

Robotics

Erwin M. Bakker | LIACS Media Lab

28-2 2019



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Leiden

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Organization and Overview

Period: February 15th - May 10th 2019
Time: Friday 09.00 – 10.45
Place: LIACS, Room 401 (Workshops Room 303)
Lecturer: Dr Erwin M. Bakker (erwin@liacs.nl)
Assistant: Andrius Bernatavicius

NB E-mail your name and student number to erwin@liacs.nl

Schedule:

15-2	Introduction and Overview
22-2	Control Space, Locomotion and Kinematics
1-3	Inverse Kinematics and Sensors
8-3	Yetiborg Introduction and <i>SLAM Workshop I</i>
15-3	<i>Project Proposals (presentation by students)</i>
22-3	Yetiborg Qualification and <i>ROS Workshop II</i>
29-3	Robotics Image Processing
5-4	Yetiborg Race and/or <i>Nao Workshop III</i>
12-4	Robotics Image Processing and Understanding
19-4	No Class
26-4	Robotics Reinforcement Learning.
3-5	Robotics Reinforcement Learning Workshop IV
10-5	Project Demos (by students)

Website: <http://liacs.leidenuniv.nl/~bakkerem2/robotics/>

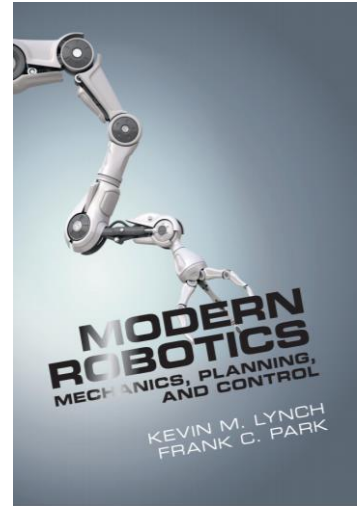


Grading (6 ECTS): Presentations and Robotics Project (60% of grade). Class discussions, attendance, workshops and assignments (40% of grade). It is necessary to be at every class and to complete every workshop.

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Overview

- Configuration Space
- Rigid Body Motion
- Forward Kinematics
- Inverse Kinematics
- Sensors
- Link: <http://modernrobotics.org>



K.M. Lynch, F.C. Park, Modern Robotics: Mechanics, Planning and Control, Cambridge University Press, 2017

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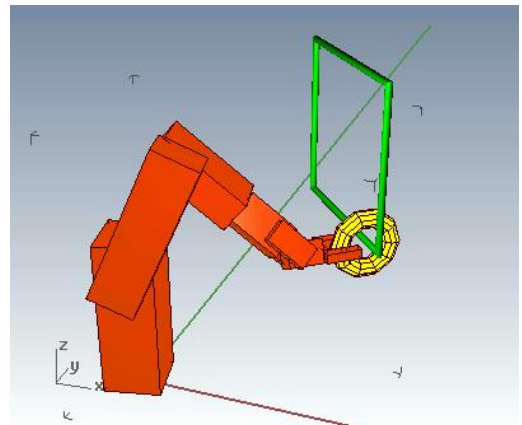
How to move to a goal?

Problem: How to move to a goal?

- Grasp, Walk, Stand, Dance, Follow, etc.

Solution:

1. *Program step by step*
 - Computer Numerical Control (CNC), Automation.
2. *Inverse kinematics:*
 - take end-points and move them to designated points.
3. *Tracing movements*
 - by specialist, human, etc.
4. *Learn the right movements*
 - **Reinforcement Learning**, give a reward when the movement resembles the designated movement.



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Configuration Space

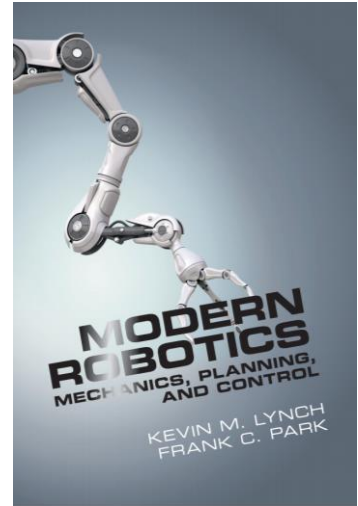
Robot Question: Where am I?

Answer:

The robot's configuration: a specification of the positions of all points of a robot.

Here we assume:

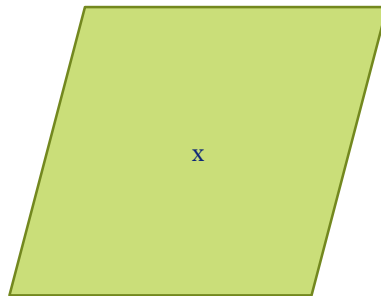
Robot links and bodies are rigid and of known shape => only a few variables needed to describe it's configuration.



K.M. Lynch, F.C. Park, Modern Robotics: Mechanics, Planning and Control, Cambridge University Press, 2017

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Configuration Space



- Degrees of Freedom of a Rigid Body: the smallest number of real-valued coordinates needed to represent its configuration

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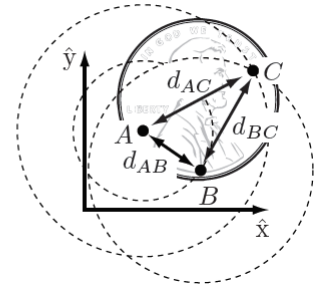
Configuration Space

Assume we have a coin with 3 points A, B, C on it.

In the plane A,B,C have 6 degrees of freedom:

$(x_A, y_A), (x_B, y_B), (x_C, y_C)$

A coin is rigid => 3 extra constraints on distances: d_{AB}, d_{AC}, d_{BC} are fixed, wherever the location of the coin would be.



1. The coin and hence A can be placed everywhere => (x_A, y_A) free to choose.
2. B can only be placed under the constraint that its distance to A would be equal to d_{AB} . => freedom to turn the coin around A with angle φ_{AB} => (x_A, y_A, φ_{AB}) are free to choose.
3. C should be placed at distance d_{AC}, d_{BC} from A and B, respectively => only 1 possibility, hence no degree of freedom added.

Degrees of Freedom (DOF) of a Coin

= sum of freedoms of the points – number of independent constraints

= number of variables – number of independent equations = $6 - 3 = 3$

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Configuration Space

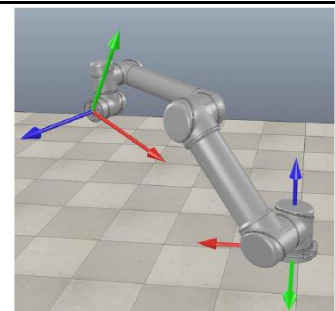
[1] Definition 2.1.

The **configuration** of a robot is a complete specification of the position of every point of the robot.

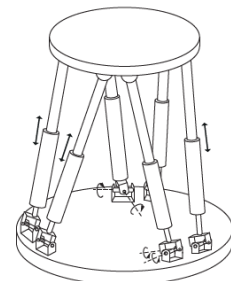
The minimum number n of real-valued coordinates needed to represent the configuration is the number of **degrees of freedom (dof)** of the robot.

The n -dimensional space containing all possible configurations of the robot is called the **Configuration Space (C-space)**.

The configuration of a robot is represented by a point in its C-space.



Open-chain robot: Manipulator (in V-REP). [1]



Closed-chain robot: Stewart-Gough platform. [1]

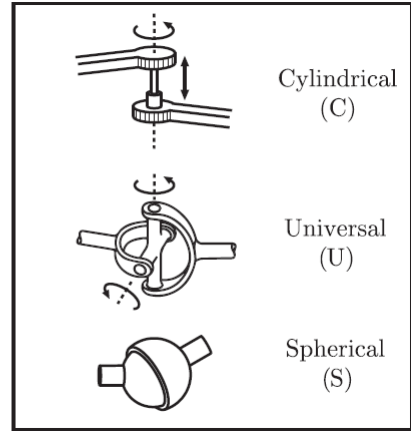
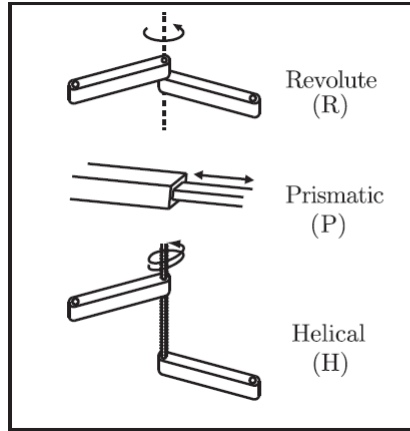
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Degrees of Freedom of a Robot

- A rigid body in 3D Space has **6 DOF**



- A joint can be seen to put constraints on the rigid bodies it connects
- It also allows freedom to move relative to the body it is attached to.



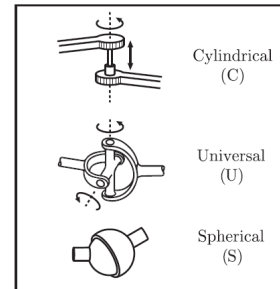
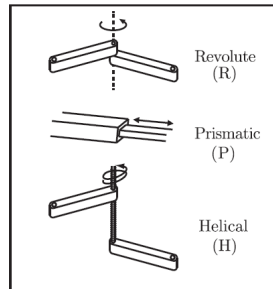
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Degrees of Freedom of a Robot

- A **rigid body** in 3D Space has 6 DOF



- A **joint** can be seen to put constraints on the rigid bodies it connects
- It also allows freedom to move relative to the body it is attached to.



Joint type	dof f	Constraints c between two planar rigid bodies	Constraints c between two spatial rigid bodies
Revolute (R)	1	2	5
Prismatic (P)	1	2	5
Helical (H)	1	N/A	5
Cylindrical (C)	2	N/A	4
Universal (U)	2	N/A	4
Spherical (S)	3	N/A	3

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Degrees of Freedom of a Robot

Planar Mechanism $DOF = 4$

Proposition (Grübler's formula)

Consider a mechanism consisting of

- N links, where ground is also regarded as a link.
- J number of joints,
- m number of degrees of freedom of a rigid body ($m = 3$ for planar mechanisms and $m = 6$ for spatial mechanisms),
- f_i the number of freedoms provided by joint i , and
- c_i the number of constraints provided by joint i , where $f_i + c_i = m$ for all i .

Then *Grübler's formula* for the number of degrees of freedom of the robot is

$$dof = m(N - 1) - \sum_{i=1}^J c_i = m(N - 1 - J) + \sum_{i=1}^J f_i$$

This formula holds only if all joint constraints are independent. If they are not independent then the formula provides a lower bound on the number of degrees of freedom.

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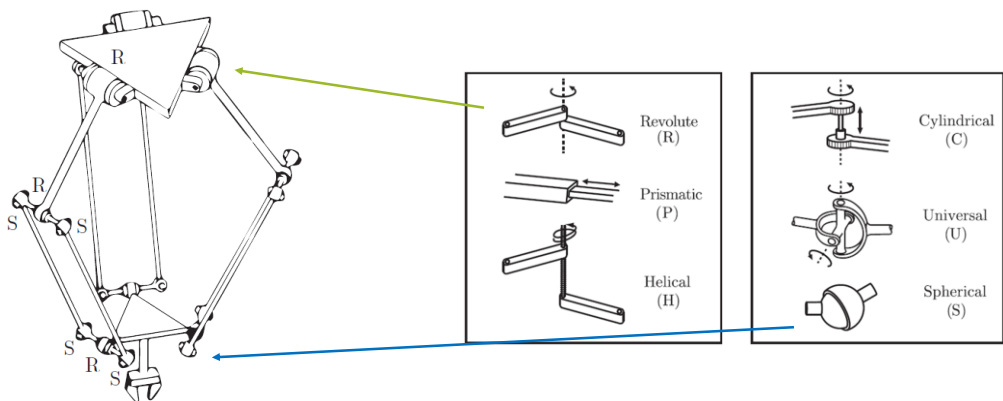
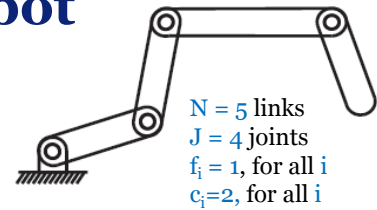


Figure 2.8: The Delta robot.

Example 2.7 (Delta robot). The Delta robot of Figure 2.8 consists of two platforms – the lower one mobile, the upper one stationary – connected by three legs. Each leg contains a parallelogram closed chain and consists of three revolute joints, four spherical joints, and five links. Adding the two platforms, there are $N = 17$ links and $J = 21$ joints (nine revolute and 12 spherical). By Grübler's formula,

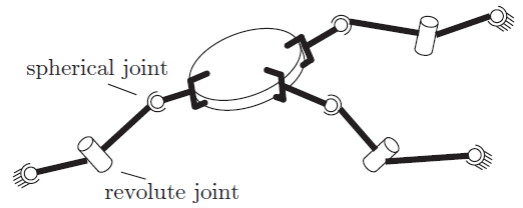
$$dof = 6(17 - 1 - 21) + 9(1) + 12(3) = 15.$$

- Links: $1 + 3 + 3 + 6 + 3 + 1 = 17$
- Joints: $21: 9 \times R(1 \text{ dof})$ and $12 \times S(3 \text{ dof})$
- $m = 6$

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Assignments

- 3x SRS
- $M=6, N = 1 + 3 \times 3 = 10, J = 3 \times 3, \text{dof per leg } 3 + 1 + 3$
 $\Rightarrow \text{dof} = 6(10 - 1 - 9) + 3 \times 7 = 21$



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Topologies

Note: $S^1 \times S^1 = T^2$ (not S^2)



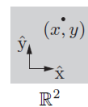
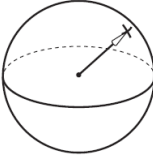

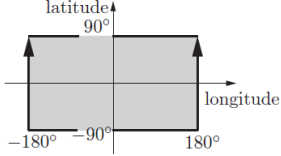
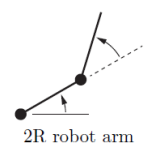

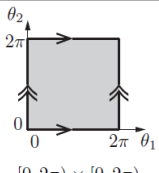
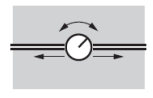

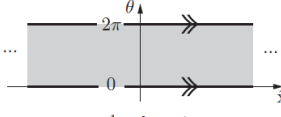
Coordinates

Explicit Coordinates

- Euclidean (x,y)
- Polar (r,φ)
- Combined $(x,y) \times (r, \varphi)$

Implicit Coordinates

- $\{(x,y,z) \mid x^2+y^2+z^2=1\}$

system	topology	sample representation
 point on a plane	 \mathbb{E}^2	 \mathbb{R}^2
 spherical pendulum	 S^2	 $[-180^\circ, 180^\circ] \times [-90^\circ, 90^\circ]$
 2R robot arm	 $T^2 = S^1 \times S^1$	 $[0, 2\pi) \times [0, 2\pi)$
 rotating sliding knob	 $\mathbb{E}^1 \times S^1$	 $\mathbb{R}^1 \times [0, 2\pi)$

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C-Space (Configuration Space)

How to describe a rigid body's position and orientation in C-Space?

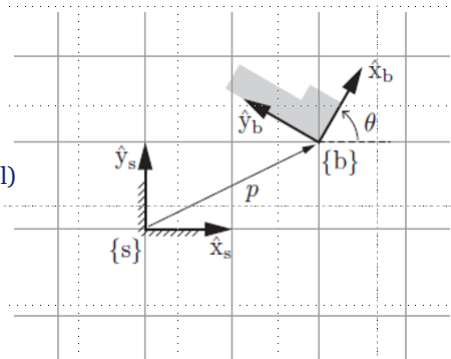
Fixed reference frame $\{s\}$

Reference frame attached to body $\{b\}$

Described by 4×4 matrix with 10 constraints (unit-length, orthogonal)

Matrix can be used to:

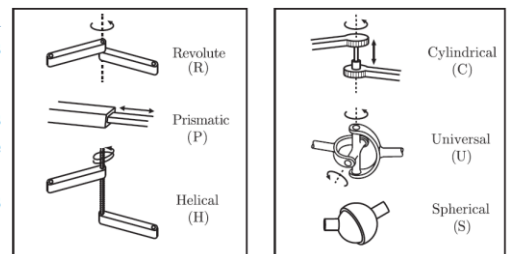
1. Translate or rotate a vector or a frame
2. Change the representation of a vector or a frame
 - for example from relative to $\{s\}$ to relative to $\{b\}$



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C-Spaces

- The C-space of a rigid body in the plane can be written as $\mathbb{R}^2 \times S^1$, since the configuration can be represented as the concatenation of the coordinates (x, y) representing \mathbb{R}^2 and an angle θ representing S^1 .
- The C-space of a PR robot arm can be written $\mathbb{R}^1 \times S^1$ (We will occasionally ignore joint limits, i.e., bounds on the travel of the joints, when expressing the topology of the C-space; with joint limits, the C-space is the Cartesian product of two closed intervals of the line.)
- The C-space of a 2R robot arm can be written $S^1 \times S^1 = T^2$, where T^n is the n -dimensional surface of a torus in an $(n+1)$ -dimensional space. (See Table 2.2.) Note that $S^1 \times S^1 \times \dots \times S^1$ (n copies of S^1) is equal to T^n , not S^n ; for example, a sphere S^2 is not topologically equivalent to a torus T^2 .
- The C-space of a planar rigid body (e.g., the chassis of a mobile robot) with a 2R robot arm can be written as $\mathbb{R}^2 \times S^1 \times T^2 = \mathbb{R}^2 \times T^3$
- As we saw in Section 2.1 when we counted the degrees of freedom of a rigid body in three dimensions, the configuration of a rigid body can be described by a point in \mathbb{R}^3 , plus a point on a two-dimensional sphere S^2 , plus a point on a one-dimensional circle S^1 , giving a total C-space of $\mathbb{R}^3 \times S^2 \times S^1$.



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Task Space and Work Space

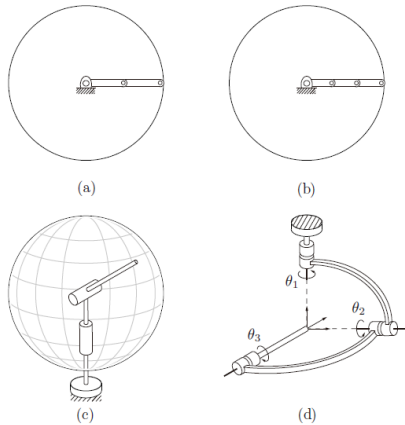
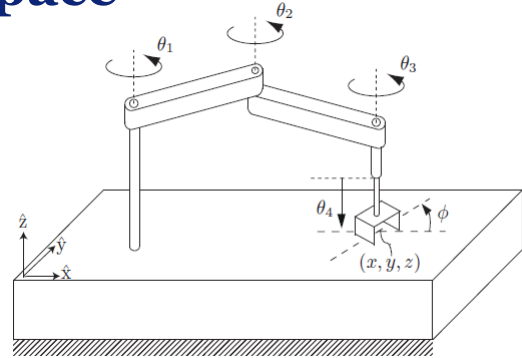


Figure 2.12: Examples of workspaces for various robots: (a) a planar 2R open chain; (b) a planar 3R open chain; (c) a spherical 2R open chain; (d) a 3R orienting mechanism.

The **workspace** is a specification of the configurations that the end-effector of the robot can reach.



The SCARA robot is an **RRRP open chain** that is widely used for tabletop pick-and-place tasks. The end-effector configuration is completely described by (x, y, z, ϕ)

⇒ **task space** $R^3 \times S^1$ and

⇒ **workspace** as the reachable points in (x, y, z) , since all orientations ϕ can be achieved at all reachable points.

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Rigid Body Motion

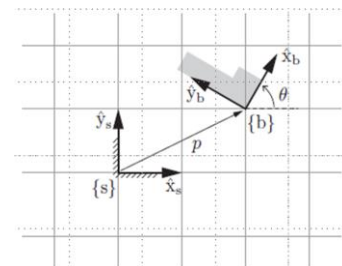
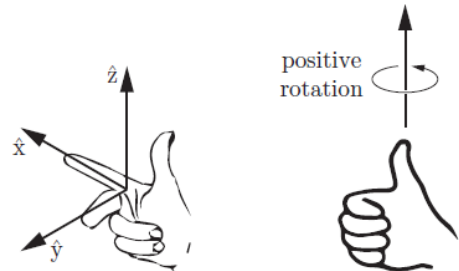
Rigid-body position and orientation $(x, y, z, \phi, \theta, \psi)$

- Can also be described by 4x4 matrix with 10 constraints.
- In general 4x4 matrices can be used for
 - Location
 - Translation + rotation of a vector or frame
 - Transformation of coordinates between frames
- Velocity of a rigid body: $(\partial x/\partial t, \partial y/\partial t, \partial z/\partial t, \partial \phi/\partial t, \partial \theta/\partial t, \partial \psi/\partial t)$

Exponential coordinates:

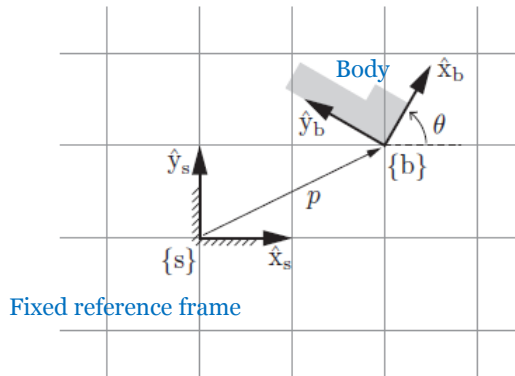
Every rigid-body configuration can be achieved by:

- Starting in the fixed home frame and integrating a constant twist for a specified time.
- Direction of a screw axis and scalar to indicate how far the screw axis must be followed



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Rigid Body Motions in the Plane



$$p = p_x \hat{x}_s + p_y \hat{y}_s.$$

$$\hat{x}_b = \cos \theta \hat{x}_s + \sin \theta \hat{y}_s,$$

$$\hat{y}_b = -\sin \theta \hat{x}_s + \cos \theta \hat{y}_s.$$

Figure 3.3: The body frame $\{b\}$ is expressed in the fixed-frame coordinates $\{s\}$ by the vector p and the directions of the unit axes \hat{x}_b and \hat{y}_b . In this example, $p = (2, 1)$ and $\theta = 60^\circ$, so $\hat{x}_b = (\cos \theta, \sin \theta) = (0.5, 1/\sqrt{2})$ and $\hat{y}_b = (-\sin \theta, \cos \theta) = (-1/\sqrt{2}, 0.5)$.

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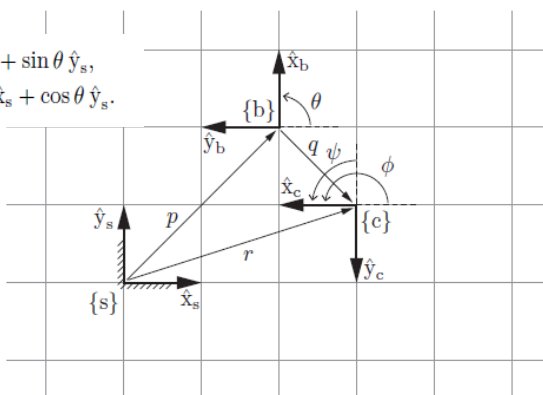
Rigid Body Motions in the Plane

Previously:

$$p = p_x \hat{x}_s + p_y \hat{y}_s.$$

$$\hat{x}_b = \cos \theta \hat{x}_s + \sin \theta \hat{y}_s,$$

$$\hat{y}_b = -\sin \theta \hat{x}_s + \cos \theta \hat{y}_s.$$



$\{b\}$ relative to $\{s\}$

$$p = \begin{bmatrix} p_x \\ p_y \end{bmatrix}$$

$$P = [\hat{x}_b \ \hat{y}_b] = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

$\{c\}$ relative to $\{s\}$

$$r = \begin{bmatrix} r_x \\ r_y \end{bmatrix}, \quad R = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix}$$

$\{c\}$ relative to $\{b\}$

$$q = \begin{bmatrix} q_x \\ q_y \end{bmatrix}, \quad Q = \begin{bmatrix} \cos \psi & -\sin \psi \\ \sin \psi & \cos \psi \end{bmatrix}$$

Figure 3.4: The frame $\{b\}$ in $\{s\}$ is given by (P, p) , and the frame $\{c\}$ in $\{b\}$ is given by (Q, q) . From these we can derive the frame $\{c\}$ in $\{s\}$, described by (R, r) . The numerical values of the vectors p , q , and r and the coordinate-axis directions of the three frames are evident from the grid of unit squares.

Note and verify: $R = PQ$, and $r = Pq + p$

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Rigid Body Motions in the Plane

{c'} described by (R,r)

$$r = \begin{bmatrix} r_x \\ r_y \end{bmatrix}, \quad R = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix}$$

Move rigid body such that {d} coincides with {d'}.

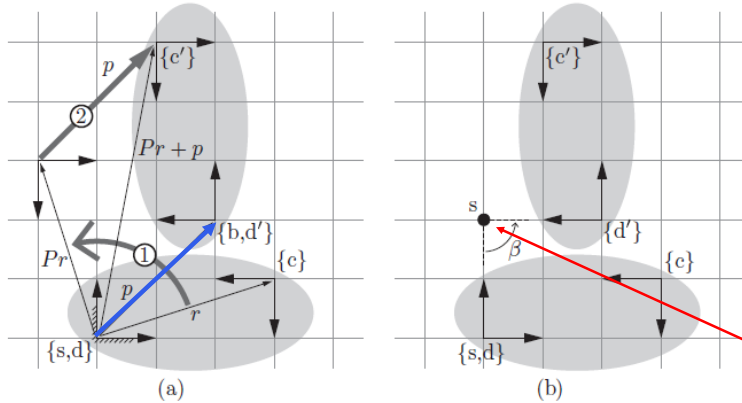
$$p = \begin{bmatrix} p_x \\ p_y \end{bmatrix} \quad P = [\hat{x}_b \ \hat{y}_b] = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

Then {c'} described by (R',r')

$$R' = PR, \\ r' = Pr + p,$$

Note: SCREW MOTION

The above rotation followed by a translation can also be expressed as a rotation of the rigid-body about a fixed point *s* by an angle β



(P, p) can be used to

1. Represent a configuration of a rigid body in {s}
2. Change the reference frame for vector representation.
3. **Displace a vector or a frame.**

Rigid Body Motions in the Plane

{c'} described by (R,r)

$$r = \begin{bmatrix} r_x \\ r_y \end{bmatrix}, \quad R = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix}$$

Move rigid body such that {d} coincides with {d'}.

$$p = \begin{bmatrix} p_x \\ p_y \end{bmatrix} \quad P = [\hat{x}_b \ \hat{y}_b] = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

Then {c'} described by (R',r')

$$R' = PR, \\ r' = Pr + p,$$

Note: SCREW MOTION

The above rotation followed by a translation can also be expressed as a rotation of the rigid-body about a fixed point *s* by an angle β

(β, s_x, s_y), where $(s_x, s_y) = (0, 2)$

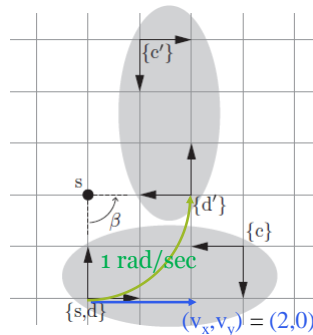
In the {s}-frame rotate 1 rad/sec with speed $(v_x, v_y) = (2, 0)$ is denoted as:

$$S = (\omega, v_x, v_y) = (1, 2, 0)$$

Following the screw-axis for an angle $\theta = \pi/2$ gives the displacement we want:

$$S\theta = (\pi/2, \pi, 0)$$

These are called the **exponential coordinates** for the planar rigid-body displacement.



Note:

- distance = vt
- distance along quarter circle with radius 2 equals π .

Forward Kinematics

The forward kinematics of 3R Planar Open Chain can be written as a product of four homogeneous transformation matrices: $T_{04} = T_{01}T_{12}T_{23}T_{34}$, where

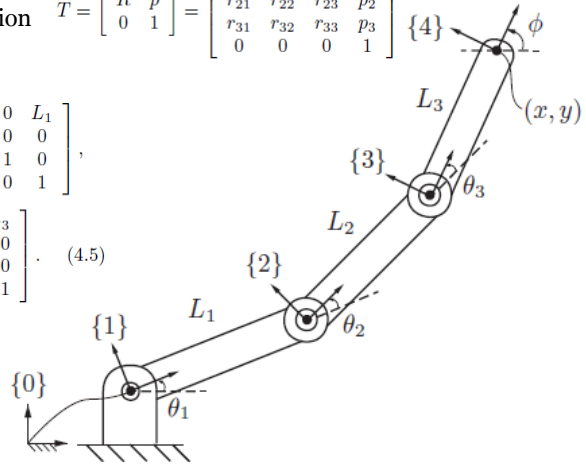
$$T = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_1 \\ r_{21} & r_{22} & r_{23} & p_2 \\ r_{31} & r_{32} & r_{33} & p_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{01} = \begin{bmatrix} \cos \theta_1 & -\sin \theta_1 & 0 & 0 \\ \sin \theta_1 & \cos \theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad T_{12} = \begin{bmatrix} \cos \theta_2 & -\sin \theta_2 & 0 & L_1 \\ \sin \theta_2 & \cos \theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

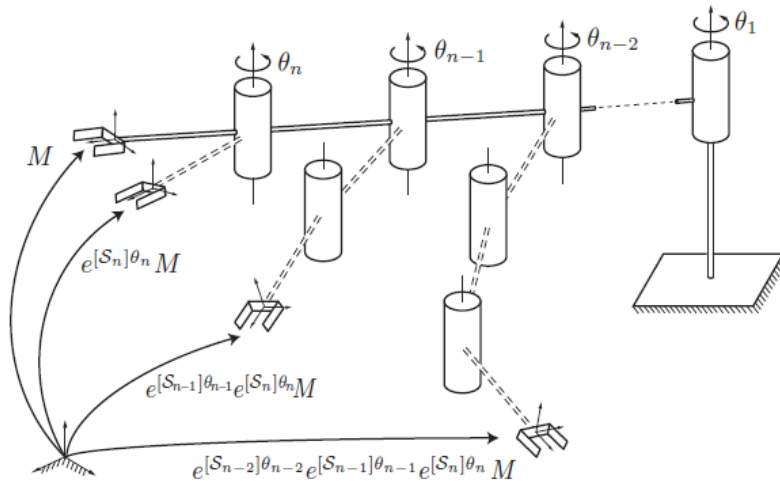
$$T_{23} = \begin{bmatrix} \cos \theta_3 & -\sin \theta_3 & 0 & L_2 \\ \sin \theta_3 & \cos \theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad T_{34} = \begin{bmatrix} 1 & 0 & 0 & L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (4.5)$$

Home position M:

$$M = \begin{bmatrix} 1 & 0 & 0 & L_1 + L_2 + L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Forward Kinematics: Product of Exponentials



PoE parameters also known as Euler-Rodrigues parameters.

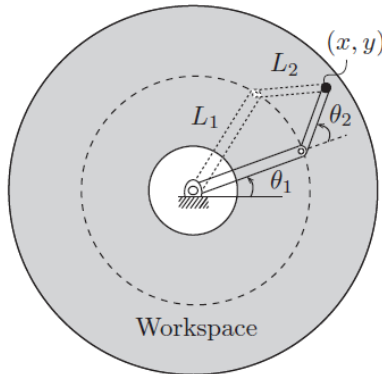
There are many other representations: - for example Denavit-Hartenberg (1955) representation is very popular, but can be cumbersome

In velocity kinematics Jacobians are used.

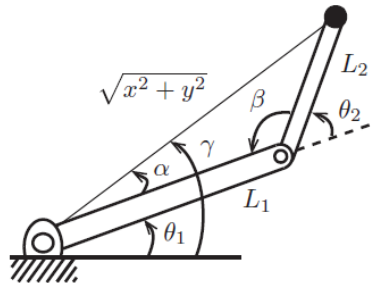
Figure 4.2: Illustration of the PoE formula for an n -link spatial open chain.

Inverse Kinematics

Which angles θ_1 , and θ_2 will lead to location (x,y) ?



(a) A workspace, and lefty and righty configurations.



(b) Geometric solution.

Law of cosines gives:

$$L_1^2 + L_2^2 - 2L_1L_2 \cos \beta = x^2 + y^2$$

, hence

$$\beta = \cos^{-1} \left(\frac{L_1^2 + L_2^2 - x^2 - y^2}{2L_1L_2} \right)$$

,and similarly

$$\alpha = \cos^{-1} \left(\frac{x^2 + y^2 + L_1^2 - L_2^2}{2L_1\sqrt{x^2 + y^2}} \right)$$

$$\gamma = \text{atan2}(y,x)$$

Answer:

$$\theta_1 = \gamma - \alpha, \quad \theta_2 = \pi - \beta$$

In general: IK-Solvers

Figure 6.1: Inverse kinematics of a 2R planar open chain.

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Real Time Physics Modelling

<https://pybullet.org/wordpress/>

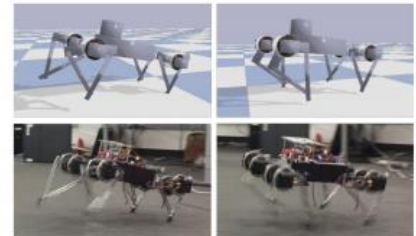
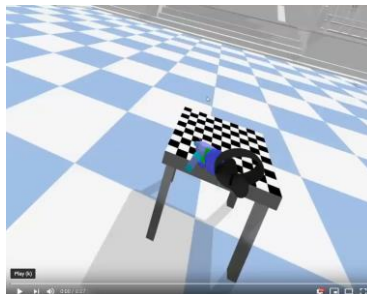
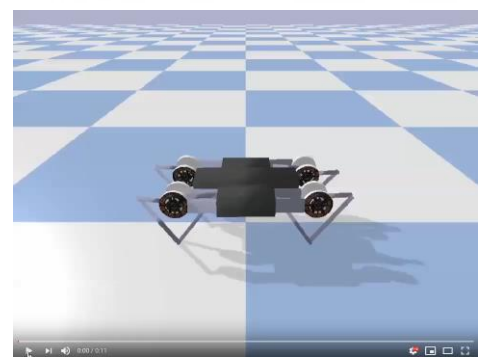
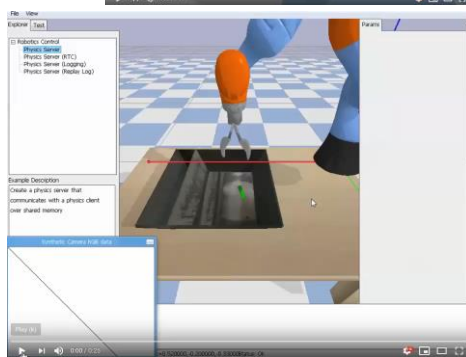


Fig. 1: The simulated and the real Minitaur learned to gallop using deep reinforcement learning.

pybullet KUKA
grasp training

Using Tensorflow
OpenAI gym
Baselines
DeepQNetworks (DQNs)



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Robotics Preparations

1) Form YetiBorg Racing Teams of 3 to 4 people

Appoint one person who will be responsible for the robot.

Email your teams to erwin@liacs.nl with subject 'Robotics YetiBorg Racing Team'.

Due: Thursday 7-3 at 14.00 PM.

2) Project Proposal Title and Abstract

Give the title and abstract of the project proposal you will present on March 15th.

Also mention the number of people that will cooperate on the project (1-4).

Email your proposal to erwin@liacs.nl with subject 'Robotics Project Proposal'.

Due: Thursday 7-3 at 14.00 PM.

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References

1. K.M. Lynch, F.C. Park, Modern Robotics: Mechanics, Planning and Control, Cambridge University Press, 2017. (DOI: 10.1017/9781316661239)
2. <https://pybullet.org/wordpress/>

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Robotics



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