to {s} to relative to {b}

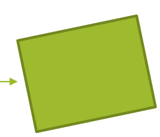



in the plane $\mathbb{R}^2 x S^1$



# C-Spaces

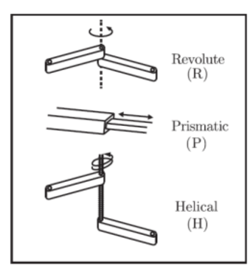
C-space of a rigid body in the plane = $\mathbb{R}^2 x S^1$ as configuration can be denoted as $(x,y,\theta)$, i.e., location $(x,y)$ in $\mathbb{R}^2$ and angle $\theta$ in $S^1$.

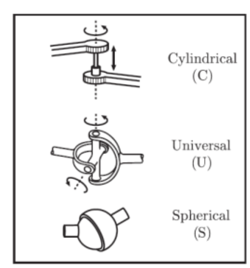


C-space of a Prismatic-Revolute (PR) robot arm is equal to $\mathbb{R}^1 x S^1$

C-space of a 2R robot arm is $S^1 x S^1 = T^2$

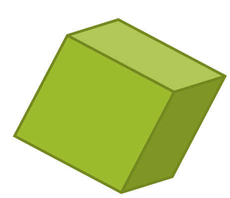
C-space of a 3R robot arm is $S^1 x S^1 x S^1 = T^3$




C-space of a planar mobile robot with a 2R robot arm is $\mathbb{R}^2 x S^1 x T^2 = \mathbb{R}^2 x T^3$

C-space of a rigid body in space is $\mathbb{R}^3 x S^2 x S^1$



# Task Space and Work Space

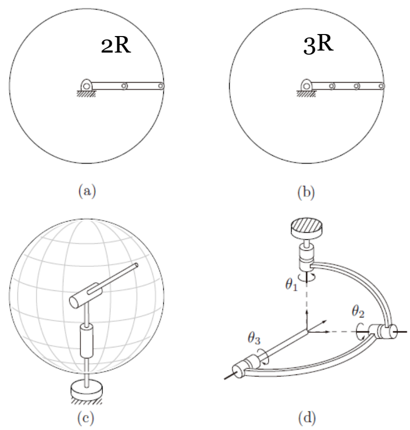
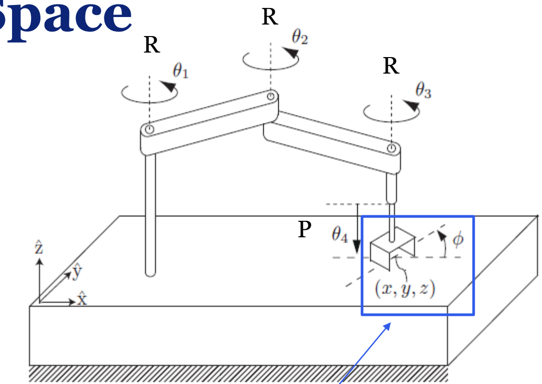



Figure 2.12: Examples of workspaces for various robots: (a) a planar 2R open chain; (b) a planar 3R open chain; (c) a spherical 2R open chain; (d) a 3R orienting mechanism.

The **workspace** is a specification of the configurations that the end-effector of the robot can reach.




The SCARA robot is an RRRP open chain that is widely used for tabletop pick-and-place tasks. The end-effector configuration is completely described by $(x, y, z, \varphi)$

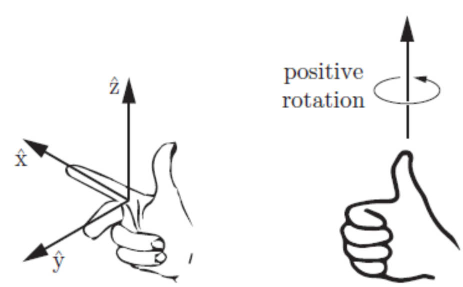
⇒ **task space** $R^3 \times S^1$ and
⇒ **workspace** as the reachable points in (x, y, z), since all orientations φ can be achieved at all reachable points.



# Rigid Body Motion

**Rigid-body position and orientation** $(x, y, z, \varphi, \theta, \psi) \in \mathbb{R}^3 \times S^2 \times S^1$

- Can also be described by 4x4 matrix with 10 constraints.
- In general 4x4 matrices can be used for
  - Location
  - Translation + rotation of a vector or frame
  - Transformation of coordinates between frames



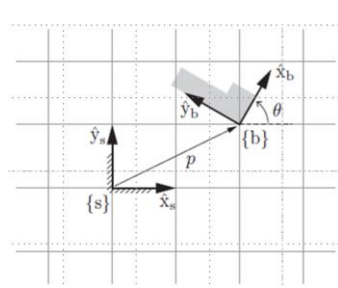

- Velocity of a rigid body: $(\partial x/\partial t,\ \partial y/\partial t,\ \partial z/\partial t,\quad \partial \phi/\partial t,\ \partial \theta/\partial t,\ \partial \psi/\partial t)$

  i.e., changes in location and orientation per unit of time

**Exponential coordinates:**

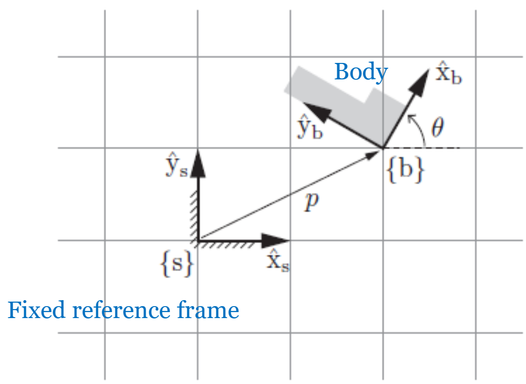
Every rigid-body configuration can be achieved by:

- Starting in the fixed home frame and integrating a constant twist for a specified time.
- Direction of a screw axis and scalar to indicate how far the screw axis must be followed




Similarly in the plane

# Rigid Body Motions in the Plane




Translation

$$p = p_x \hat{x}_s + p_y \hat{y}_s.$$

Rotation

$$\begin{aligned} \hat{x}_b &= \cos\theta\, \hat{x}_s + \sin\theta\, \hat{y}_s, \\ \hat{y}_b &= -\sin\theta\, \hat{x}_s + \cos\theta\, \hat{y}_s. \end{aligned}$$

**Figure 3.3:** The body frame {b} is expressed in the fixed-frame coordinates {s} by the vector $p$ and the directions of the unit axes $\hat{x}_b$ and $\hat{y}_b$. In this example, $p = (2, 1)$ and $\theta = 60°$, so $\hat{x}_b = (\cos\theta, \sin\theta) = (0.5, 1/\sqrt{2})$ and $\hat{y}_b = (-\sin\theta, \cos\theta) = (-1/\sqrt{2}, 0.5)$.



# Rigid Body Motions in the Plane

Previously:

$$p = p_x \hat{x}_s + p_y \hat{y}_s.$$

$$\begin{aligned} \hat{x}_b &= \cos\theta\, \hat{x}_s + \sin\theta\, \hat{y}_s, \\ \hat{y}_b &= -\sin\theta\, \hat{x}_s + \cos\theta\, \hat{y}_s. \end{aligned}$$

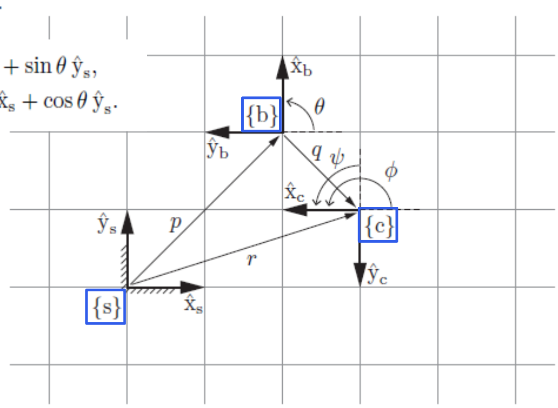



**Figure 3.4:** The frame {b} in {s} is given by $(P, p)$, and the frame {c} in {b} is given by $(Q, q)$. From these we can derive the frame {c} in {s}, described by $(R, r)$. The numerical values of the vectors $p$, $q$, and $r$ and the coordinate-axis directions of the three frames are evident from the grid of unit squares.

{b} relative to {s}

$$p = \begin{bmatrix} p_x \\ p_y \end{bmatrix}$$

$$P = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

{c} relative to {b}

$$q = \begin{bmatrix} q_x \\ q_y \end{bmatrix}, \quad Q = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix}$$

{c} relative to {s}

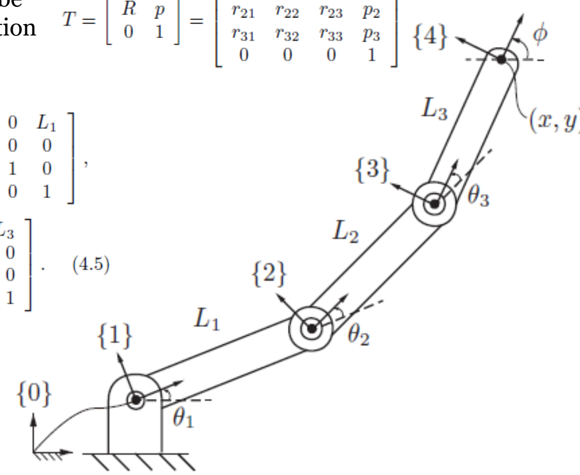
$$r = \begin{bmatrix} r_x \\ r_y \end{bmatrix}, \quad R = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}$$

**Note and verify:**
R = PQ, convert Q to {s}-frame
r = Pq+p, convert q to {s}-frame and add p

# Forward Kinematics

The forward kinematics of 3R Planar Open Chain can be written as a product of four homogeneous transformation matrices: $T_{04} = T_{01}T_{12}T_{23}T_{34}$, where

$$T = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_1 \\ r_{21} & r_{22} & r_{23} & p_2 \\ r_{31} & r_{32} & r_{33} & p_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$




$$T_{01} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad T_{12} = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & L_1 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$T_{23} = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & L_2 \\ \sin\theta_3 & \cos\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad T_{34} = \begin{bmatrix} 1 & 0 & 0 & L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (4.5)$$

Home position M:

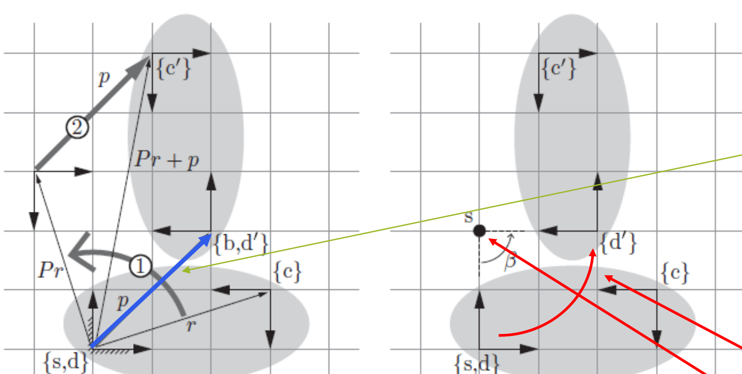
$$M = \begin{bmatrix} 1 & 0 & 0 & L_1 + L_2 + L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



# Rigid Body Motions in the Plane

{c} described by (R,r)

$$r = \begin{bmatrix} r_x \\ r_y \end{bmatrix}, \quad R = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}$$




(a) (b)

Move rigid body such that {d} coincides with {d'}.

$$p = \begin{bmatrix} p_x \\ p_y \end{bmatrix} \quad P = [\hat{x}_b \ \hat{y}_b] = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Then {c'} described by (R',r'):

$$R' = PR,$$
$$r' = Pr + p,$$

(P, p) can be used to
Displace an elliptical rigid body {d} coinciding with {s} to {d'} coinciding with {b}: Rotation **P** followed by translation **p**

Note: SCREW MOTION
The above **rotation** (1) followed by a **translation** (2) can also be expressed as

a rotation of the rigid-body about a fixed point s by an angle β

# Rigid Body Motions in the Plane

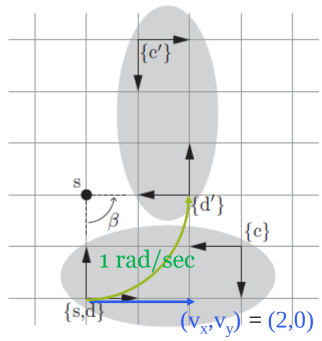
Note: SCREW MOTION
The above rotation followed by a translation can also be expressed as a rotation of the rigid-body about a fixed point s by an angle $\beta$

$(\beta, s_x, s_y)$ , where $(s_x, s_y) = (0, 2)$

In the {s}-frame rotate 1 rad/sec with speed $(v_x,v_y) = (2, 0)$ is denoted as:
$S = (\omega, v_x, v_y) = (1, 2, 0)$

Following the screw-axis for an angle $\theta = \pi/2$ gives the displacement we want:
$S\theta = (\pi/2, \pi, 0)$

These are called the **exponential coordinates** for the planar rigid-body displacement.




{c} described by (R,r)

$$r = \begin{bmatrix} r_x \\ r_y \end{bmatrix}, \qquad R = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}$$

Move rigid body such that {d} coincides with {d'}.

$$p = \begin{bmatrix} p_x \\ p_y \end{bmatrix} \qquad P = [\hat{x}_b \ \ \hat{y}_b] = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Then {c'} described by (R',r'):

$$R' = PR,$$
$$r' = Pr + p,$$

Note:
- distance = vt
- distance along quarter circle with radius 2 equals $\pi$.



# Forward Kinematics: Product of Exponentials

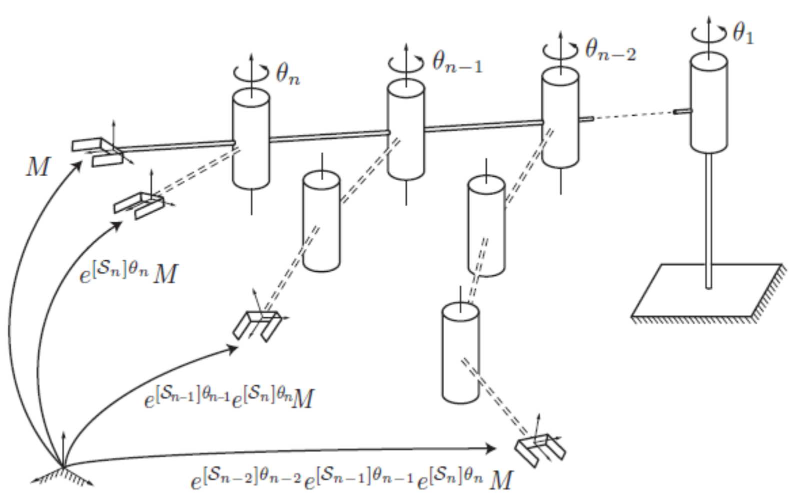



**Figure 4.2:** Illustration of the PoE formula for an $n$-link spatial open chain.

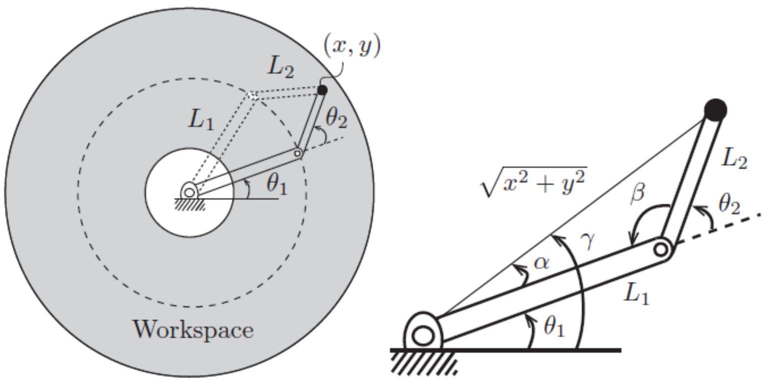
PoE parameters also known as Euler-Rodrigues parameters.

There are many other representations:
- for example Denavit-Hartenberg (1955) representation is very popular, but can be cumbersome

In velocity kinematics Jacobians are used.

# Inverse Kinematics

Which angles $\theta_1$, and $\theta_2$ will lead to location (x,y)?




(a) A workspace, and lefty and righty configurations.

(b) Geometric solution.

**Figure 6.1:** Inverse kinematics of a 2R planar open chain.

Law of cosines gives:

$$L_1^2 + L_2^2 - 2L_1L_2\cos\beta = x^2 + y^2$$

, hence

$$\beta = \cos^{-1}\left(\frac{L_1^2 + L_2^2 - x^2 - y^2}{2L_1L_2}\right)$$

,and similarly

$$\alpha = \cos^{-1}\left(\frac{x^2 + y^2 + L_1^2 - L_2^2}{2L_1\sqrt{x^2+y^2}}\right)$$

γ= atan2(y,x)

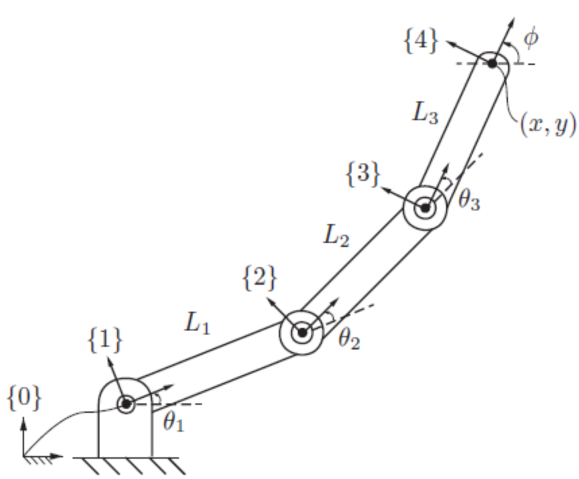
Answer:

$$\theta_1 = \gamma - \alpha, \qquad \theta_2 = \pi - \beta$$

In general: IK-Solvers, Newton-Raphson, etc.



# Inverse Kinematics

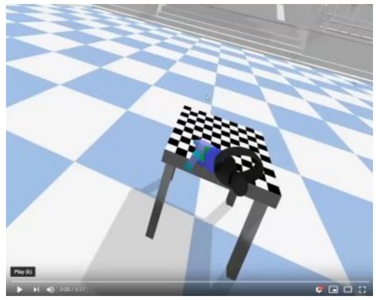



How would you solve this?

Which angles $\theta_1$, $\theta_2$, and $\theta_3$ will lead to location (x,y)?

# Real Time Physics Modelling

https://pybullet.org/wordpress/



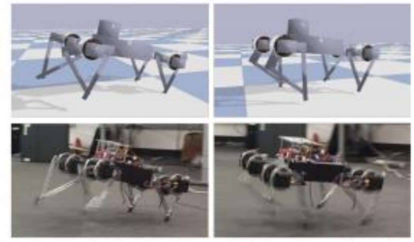
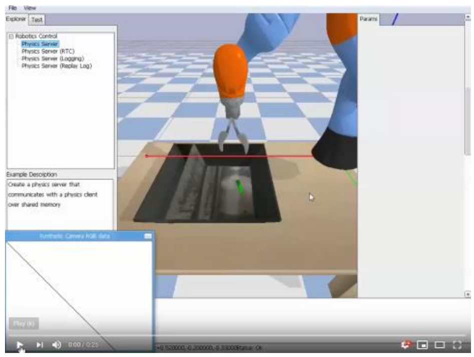
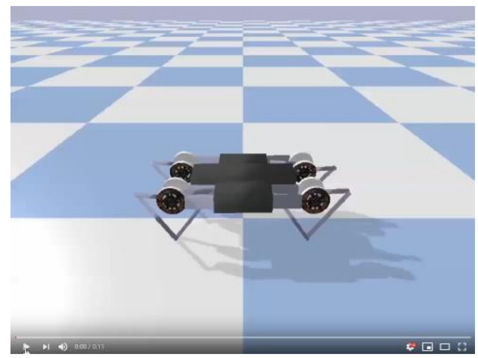


Fig. 1: The simulated and the real Minitaurs learned to gallop using deep reinforcement learning.

pybullet KUKA
grasp training

Using Tensorflow
OpenAI gym
Baselines
DeepQNetworks (DQNs)





Universiteit Leiden. Bij ons leer je de wereld kennen

# Organization and Overview

**Lecturer:**
Dr Erwin M. Bakker ( erwin@liacs.nl )
Room 126a and LIACS Media Lab (LML)

**Teaching assistant:**
Mor Puigventos ( email )
TBA (email)

**Period:** February 6th - May 22nd 2023
**Time:** Monday 15.15 - 17.00
**Place (Rooms):**
a) Gorlaeus - Lecture Hall C3
b) Sylvius - 15.31
c) Van Steenis - E0.04
d) Oortgebouw - Sitterzaal

**Schedule (tentative, visit regularly):**

| Date | Room | Subject |
|---|---|---|
| 6-2 | a | Introduction and Overview |
| 13-2 | a | Locomotion and Inverse Kinematics |
| 20-2 | b | Robotics Sensors and Image Processing |
| 27-2 | a | SLAM + SLAM Workshop |
| 6-3 | c | Mobile Robot Challenge Introduction |
| 13-3 | a | Project Proposals I (by students) |
| 20-3 | d | Project Proposals II (by students) |
| 27-3 | d | Robotics Vision (Week 13, start 15.30) |
| 3-4 | d | Robotics Reinforcement Learning&Workshop |
| 10-4 | | No Class (Eastern) |
| 17-4 | d | Project Progress I (by students) |
| 24-4 | d | Project Progress II (by students) |
| 1-5 | d | Mobile Robot Challenge I |
| 8-5 | a | Mobile Robot Challenge II |
| 15-5 | d | Project Demos I |
| 22-5 | d | Project Demos II |
| 29-5 | | Whit Monday |
| 5-6 | | Project Deliverables |

Website: http://liacs.leidenuniv.nl/~bakkerem2/robotics/

Grading (6 ECTS):
- Presentations and Robotics Project (60% of grade).
- Class discussions, attendance, assignments (pass/no pass) 2 workshops (0-10) (2 x 20% = 40% of grade).
- It is necessary to be at every class and to complete every workshop and assignment.

Universiteit Leiden. Bij ons leer je de wereld kennen

# Robotics Homework II

Visit http://modernrobotics.org and obtain the pdf of the book.

**Read Chapters 1 and 2,** and answer the following exercises: TBA

The exercises of Homework II will be available on Tuesday (14-2) on

BrightSpace
and
https://liacs.leidenuniv.nl/~bakkerem2/robotics/



# References

1. K.M. Lynch, F.C. Park, Modern Robotics: Mechanics, Planning and Control, Cambridge University Press, 2017. ( DOI: 10.1017/9781316661239 )
2. https://pybullet.org/wordpress/