

Robotics

Erwin M. Bakker | LIACS Media Lab

14-2 2022



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Leiden

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Organization and Overview

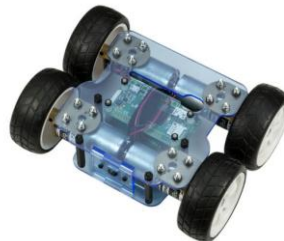
Period: February 7th – May 23rd 2022
Time: Monday 16.15 – 18.00
Place: Room 407 - 409
Lecturer: Erwin M. Bakker (erwin@liacs.nl)
Assistant: Hainan Yu (h.yu@liacs.leidenuniv.nl)

NB Register on Brightspace

Schedule:

7-2	Introduction and Overview
14-2	Locomotion and Inverse Kinematics
21-2	Robotics Sensors and Image Processing
28-2	SLAM + SLAM Workshop
7-3	Mobile Robot Challenge Introduction
14-3	Project Proposals I (presentation by students)
21-3	Project Proposals II (presentation by students)
28-3	Robotics Vision
4-4	Robotics Reinforcement Learning
11-4	Robotics Reinforcement Learning Workshop II
18-4	No Class (Eastern)
25-4	Project Progress I (presentations by students)
2-5	Project Progress II (presentations by students)
9-5	Mobile Robot Challenge
16-5	Project Demos I
23-5	Project Demos II

Website: <http://liacs.leidenuniv.nl/~bakkerem2/robotics/>



Grading (6 ECTS):

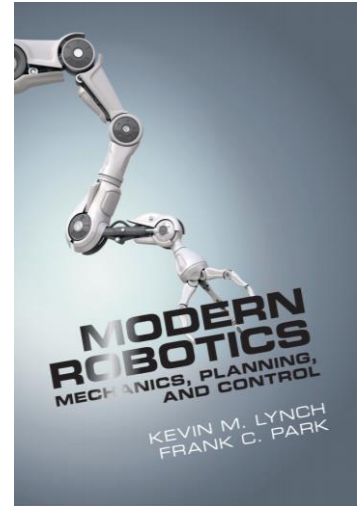
- Presentations and Robotics Project (60% of grade).
- Class discussions, attendance, workshops and assignments (40% of grade).
- It is necessary to be at every class and to complete every workshop and assignment.

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Overview

- Robotic Actuators
- Configuration Space
- Rigid Body Motion
- Forward Kinematics
- Inverse Kinematics

- Link: <http://modernrobotics.org>

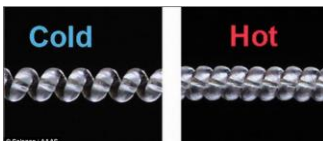
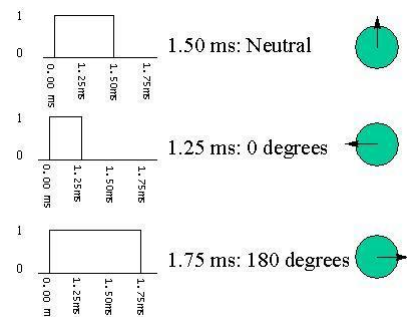


K.M. Lynch, F.C. Park, Modern Robotics: Mechanics, Planning and Control, Cambridge University Press, 2017

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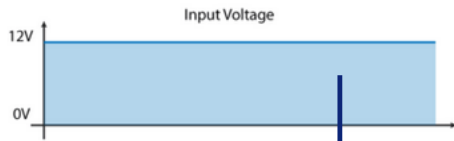
Robotics Actuators

- **Electro motors**
- **Servo's**
- **Stepper Motors**
- **Brushless motors**
- Solenoids
- Hydraulic, pneumatic actuator's
- Magnetic actuators
- Artificial Muscles
- Etc.

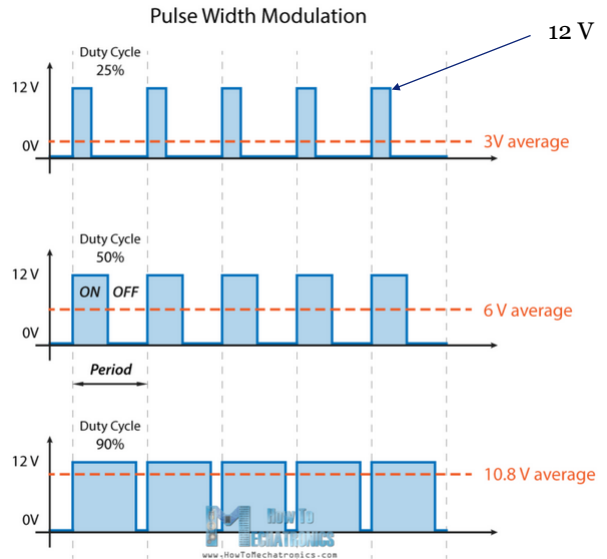


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DC Motors



Speed



<https://howtomechanics.com/how-it-works/electronics/how-to-make-pwm-dc-motor-speed-controller-using-555-timer-ic/>

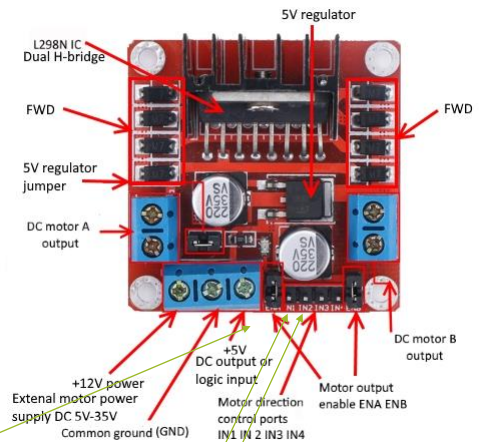
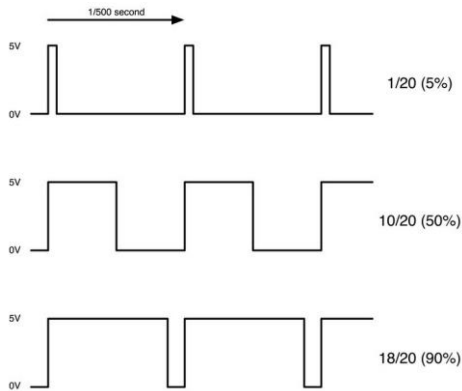
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DC Electro Motors:

- Duty Cycle



Brushed DC (BDC) motor controller Using L298N



```










Loop:
  PWM(ENA,128);
  DigitalWrite(IN1, HIGH);
  DigitalWrite(IN2, LOW);
  PWM(ENB,64);
  DigitalWrite(IN3, HIGH);
  DigitalWrite(IN4, LOW);
  
```

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DC Motor Controllers

Pololu Simple Motor Controllers

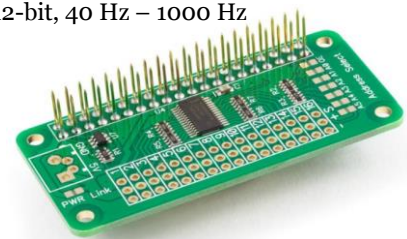
- USB, TTL Serial, Analog, RC Control, I2C

	Original versions, not recommended for new designs (included for comparison purposes)					G2 versions, released November 2018			
	 SMC 18v7	 SMC 18v15	 SMC 24v12	 SMC 18v25	 SMC 24v23	 SMC G2 18v15	 SMC G2 24v12	 SMC G2 18v25	 SMC G2 24v19
Minimum operating voltage:	5.5 V	5.5 V	5.5 V	5.5 V	5.5 V	6.5 V	6.5 V	6.5 V	6.5 V
Recommended max operating voltage:	24 V(1)	24 V(1)	34 V(2)	24 V(1)	34 V(2)	24 V(1)	34 V(2)	24 V(1)	34 V(2)
Max nominal battery voltage:	18 V	18 V	28 V	18 V	28 V	18 V	28 V	18 V	28 V
Max continuous current (no additional cooling):	7 A	15 A	12 A	25 A	23 A	15 A	12 A	25 A	19 A
USB, TTL serial, Analog, RC control:	✓	✓	✓	✓	✓	✓	✓	✓	✓
I ² C control:						✓	✓	✓	✓
Hardware current limiting:						✓	✓	✓	✓
Reverse voltage protection:						✓	✓	✓	✓

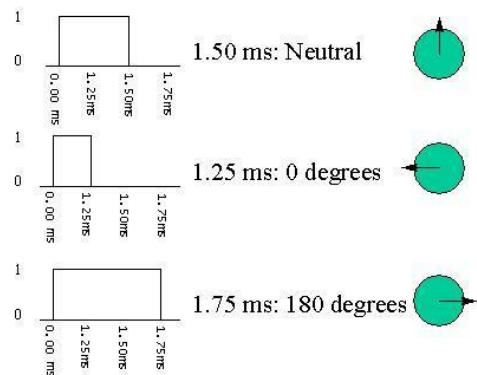
<https://www.pololu.com/category/94/pololu-simple-motor-controllers>

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Servo's



16-ch, 12-bit, 40 Hz – 1000 Hz



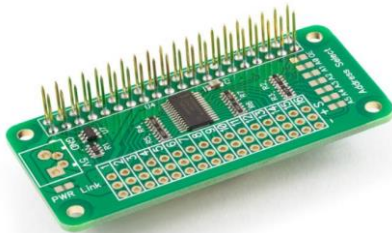
www.pololu.com

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Servo's

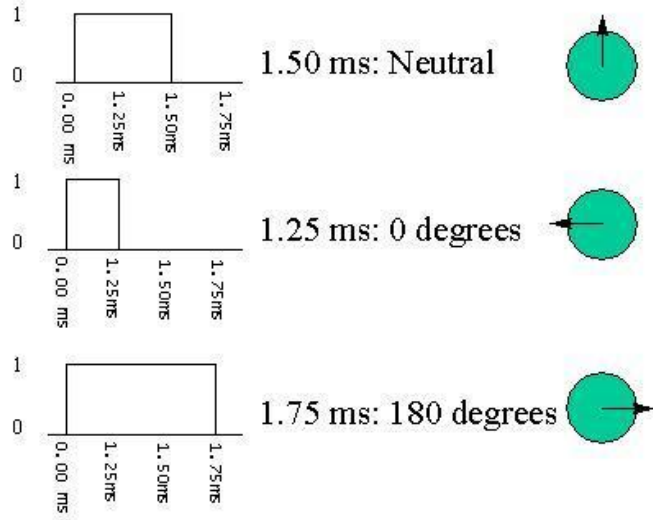


www.potoku.com



16-ch, 12-bit, 40 Hz – 1000 Hz

Pulse Width Modulated (PWM) Signal: 50 Hz, i.e. 20ms periods



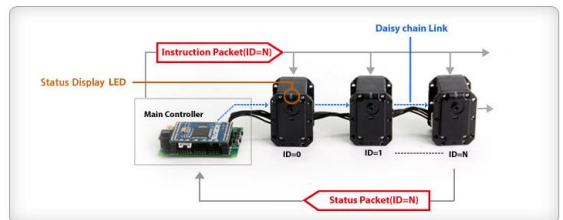
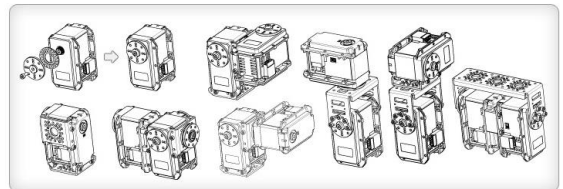
Note: often trimmed Pulse Width

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Dynamixel Servo's



Flexible Construction and Modular Structures

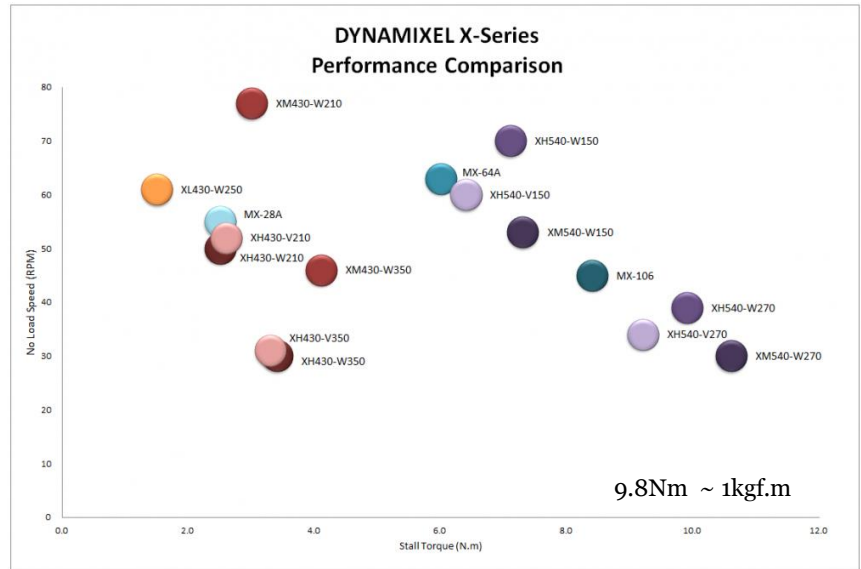


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Servo's



Performance Comparison



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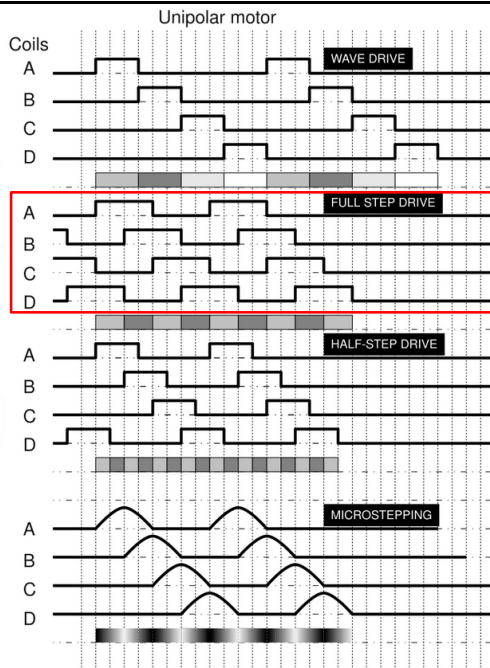
Stepper Motors



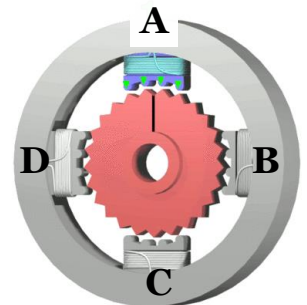
www.pololu.com



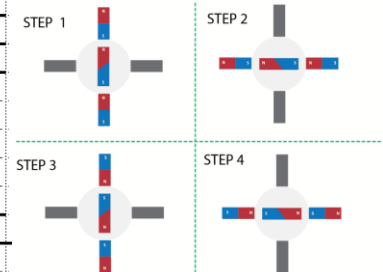
Drivers: low-level, high level



By Wapcaplet: Teravolt. (Wikipedia)

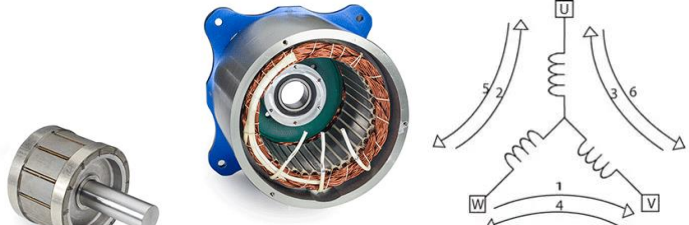
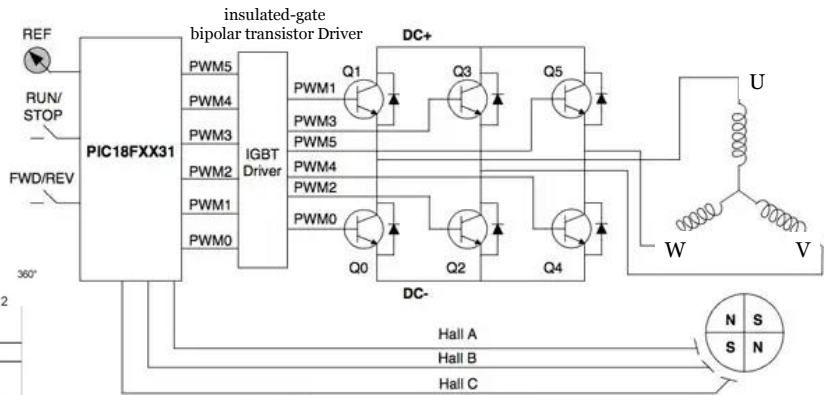
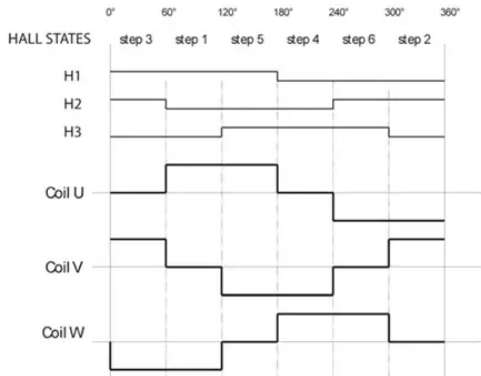
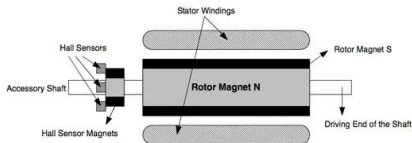


Full step operation



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Brushless Motors

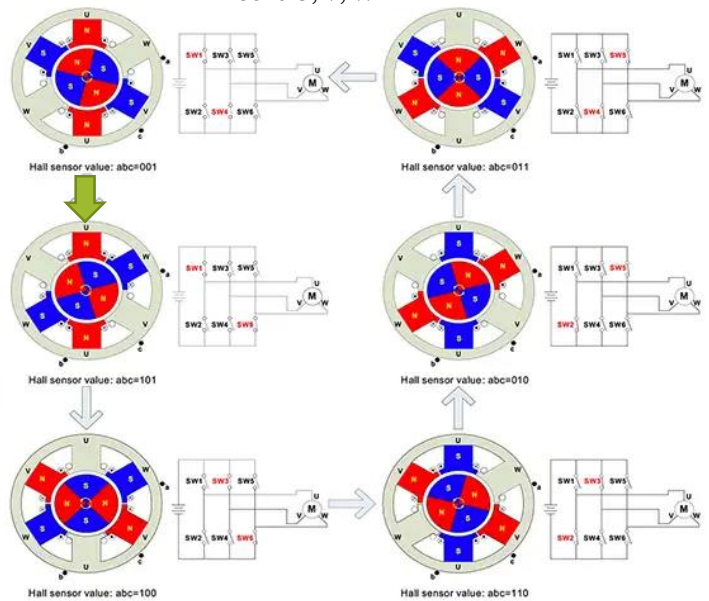
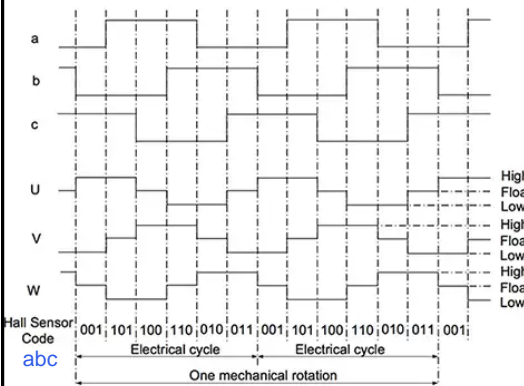


<https://www.digikey.com>

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Brushless Motors

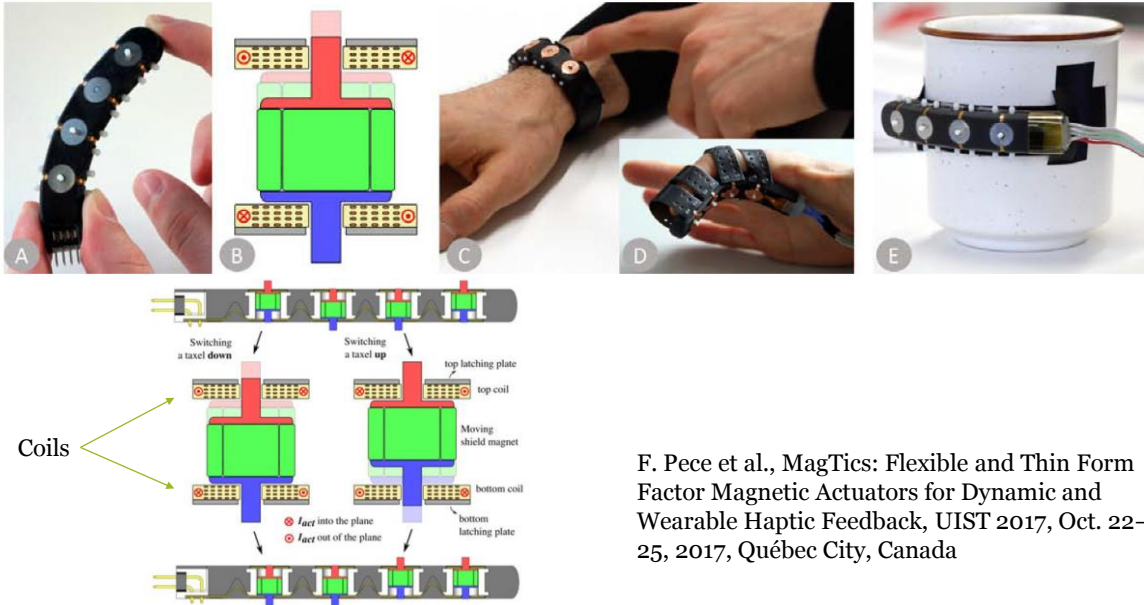
Coils U, V, W



<https://www.digikey.com/en/articles/techzone/2016/dec/how-to-power-and-control-brushless-dc-motors>

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Flexible Magnetic Actuators



F. Pece et al., MagTics: Flexible and Thin Form Factor Magnetic Actuators for Dynamic and Wearable Haptic Feedback, UIST 2017, Oct. 22–25, 2017, Québec City, Canada

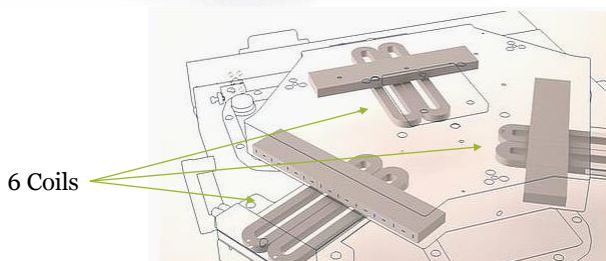
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Robotics Actuators

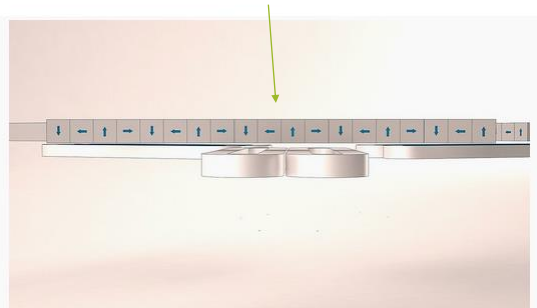


- 6D Magnetic Control
- <https://www.pi-usa.us>
- pimag-6d-magnetic-levitation

Halbach arrangement of magnets



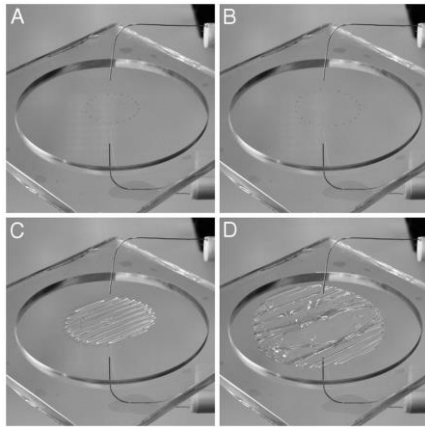
Simple structure: The platform levitates on a magnetic field generated by only six planar coils in the stator



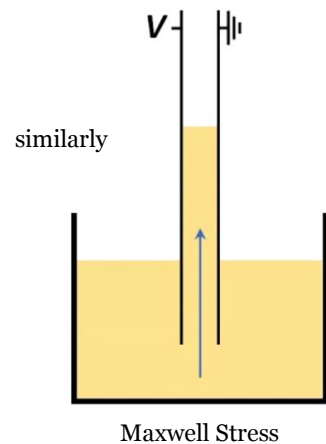
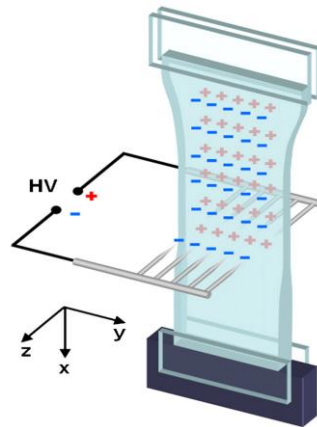
The Halbach arrangement of the magnets makes it possible, to minimize the energy required by the active coils in the stator for carrying the platform, to increase the load carrying capacity and to reduce thermal load

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Artificial Muscles



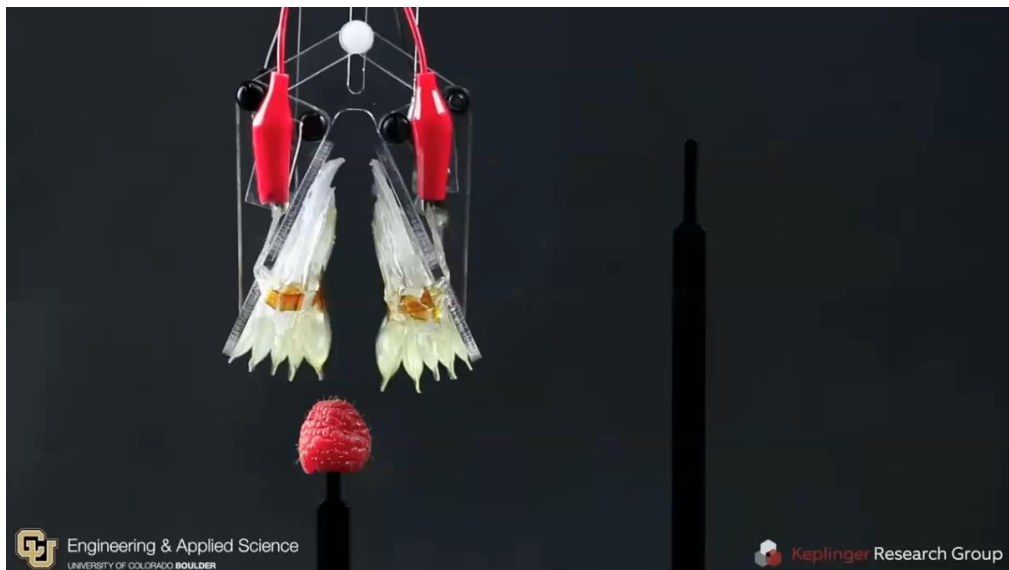
0 – 25 kV



Röntgen's electrode-free elastomer actuators without electromechanical pull-in instability by C. Keplinger, et al. PNAS March 9, 2010 107 (10) 4505-4510; <https://doi.org/10.1073/pnas.0913461107>

Röntgen WC (1880) Ueber die durch Electricität bewirkten Form- und Volumenänderungen von dielectricischen Körpern. Ann Phys Chem 11:771-786.

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See also TED Talk **The artificial muscles that will power robots of the future** by **Christoph Keplinger** <https://www.youtube.com/watch?v=ER15KmrB8h8>

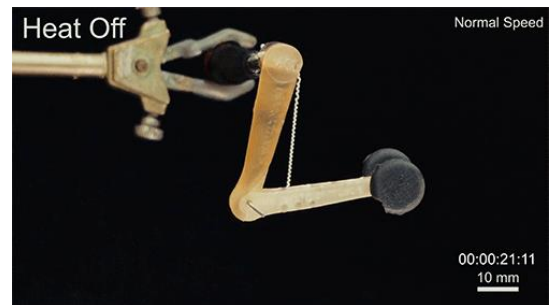
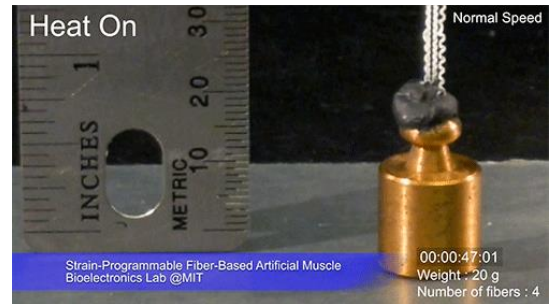
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MIT Artificial Muscles

- Combination of two dissimilar polymers into a single fiber
- The polymers have very different thermal expansion coefficients (as in bimetal)
- Developed by Mehmet Kanik, Sirma Örgüç, working with Polina Anikeeva, Yoel Fink, Anantha Chandrakasan, and C. Cem Taşan, and five others

<http://news.mit.edu/2019/artificial-fiber-muscles-0711>



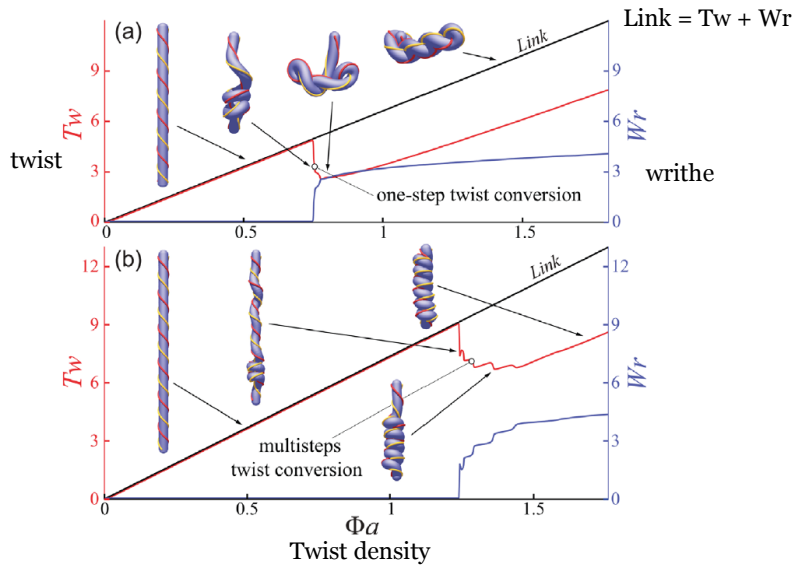
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Spanish Dancer by Micha Heilman and Stella Tsilia



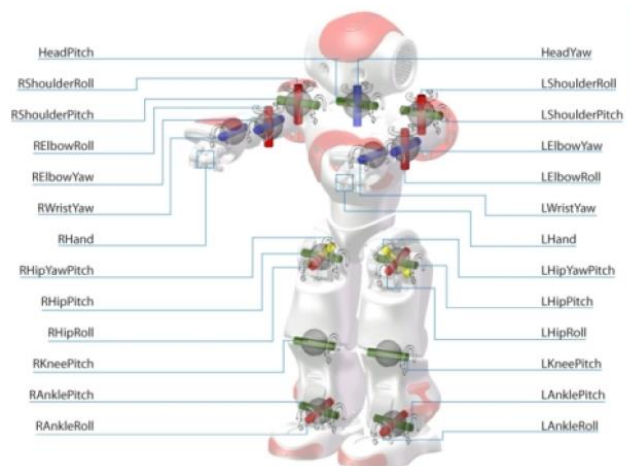
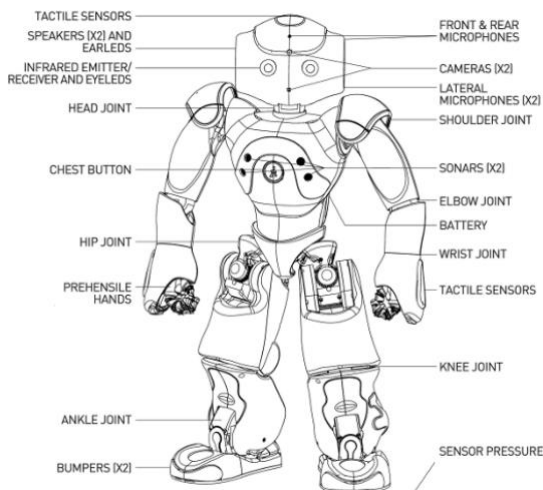
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N. Charles, M. Gazzola, and L. Mahadevan, **Topology, Geometry, and Mechanics of Strongly Stretched and Twisted Filaments: Solenoids, Plectonemes, and Artificial Muscle Fibers**
 PHYSICAL REVIEW LETTERS 123, 208003 (2019)



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NAO



http://doc.aldebaran.com/2-1/family/nao_dcm/actuator_sensor_names.html

Hexapod: S.P.I.N. by M. Huijben, M. Swenne, R. Voeter, S. Alvarez Rodriguez.

S.P.I.N. - Spider Python INator

Marcel Huijben (s1780107)
 Martijn Swenne (s1923889)
 Sebastiaan Alvarez Rodriguez (s1810979)
 Robin Voetter (s1835130)

2/13/2022

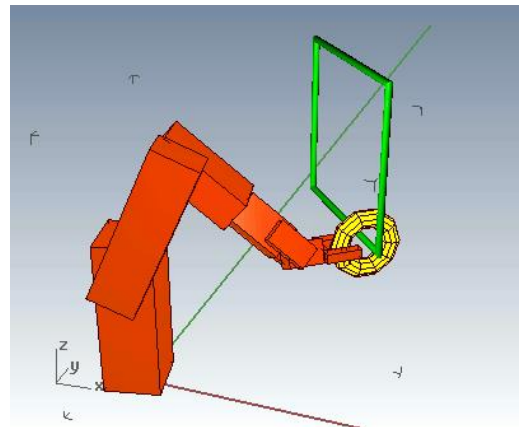
How to move to a goal?

Problem: How to move to a goal?

- Grasp, Walk, Stand, Dance, Follow, etc.

Solution:

1. *Program step by step*
 - Computer Numerical Control (CNC), Automation.
2. *Inverse kinematics:*
 - take end-points and move them to designated points.
3. *Tracing movements*
 - by specialist, human, etc.
4. *Learn the right movements*
 - **Reinforcement Learning**, give a reward when the movement resembles the designated movement.



<https://pybullet.org/wordpress/>

Configuration Space

Robot Question: Where am I?

Answer:

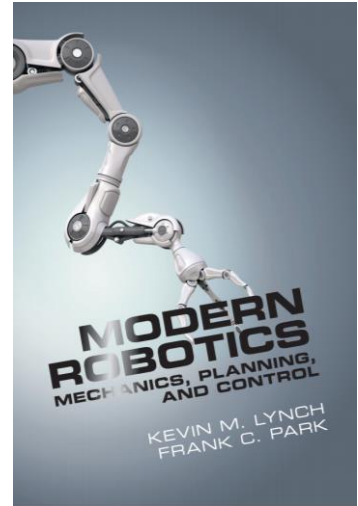
The robot's configuration: a specification of the positions of all points of a robot.

Here we assume:

Robot links and bodies are rigid and of known shape

=>

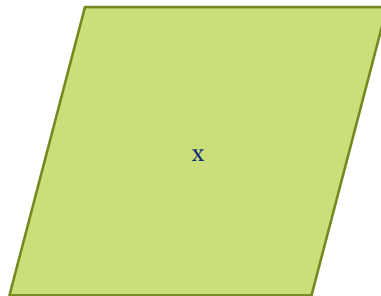
only a few variables needed to describe it's configuration.



K.M. Lynch, F.C. Park, Modern Robotics: Mechanics, Planning and Control, Cambridge University Press, 2017

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Configuration Space



Degrees of Freedom of a Rigid Body:

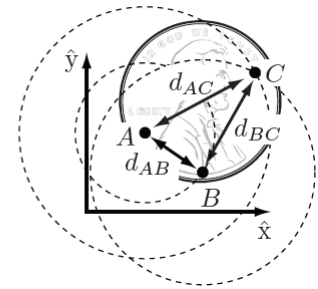
the smallest number of real-valued coordinates needed to represent its configuration

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Configuration Space

In the plane:

Assume a coin (heads) with 3 points A, B, C on it.



In the plane A,B,C have 6 degrees of freedom: $(x_A, y_A), (x_B, y_B), (x_C, y_C)$ (6 variables) B

A coin is rigid => 3 extra constraints on distances: d_{AB}, d_{AC}, d_{BC} (3 constraints)

These are fixed, wherever the location of the coin.

1. The coin and hence **A** can be placed everywhere => (x_A, y_A) free to choose.
2. **B** can only be placed under the constraint that its distance to **A** would be equal to d_{AB} .
=> freedom to turn the coin around **A** with angle φ_{AB} => (x_A, y_A, φ_{AB}) are free to choose.
3. **C** should be placed at distance d_{AC}, d_{BC} from **A** and **B**, respectively
=> only 1 possibility, hence no degree of freedom added.

Degrees of Freedom (DOF) of a Coin

= sum of freedoms of the points – number of independent constraints

= number of variables – number of independent equations

$$= 6 - 3 = 3$$

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Configuration Space

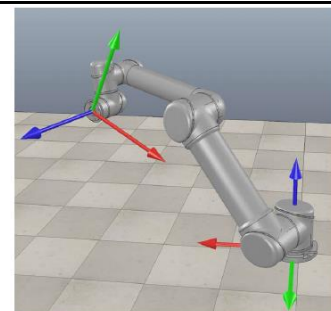
[1] Definition 2.1.

The **configuration** of a robot is a complete specification of the position of every point of the robot.

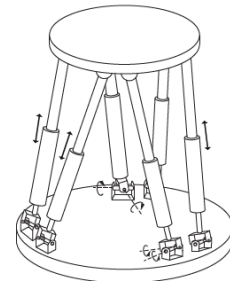
The minimum number n of real-valued coordinates needed to represent the configuration is the number of **degrees of freedom (dof)** of the robot.

The n -dimensional space containing all possible configurations of the robot is called the **Configuration Space (C-space)**.

The configuration of a robot is represented by a point in its C-space.



Open-chain robot: Manipulator (in V-REP). [1]



Closed-chain robot: Stewart-Gough platform. [1]

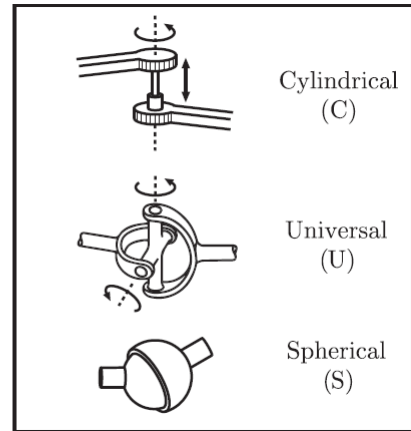
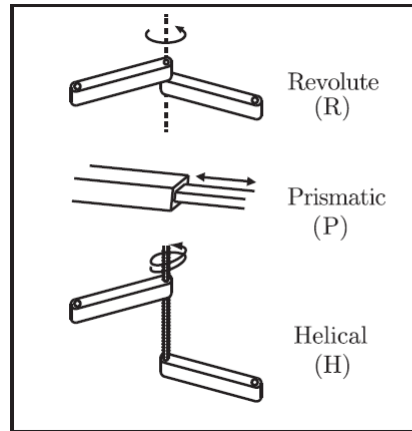
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Degrees of Freedom of a Robot

- A rigid body in 3D Space has **6 DOF**



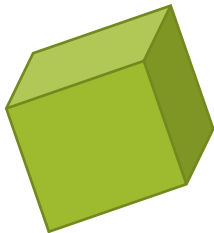
- A joint can be seen to put constraints on the rigid bodies it connects
- It also allows freedom to move relative to the body it is attached to.



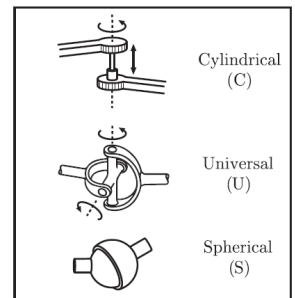
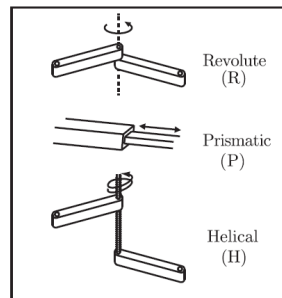
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Degrees of Freedom of a Robot

- A **rigid body** in 3D Space has 6 DOF



- A **joint** can be seen to put constraints on the rigid bodies it connects
- It also allows freedom to move relative to the body it is attached to.



Joint type	dof f	Constraints c between two planar 2D rigid bodies	Constraints c between two spatial 3D rigid bodies
Revolute (R)	1	2	5
Prismatic (P)	1	2	5
Helical (H)	1	N/A	5
Cylindrical (C)	2	N/A	4
Universal (U)	2	N/A	4
Spherical (S)	3	N/A	3

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Degrees of Freedom of a Robot

Planar Mechanism DOF = 4

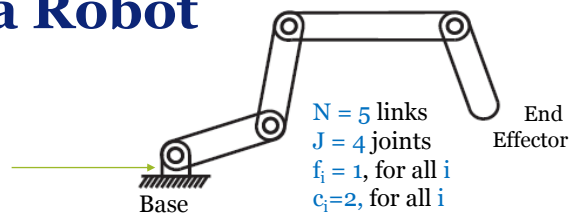
Proposition (Grübler's formula)

Consider a mechanism consisting of

- N links, where ground (!) is also regarded as a link
- J number of joints
- m number of degrees of freedom of a rigid body
($m = 3$ for planar mechanisms and $m = 6$ for spatial mechanisms)
- f_i the number of freedoms provided by joint i
- c_i the number of constraints provided by joint i , where $f_i + c_i = m$ for all i .

Then *Grübler's formula* for the number of degrees of freedom of the robot is

$$dof = m(N - 1) - \sum_{i=1}^J c_i = m(N - 1 - J) + \sum_{i=1}^J f_i$$



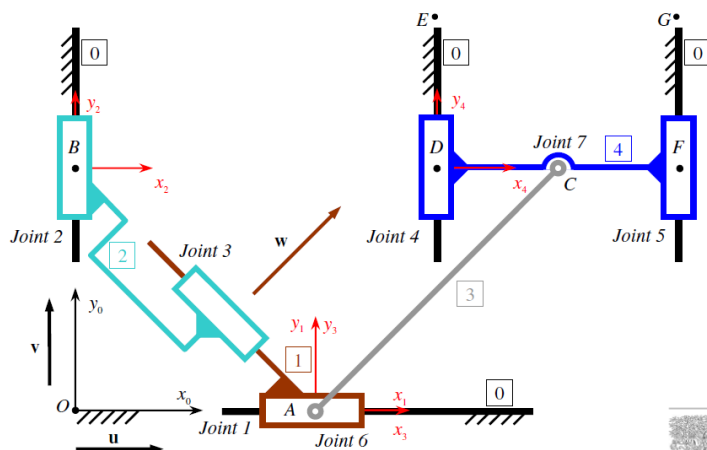
This formula holds only if all joint constraints are independent. If they are not independent then the formula provides a lower bound on the number of degrees of freedom.

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Joint reactions in rigid body mechanisms with dependent constraints

Marek Wojtyra*

Warsaw University of Technology, Institute of Aeronautics and Applied Mechanics, ul. Nowowiejska 24, 00-665 Warsaw, Poland



$$dof = m(N - 1) - \sum_{i=1}^J c_i = m(N - 1 - J) + \sum_{i=1}^J f_i$$

Fig. 1. Planar mechanism.

Mechanism and Machine Theory 44 (2009) 2265–2278

Contents lists available at ScienceDirect

Mechanism and Machine Theory

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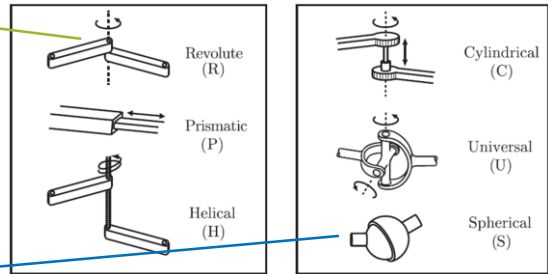
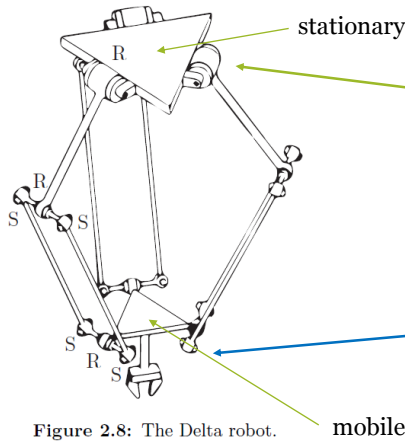


Figure 2.8: The Delta robot.

Example 2.7 (Delta robot). The Delta robot of Figure 2.8 consists of two platforms – the lower one mobile, the upper one stationary – connected by three legs. Each leg contains a parallelogram closed chain and consists of three revolute joints, four spherical joints, and five links. Adding the two platforms, there are $N = 17$ links and $J = 21$ joints (nine revolute and 12 spherical). By Grübler's formula,

$$\text{dof} = 6(17 - 1 - 21) + 9(1) + 12(3) = 15.$$

- Links: $1 + 3 + 3 + 6 + 3 + 1 = 17$
- Joints: $21: 9 \times R(1 \text{ dof})$ and $12 \times S(3 \text{ dof})$
- $m = 6$

$$\text{dof} = m(N - 1) - \sum_{i=1}^J c_i = m(N - 1 - J) + \sum_{i=1}^J f_i$$

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Systems and their Topologies

Note: $S^1 \times S^1 = T^2$ (not S^2)



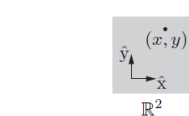
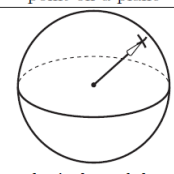
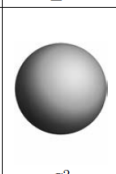
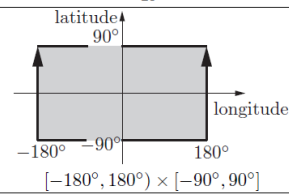
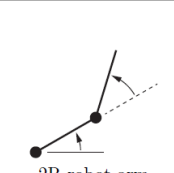

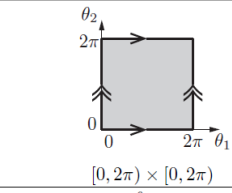
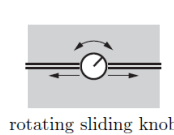
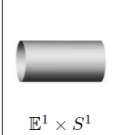
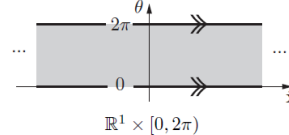
Coordinates can be:

Explicit Coordinates

- Euclidean (x,y)
- Polar (r,φ)
- Combined $(x,y) \times (r, \varphi)$

Implicit Coordinates

- $\{ (x,y,z) \mid x^2+y^2+z^2=1 \}$

system	topology	sample representation
 point on a plane	 \mathbb{E}^2	 \mathbb{R}^2
 spherical pendulum	 S^2	 latitude 90° -180° -90° 180° longitude $[-180^\circ, 180^\circ] \times [-90^\circ, 90^\circ]$
 2R robot arm	 $T^2 = S^1 \times S^1$	 θ_2 2π 0 0 2π θ_1 $[0, 2\pi) \times [0, 2\pi)$
 rotating sliding knob	 $\mathbb{E}^1 \times S^1$	 θ 2π 0 \dots \dots $\mathbb{R}^1 \times [0, 2\pi)$

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C-Space (Configuration Space)

How to describe a rigid body's position and orientation in C-Space?

Fixed reference frame $\{s\}$

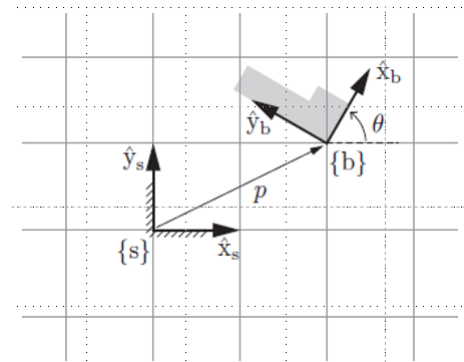
Reference frame attached to body $\{b\}$

In \mathbb{R}^3 described by a 4×4 matrix with 10 constraints
(constraints, e.g.: unit-length, orthogonal)

Note: a point in $\mathbb{R}^3 \times S^2 \times S^1$

Matrix can be used to:

1. Translate or rotate a vector or a frame
2. Change the representation of a vector or a frame
 - for example from relative to $\{s\}$ to relative to $\{b\}$

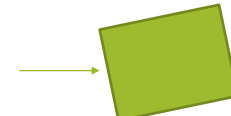


in the plane $\mathbb{R}^2 \times S^1$

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C-Spaces

C-space of a rigid body in the plane = $\mathbb{R}^2 \times S^1$ as configuration can be denoted as (x, y, θ) , i.e., location (x, y) in \mathbb{R}^2 and angle θ in S^1 .



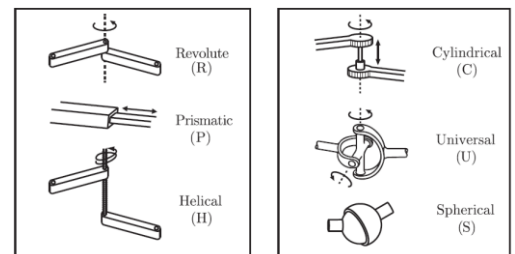
C-space of a Prismatic-Revolute (PR) robot arm is equal to $\mathbb{R}^1 \times S^1$

C-space of a 2R robot arm is $S^1 \times S^1 = T^2$

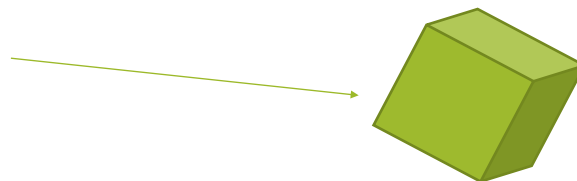
C-space of a 3R robot arm is $S^1 \times S^1 \times S^1 = T^3$

C-space of a planar mobile robot with a 2R robot arm

is $\mathbb{R}^2 \times S^1 \times T^2 = \mathbb{R}^2 \times T^3$



C-space of a rigid body in space is $\mathbb{R}^3 \times S^2 \times S^1$



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Task Space and Work Space

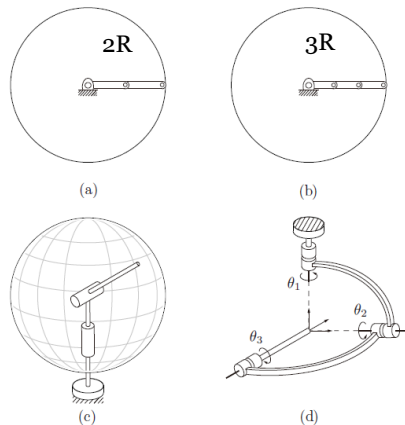
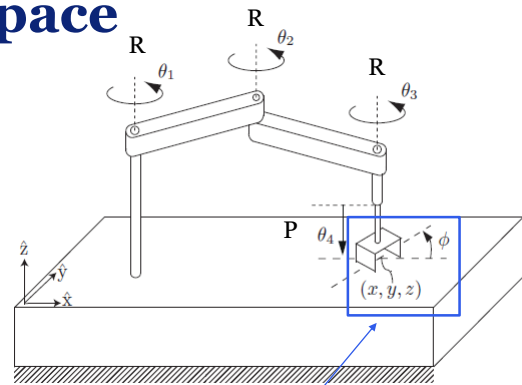


Figure 2.12: Examples of workspaces for various robots: (a) a planar 2R open chain; (b) a planar 3R open chain; (c) a spherical 2R open chain; (d) a 3R orienting mechanism.

The **workspace** is a specification of the configurations that the end-effector of the robot can reach.



The SCARA robot is an **RRRP open chain** that is widely used for tabletop pick-and-place tasks. The end-effector configuration is completely described by (x, y, z, ϕ)

⇒ **task space** $R^3 \times S^1$ and

⇒ **workspace** as the reachable points in (x, y, z) , since all orientations ϕ can be achieved at all reachable points.

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Rigid Body Motion

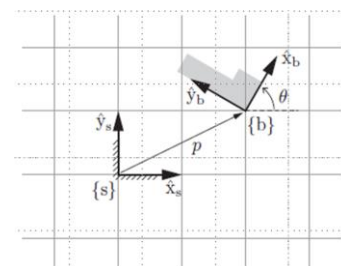
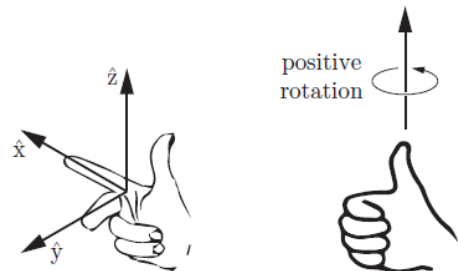
Rigid-body position and orientation $(x, y, z, \phi, \theta, \psi) \in \mathbb{R}^3 \times S^2 \times S^1$

- Can also be described by 4x4 matrix with 10 constraints.
- In general 4x4 matrices can be used for
 - Location
 - Translation + rotation of a vector or frame
 - Transformation of coordinates between frames
- **Velocity of a rigid body:** $(\partial x/\partial t, \partial y/\partial t, \partial z/\partial t, \partial \phi/\partial t, \partial \theta/\partial t, \partial \psi/\partial t)$
i.e., changes in location and orientation per unit of time

Exponential coordinates:

Every rigid-body configuration can be achieved by:

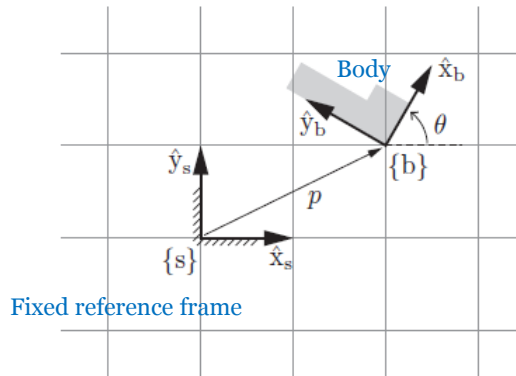
- Starting in the fixed home frame and integrating a constant twist for a specified time.
- Direction of a screw axis and scalar to indicate how far the screw axis must be followed



Similarly in the plane

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Rigid Body Motions in the Plane



Translation

$$p = p_x \hat{x}_s + p_y \hat{y}_s.$$

Rotation

$$\hat{x}_b = \cos \theta \hat{x}_s + \sin \theta \hat{y}_s,$$

$$\hat{y}_b = -\sin \theta \hat{x}_s + \cos \theta \hat{y}_s.$$

Figure 3.3: The body frame $\{b\}$ is expressed in the fixed-frame coordinates $\{s\}$ by the vector p and the directions of the unit axes \hat{x}_b and \hat{y}_b . In this example, $p = (2, 1)$ and $\theta = 60^\circ$, so $\hat{x}_b = (\cos \theta, \sin \theta) = (0.5, 1/\sqrt{2})$ and $\hat{y}_b = (-\sin \theta, \cos \theta) = (-1/\sqrt{2}, 0.5)$.

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Rigid Body Motions in the Plane

$\{b\}$ relative to $\{s\}$

$$p = \begin{bmatrix} p_x \\ p_y \end{bmatrix}$$

$$P = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

$\{c\}$ relative to $\{b\}$

$$q = \begin{bmatrix} q_x \\ q_y \end{bmatrix}, \quad Q = \begin{bmatrix} \cos \psi & -\sin \psi \\ \sin \psi & \cos \psi \end{bmatrix}$$



$\{c\}$ relative to $\{s\}$

$$r = \begin{bmatrix} r_x \\ r_y \end{bmatrix}, \quad R = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix}$$

Previously:

$$p = p_x \hat{x}_s + p_y \hat{y}_s.$$

$$\hat{x}_b = \cos \theta \hat{x}_s + \sin \theta \hat{y}_s,$$

$$\hat{y}_b = -\sin \theta \hat{x}_s + \cos \theta \hat{y}_s.$$

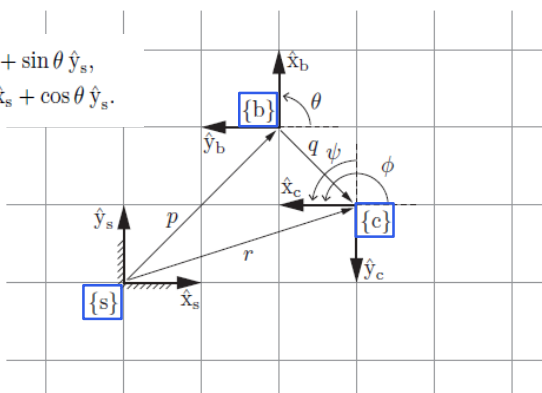


Figure 3.4: The frame $\{b\}$ in $\{s\}$ is given by (P, p) , and the frame $\{c\}$ in $\{b\}$ is given by (Q, q) . From these we can derive the frame $\{c\}$ in $\{s\}$, described by (R, r) . The numerical values of the vectors p , q , and r and the coordinate-axis directions of the three frames are evident from the grid of unit squares.

Note and verify:

$R = PQ$, convert Q to $\{s\}$ -frame
 $r = Pq + p$, convert q to $\{s\}$ -frame
 and add p

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Forward Kinematics

The forward kinematics of 3R Planar Open Chain can be written as a product of four homogeneous transformation matrices: $T_{04} = T_{01}T_{12}T_{23}T_{34}$, where

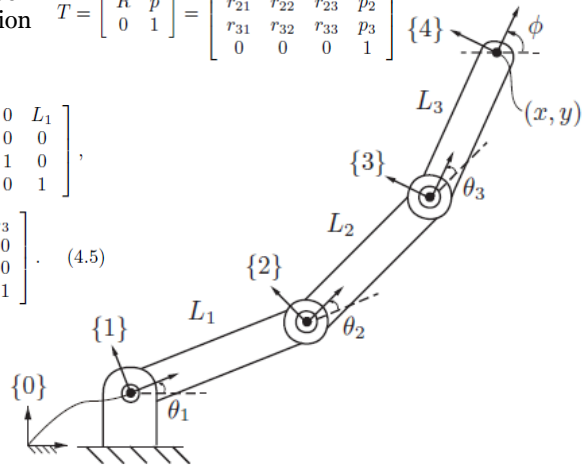
$$T = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_1 \\ r_{21} & r_{22} & r_{23} & p_2 \\ r_{31} & r_{32} & r_{33} & p_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{01} = \begin{bmatrix} \cos \theta_1 & -\sin \theta_1 & 0 & 0 \\ \sin \theta_1 & \cos \theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$T_{12} = \begin{bmatrix} \cos \theta_2 & -\sin \theta_2 & 0 & L_1 \\ \sin \theta_2 & \cos \theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$T_{23} = \begin{bmatrix} \cos \theta_3 & -\sin \theta_3 & 0 & L_2 \\ \sin \theta_3 & \cos \theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$T_{34} = \begin{bmatrix} 1 & 0 & 0 & L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (4.5)$$



Home position M:

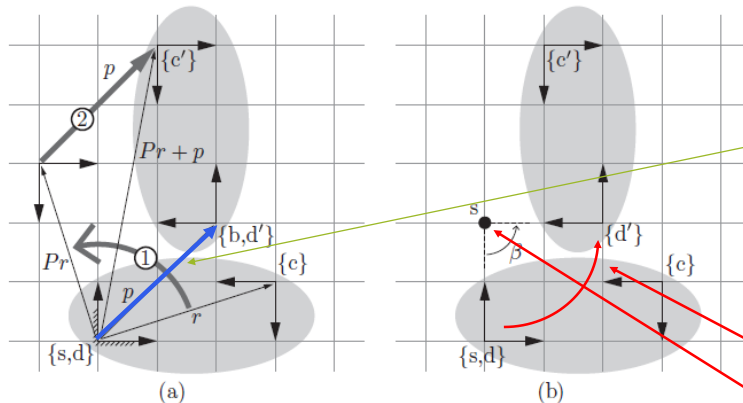
$$M = \begin{bmatrix} 1 & 0 & 0 & L_1 + L_2 + L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rigid Body Motions in the Plane

{c} described by (R,r)

$$r = \begin{bmatrix} r_x \\ r_y \end{bmatrix}, \quad R = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix}$$

Move rigid body such that {d} coincides with {d'}.



$$p = \begin{bmatrix} p_x \\ p_y \end{bmatrix} \quad P = [\hat{x}_b \ \hat{y}_b] = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

Then {c} described by (R',r')

$$R' = PR, \\ r' = Pr + p,$$

Note: SCREW MOTION

The above **rotation (1)** followed by a **translation (2)** can also be expressed as

a rotation of the rigid-body about a fixed point s by an angle β

(P, p) can be used to

Displace an elliptical rigid body {d} coinciding with {s} to {d'} coinciding with {b}: **Rotation P** followed by **translation p**

Rigid Body Motions in the Plane

Note: **SCREW MOTION**

The above rotation followed by a translation can also be expressed as a rotation of the rigid-body about a fixed point s by an angle β

(β, s_x, s_y) , where $(s_x, s_y) = (0, 2)$

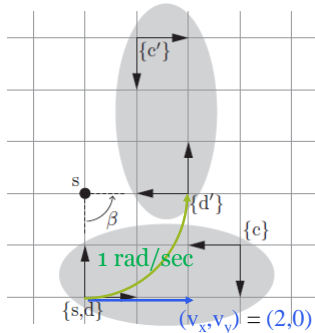
In the $\{s\}$ -frame rotate 1 rad/sec with speed $(v_x, v_y) = (2, 0)$ is denoted as:

$$S = (\omega, v_x, v_y) = (1, 2, 0)$$

Following the screw-axis for an angle $\theta = \pi/2$ gives the displacement we want:

$$S\theta = (\pi/2, \pi, 0)$$

These are called the **exponential coordinates** for the planar rigid-body displacement.



$\{c\}$ described by (R,r)

$$r = \begin{bmatrix} r_x \\ r_y \end{bmatrix}, \quad R = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix}$$

Move rigid body such that $\{d\}$ coincides with $\{d'\}$.

$$p = \begin{bmatrix} p_x \\ p_y \end{bmatrix} \quad P = [\hat{x}_b \ \hat{y}_b] = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

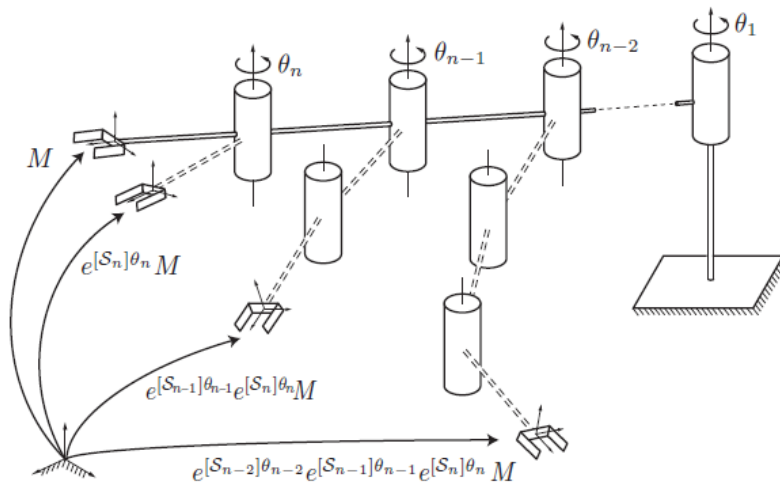
Then $\{c'\}$ described by (R',r') :

$$R' = PR, \\ r' = Pr + p,$$

Note:

- distance = vt
- distance along quarter circle with radius 2 equals π .

Forward Kinematics: Product of Exponentials



PoE parameters also known as Euler-Rodrigues parameters.

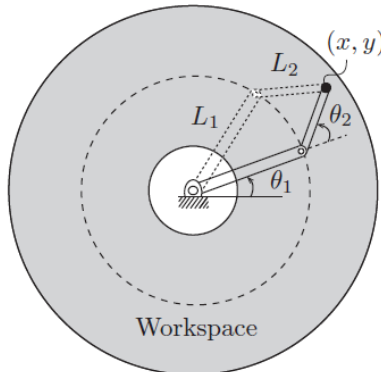
There are many other representations: - for example Denavit-Hartenberg (1955) representation is very popular, but can be cumbersome

In velocity kinematics Jacobians are used.

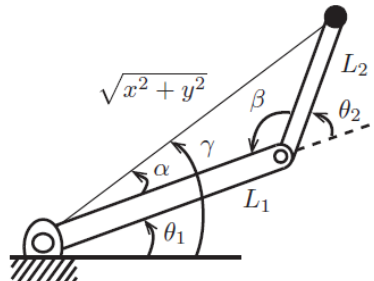
Figure 4.2: Illustration of the PoE formula for an n -link spatial open chain.

Inverse Kinematics

Which angles θ_1 , and θ_2 will lead to location (x,y) ?



(a) A workspace, and lefty and righty configurations.



(b) Geometric solution.

Law of cosines gives:

$$L_1^2 + L_2^2 - 2L_1L_2 \cos \beta = x^2 + y^2$$

, hence

$$\beta = \cos^{-1} \left(\frac{L_1^2 + L_2^2 - x^2 - y^2}{2L_1L_2} \right)$$

,and similarly

$$\alpha = \cos^{-1} \left(\frac{x^2 + y^2 + L_1^2 - L_2^2}{2L_1\sqrt{x^2 + y^2}} \right)$$

$$\gamma = \text{atan2}(y,x)$$

Answer:

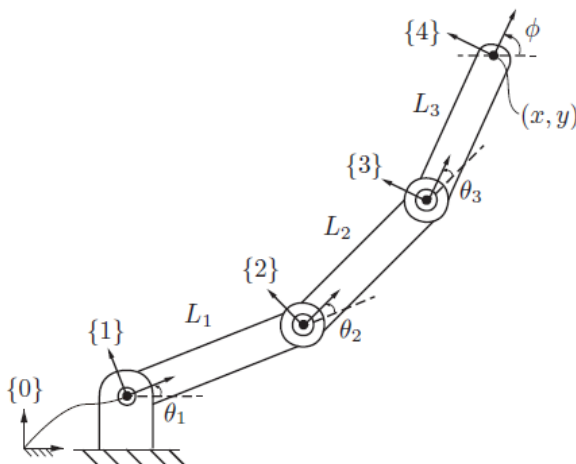
$$\theta_1 = \gamma - \alpha, \quad \theta_2 = \pi - \beta$$

Figure 6.1: Inverse kinematics of a 2R planar open chain.

In general: IK-Solvers, Newton-Raphson, etc.

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Inverse Kinematics



How would you solve this?

Which angles θ_1 , θ_2 , and θ_3 will lead to location (x,y) ?

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Real Time Physics Modelling

<https://pybullet.org/wordpress/>

pybullet KUKA
grasp training

Using Tensorflow
OpenAI gym
Baselines
DeepQNetworks (DQNs)

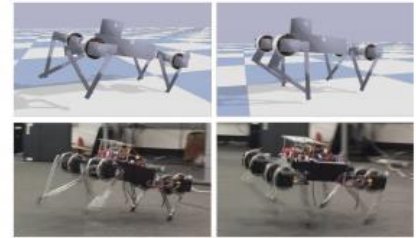
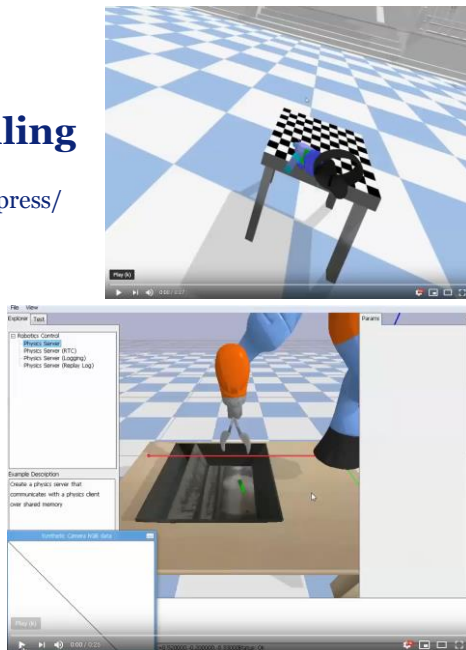
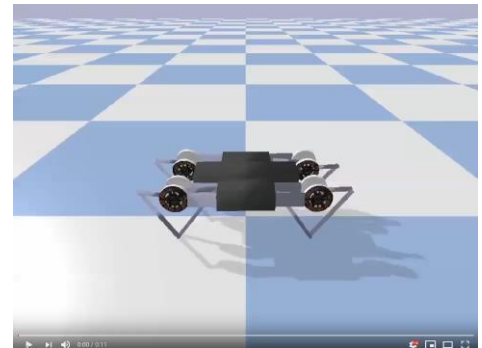


Fig. 1: The simulated and the real Minitaurs learned to gallop using deep reinforcement learning.



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Organization and Overview

Period: February 7th – May 23rd 2022
Time: Monday 16.15 – 18.00
Place: Room 407 - 409
Lecturer: Erwin M. Bakker (erwin@liacs.nl)
Assistant: Hainan Yu (h.yu@liacs.leidenuniv.nl)

NB Register on Brightspace

Schedule:

7-2	Introduction and Overview
14-2	Locomotion and Inverse Kinematics
21-2	Robotics Sensors and Image Processing
28-2	SLAM + SLAM Workshop
7-3	Mobile Robot Challenge Introduction
14-3	Project Proposals I (presentation by students)
21-3	Project Proposals II (presentation by students)
28-3	Robotics Vision
4-4	Robotics Reinforcement Learning
11-4	Robotics Reinforcement Learning Workshop II
18-4	No Class (Eastern)
25-4	Project Progress I (presentations by students)
2-5	Project Progress II (presentations by students)
9-5	Mobile Robot Challenge
16-5	Project Demos I
23-5	Project Demos II

Website: <http://liacs.leidenuniv.nl/~bakkerem2/robotics/>



Grading (6 ECTS):

- Presentations and Robotics Project (60% of grade).
- Class discussions, attendance, workshops and assignments (40% of grade).
- It is necessary to be at every class and to complete every workshop and assignment.

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Robotics Homework II

Visit <http://modernrobotics.org> and obtain the pdf of the [book](#).

Read Chapters 1 and 2, and answer the following exercises: TBA

The exercises of Homework II will be available on Tuesday (15-2) on

BrightSpace

and

<https://liacs.leidenuniv.nl/~bakkerem2/robotics/>

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References

1. K.M. Lynch, F.C. Park, Modern Robotics: Mechanics, Planning and Control, Cambridge University Press, 2017. (DOI: 10.1017/9781316661239)
2. <https://pybullet.org/wordpress/>

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