Biomedical Security

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Some Security News

Google's Project Zero Finds a Nation-State Zero-Day Operation

Google's Project Zero discovered, and caused to be patched, eleven zero-day exploits against Chrome, Safari, Microsoft Windows, and iOS. This seems to have been exploited by "Western government operatives actively conducting a counterterrorism operation":

> The exploits, which went back to early 2020 and used never-before-seen techniques, were "watering hole" attacks that used infected websites to deliver malware to visitors. They caught the attention of cybersecurity experts thanks to their scale, sophistication, and speed.

Wi-Fi Devices as Physical Object Sensors

The new 802.11bf standard will turn Wi-Fi devices into object sensors:

> In three years or so, the Wi-Fi specification is scheduled to get an upgrade that will turn wireless devices into sensors capable of gathering data about the people and objects bathed in their signals.
>
> "When 802.11bf will be finalized and introduced as an IEEE standard in September 2024, Wi-Fi will cease to be a communication-only standard and will legitimately become a full-fledged sensing paradigm," explains Francesco Restuccia, assistant professor of electrical and computer engineering at Northeastern University, in a paper summarizing the state of the Wi-Fi Sensing project (SENS) currently being developed by the Institute of Electrical and Electronics Engineers (IEEE).
>
> SENS is envisioned as a way for devices capable of sending and receiving wireless data to use Wi-Fi signal interference differences to measure the range, velocity, direction, motion, presence, and proximity of people and objects.

More detail in the article. Security and privacy controls are still to be worked out, which means that there probably won't be any.

Tags: academic papers, sensors, Wi-Fi, wireless
Posted on April 5, 2021 at 6:15 AM • 31 Comments

April 8th 2021, http://www.scheier.com

Shared cash in DDIO makes Keystroke timing attack possible for untrusted program by monitoring/timimg SSH packets.

Overview

- Cryptography: Classical Algorithms,
- Cryptography: Public Key Algorithms
- Cryptography: Protocols
- Cryptography Workshop
- Biomedical Security and Applications
- Student Presentations

Grading:
Class participation, assignments (3 out of 4)
(workshop + presentation + technical survey)/3

Cryptography: Sharing Secrets

Crypto-text C

Alice

$C = E_K$ ('HELLO BOB')

Secret key K

K?

Eve

Bob

$D_K(C)$ = 'HELLO BOB'

Secret key K

How?

Crypto-Analyst Eve
- Crypto-text only
- Known Plaintext
- Chosen Plaintext

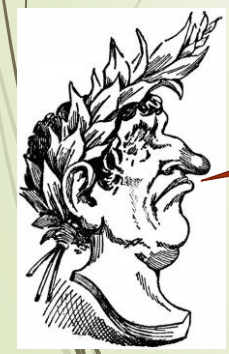
Cryptography: Sharing Secrets

- CAESAR a substitution cipher

Secret Key: 3

Plain Text: A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

Cipher Text: D E F G H I J K L M N O P Q R S T U V W X Y Z A B C



DWWDFFKL

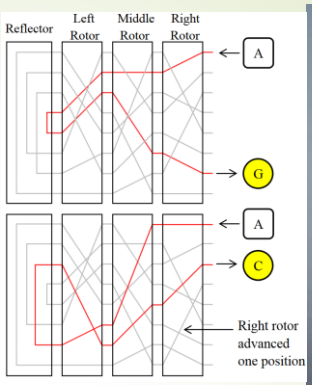
E_3(HELP) = KHOS
D_3(KHOS) = HELP

$D_3 = E_{26-k}$

Mafia boss Bernardo Provenzano's cipher: 'A' -> 4, 'B' -> 5, etc. In April 2006, Provenzano was captured in Sicily partly because messages encrypted using his cipher, were broken.

https://www.theregister.co.uk/2006/04/19/mafia_don_clueless_crypto/

Enigma



Encryption as a product of permutations:

- P the plug-board transformation
- U the reflector
- L, M, and R the three rotors
- Then encryption is $E = PRMLUL^{-1}M^{-1}R^{-1}P^{-1}$
- After each key press the rotors turn i positions changing the transformation: R becomes C^iRC^{-i}, where C is the cyclic permutation (A->B, B-> C, etc. …)
- the military Enigma has 158,962,555,217,826,360,000 settings

https://en.wikipedia.org/wiki/Enigma_machine

http://enigmamuseum.com/replica/

ONE-TIME PAD

- A crypto system with perfect secrecy

Plaintext: 01000110101110100110
Key: 11010100001100010010
Crypto-text: 10010010100010110100

Uses XOR for both encryption and decryption.

Classical Symmetric or Two-way Crypto Systems

- A shared secret key K used for both encryption as well as decryption.

Plaintext P
Crypto-text C

$C = E_K(P)$
$P = E^{-1}_K(C) = D_K(C)$

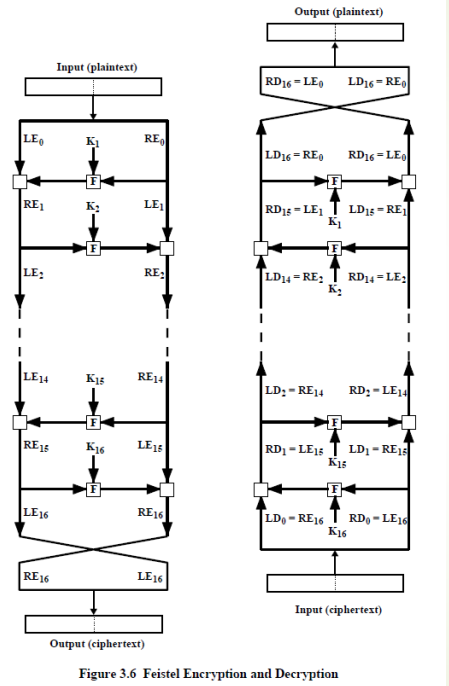
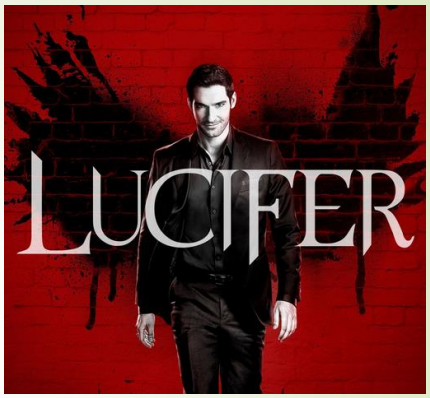


Figure 3.6 Feistel Encryption and Decryption



IBM's Cipher LUCIFER designed by H. Feistel and D. Coppersmith in 1973 used Feistel Networks for encryption and decryption.

LUCIFER is one of the first commercial block ciphers on which DES is based.

Classical Symmetric Crypto System: Data Encryption Standard (DES)

- March 17, 1975 published by the National Bureau of Standards (NBS)
- NSA reduced key-size from the original 128-bit to 56-bit
- At the time NSA studied it and said it was secure to use as a standard SKCS.
- Next government standard was classified: Skipjack

- Block cipher encrypting data in 64-bit blocks
- Key length 56-bits
- 16 rounds: in each round a substitution followed by a permutation

Advanced Encryption Standard (AES)

- Block-size: 128
- Key-sizes: 128, 192, 256
- NIST Specification 2001
- Origin: a subset of 3 out of the Rijndael Cipher by V. Rijmen and J. Daemen (NIST paper 2003)
- Substitution-permutation network
- From the cipher key the keys per round are derived.
- Each round
  - has a non-linear substitution step implemented using a lookup table
  - Followed by transposition using cyclic shifts
  - And a mixing step on the columns of the internal state matrix.
  - The round ends with an add key operation.

Secret Key Crypto Systems

Alice

$C = E_K$ ('HELLO BOB')

Secret key K

Crypto-text C

Bob

$D_K(C)$ = 'HELLO BOB'

Secret key K

K?

Eve

How?

Crypto-Analyst Eve
- Crypto-text only
- Known Plaintext
- Chosen Plaintext

Public Key Crypto Systems

Idea by Diffie and Hellman [1976]

- Encryption method made public.
- Decryption method kept secret.

Closely related to the idea of cryptographic one-way functions:

But there exists a trapdoor that makes it easy to calculate x given f(x).

Note: No known cryptographic one-way functions. Only likely to be intractable!

Intractability and P = NP?

'Definition' P the class of problems solvable in polynomial time

'Definition' NP the class of problems solvable in non-deterministic polynomial time.
i.e., guess the solution in O(1) time and verify in polynomial time.

- SAT, 3-SAT, Traveling Salesman Problem, Knapsack Problem, etc, are in NP

'Definition' Assume H ϵ NP. Then H is NP-Hard if any NP problem can be reduced to H in polynomial time.

- SAT is NP-Hard

'Definition' Assume H ϵ NP, then H is NP-Complete if there exists an NP-Complete problem L that reduces in polynomial time to H.

- SAT is NP-Complete (S.A. Cook, ACM STOC, 1971)
- If there exists a polynomial time algorithm for any NP-Complete problem then P = NP.

See for a more formal introduction to the theory of NP-Completeness:
Michael R. Garey and David S. Johnson (1979).
Computers and Intractability: A Guide to the Theory of NP-Completeness. W.H. Freeman. ISBN 0-7167-1045-5.

Public Key Crypto Systems

Alice

$C = E_{Bob}$ ('HELLO BOB')

Get Public Key E_{Bob}
From Bob or
from Public Key Register

Crypto-text C

Public Key Register

E_{Bob}
E_{Alice}
E_{Eve}
E_{You}
…
E_{Me}

Bob

D_{Bob} (C) = 'HELLO BOB'

Secret Key D_{Bob}
Public Key E_{Bob}
Publish Public Key **E_{Bob}** on Public Key Register or send to Alice

Digital Signatures

Alice

M = 'Message from Alice'
S = D_{Alice} ('Message from Alice')

Secret Key D_{Alice}
Public Key E_{Alice}
on Public Key Register, or send to Bob

Message M, **Signature S**

Public Key Register

E_{Bob}
E_{Alice}
E_{Eve}
E_{You}
…
E_{Me}

Bob

Verify:
E_{Alice} (**S**) = 'Message from Alice' = M

Get Public Key E_{Alice}

Public Key Crypto System: RSA

*Rivest, R.; Shamir, A.; Adleman, L. "A Method for Obtaining Digital Signatures and Public-Key Cryptosystems" . Communications of the ACM. **21** (2): 120–126. (Feb. 1978).*

- Key Generation

  Select p, q, both primes.

  Calculate $n = p * q$

  Calculate $\varphi(n) = (p-1)(q-1)$

  Select e such that $\gcd(\varphi(n), e) = 1$ and $1 < e < \varphi(n)$

  Calculate d such that $d = e^{-1} \bmod \varphi(n)$

  Public key is (e, n)

  Secret key is (d, n)

- Encryption of plaintext $M < n$

  $C = M^e \bmod n$

- Decryption of ciphertext/crypto-text C

  $M = C^d \bmod n$

A Short Introduction to Number Theory

- Primes
- Factorization
- Euclid's Algorithm
- Modular Arithmetic and Groups
- Fast Exponentiation
- Discrete Logarithms
- Euler Phi

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Number Theory

Definition (Divisors):
$b \neq 0$ divides a, if $a = mb$ for some m (where a, b, and m are integers)

Notation: $b \mid a$

Example: divisors of 24 are
1, 2, 3, 4, 6, 8, 12, and 24

Question: does $-4 \mid 24$ hold?

The following relations hold:

- if $a \mid 1$, then $a = \pm 1$
- if $a \mid b$ and $b \mid a$, then $a = \pm b$
- any $b \neq 0$ divides 0
- if $b \mid g$ and $b \mid h$, then $b \mid (mg+nh)$ for arbitrary integers m and n

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Number Theory

Definition (Prime Numbers):
An integer $p>1$ is a prime number if its only divisors are ± 1 and $\pm p$.

Theorem: Any positive integer $a>1$ can be factored in a unique way as:

$$a = p_1^{a1} \cdot p_2^{a2} \ldots p_t^{at},$$
where $p_1 > p_2 > \ldots > p_t$ are prime,
and $a_i > 0$

or $a = \prod_{i=1\ldots t} p_i^{a_i}$, where $p_1 > p_2 > \ldots > p_t$ are prime and each $a_i \geq 0$

Example: $91 = 7 \times 13$,
$11011 = 7 \times 11^2 \times 13$

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Number Theory

Definition1 (GCD):

The positive integer c is said to be the greatest common divisor of a and b if:

1) c | a and c | b
2) if d | a and d | b, then d | c

Notation: c = gcd(a,b)

Definition2 (GCD):

gcd(a,b) = max[k, such that k | a and k | b]

Example: $192 = 2^2 \times 3^1 \times 4^2$

$18 = 2^1 \times 3^2$

$gcd(18,192) = 2^1 \times 3^1 \times 4^0 = 6$

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Number Theory

Definition1 (Relative Prime):

The integers a and b are said to be relatively prime if gcd(a,b) = 1.

Example:

192 and 18 are not relatively prime:

$192 = 2^2 \times 3^1 \times 4^2$

$18 = 2^1 \times 3^2$

$gcd(18,192) = 2^1 \times 3^1 \times 4^0 = 6$

74 and 75 are relatively prime:

$74 = 2 \times 37$

$75 = 3 \times 5^2$

$gcd(74,75) = 1$

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Number Theory: Modular Arithmetic

Given any positive integer n and any integer a we can write:

$a = qn + r$, where $0 \leq r < n$, $q = \lfloor a/n \rfloor$

r is called the **residue** (mod n)

Definition: If a is an integer and n is a positive integer we define *a mod n* to be the remainder when a is divided by n.
Thus, $a = \lfloor a/n \rfloor \times n + (a \bmod n)$

Definition: Two integers a and b are said to be congruent modulo n if
$(a \bmod n) = (b \bmod n)$

Notation: $a \equiv b \bmod n$

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Number Theory: Modular Arithmetic

Examples: $73 \equiv 4 \bmod 23$ as
$73 = 3 \times 23 + 4$, hence
$(73 \bmod 23) = 4$, and clearly $4 = (4 \bmod 23)$, thus
$(73 \bmod 23) = (4 \bmod 23) \Rightarrow 73 \equiv (4 \bmod 23)$

$21 \equiv -9 \bmod 10$ as
$1 = (21 \bmod 10)$ and
$1 = (-9 \bmod 10)$

Properties (Check):

- $a \equiv b \bmod n$ if $n \mid (a-b)$
- $(a \bmod n) = (b \bmod n)$ implies $a \equiv b \bmod n$
- $a \equiv b \bmod n$ implies $b \equiv a \bmod n$
- $a \equiv b \bmod n$ and $b \equiv c \bmod n$ implies $a \equiv c \bmod n$

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Number Theory: Modular Arithmetic

The mod n operator maps all integers into the set of integers $\mathbb{Z}_n = \{0,1,\ldots,(n-1)\}$, the set of all residues modulo n.

The following properties hold for modular arithmetic within $\mathbb{Z}_n$:

- $(w + x) \bmod n = (x + w) \bmod n$
- $((w+x)+y) \bmod n = (w+(x+y)) \bmod n$
- $(0+w) \bmod n = w \bmod n$
- $\forall w \in \mathbb{Z}_n \; \exists z \in \mathbb{Z}_n$ such that $w + z \equiv 0 \bmod n$

- $(w \times x) \bmod n = (x \times w) \bmod n$
- $((w \times x) \times y) \bmod n = (w \times (x \times y)) \bmod n$
- $(1 \times w) \bmod n = w \bmod n$
- $(w \times (x+y)) \bmod n = ((w \times x)+(w \times y)) \bmod n$

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Number Theory: Modular Arithmetic

Z_8:	0	1	2	3	4	5	6	7
×6:	0	6	12	18	24	30	36	42
mod 8:	0	6	4	2	0	6	4	2

Z_8:	0	1	2	3	4	5	6	7
×5:	0	5	10	15	20	25	30	35
mod 8:	0	5	2	7	4	1	6	3

Note: gcd(6,8) = 2, and gcd(5,8) = 1

Notation: $Z_p^* = \{1,2,\ldots,(p-1)\}$

Theorem: Let p prime, then for each $w \in Z_p^*$ there exists a number z such that $w \times z \equiv 1 \bmod p$,

z is equal to the multiplicative inverse w^{-1} of w in Z_p^*

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Fast Exponentiation

Calculate $a^b \bmod n = 7^{560} \bmod 561$

$a = 7$, $b = 560 = 1000110000$, $n = 561$

I	bit b_i	Exponent c	result d	->7^{560}
9	1	1	7	7^1
8	0	2	49	7^2
7	0	4	157	7^4
6	0	8	526	7^8
5	1	17	160	7^{16+1}
4	1	35	241	7^{32+2+1}
3	0	70	298	7^{64+4+2}
2	0	140	166	$7^{128+8+4}$
1	0	280	67	$7^{256+16+8}$
0	0	560	1	$7^{512+32+16}$

```
c ← 0; d ← 1
for i ← k downto 0
    do  c ← 2 × c
        d ← (d × d) mod n
        if  bi = 1
            then  c ← c + 1
                  d ← (d × a) mod n
return d
```

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Number Theory: Euler Totient Function

Definition: The Euler's totient function $\Phi(n)$ of n is equal to the number of positive integers $<n$ that are relative prime to n.

Examples:

8: {1,3,5,7} are relative prime to 8 and <8, thus $\Phi(8) = 4$

11: {1,2,3,4,5,6,7,8,9,10} are all relative prime to 11 and <11, thus $\Phi(11) = 10$

Lemma: If p is prime, then $\Phi(p) = p - 1$.

Lemma: If $n = pq$, with p and q prime, then $\Phi(n) = (p-1)(q-1)$.

Proof: {p,2p,…,(q-1)p}, {q,2q,…,(p-1)q}, and 0 are not relatively prime to n. Thus $\Phi(n) = pq - (q-1) - (p-1) - 1 = (p-1)(q-1)$.

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Number Theory: Euler's Totient Function

Fermat's Theorem (1640): For every prime p and any integer a, the following holds:

$$a^{p-1} \equiv 1 \bmod p$$

Euler's Theorem (~1740): For any positive integer n, and any integer a relative prime to n, the following holds:

$$a^{\Phi(n)} \equiv 1 \bmod n$$

Corollary: Let p,q be prime, and n = pq, and m an integer such that gcd(m,n)=1, then

$$m^{(p-1)(q-1)} \equiv 1 \bmod n$$

Examples:

$2^6 = 64 = 63 + 1 \equiv 1 \bmod 7$

$4^{(5-1)(7-1)} = 4^{24} = (4^8)^3 \bmod 35 \equiv 16^3 \bmod 35 \equiv 4096 \bmod 35 \equiv 1 \bmod 35$

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Number Theory: Testing for Primality

[Miller'75, Rabin'80]

Procedure Witness(a,n) n is to be tested for primality, a is some integer less than n.

if (not $a^{n-1} \equiv 1 \bmod n$) or ($\exists x: x^2 \equiv 1 \bmod n$ and $x \neq \pm 1$)

then return TRUE { n is no prime }

else return FALSE { n may be prime }

If n is no prime the probability that Witness returns FALSE is <0.5.

Thus, if Witness returns FALSE s times the probability that n is prime is at least $1 - 2^{-s}$.

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Number Theory: Number of Primes

Definition: $\pi(n)$ is equal to the number of primes p that satisfy $2 \le p \le n$.

Theorem: (The Prime Number Theorem)

Conjectured by Legendre, Gauss, Dirichlet, Chebyshev, and Riemann; proven by Hadamard and de la Vallee Poussin in 1896.

$$\pi(n) \sim n/\ln(n)$$

Thus there are about

$$10^{100}/\ln(10^{100}) - 10^{99}/\ln(10^{99}) = 0.039 \times 10^{99}$$ 100-digit primes

There are 4.5×10^{99} 100-digit odd numbers.

That is, about 1 of every 115 ***100-digit odd numbers*** is prime.

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Number Theory: Euclid's Algorithm (~300 BC) Finding the Greatest Common Divisor

Theorem: For any integer $a \ge 0$, and any integer $b > 0$: ***gcd(a,b) = gcd(b,a mod b)***

Proof: Let $d = \gcd(a,b)$ => $d \mid a$ and $d \mid b$

=> $a = kb + a \bmod b$ for some integer k

=> $(a \bmod b) = a - kb$

=> $d \mid (a \bmod b)$ (as $d \mid a$ and $d \mid kb$).

Thus d is a common divisor of b and $(a \bmod b)$.

Conversely, if $d = \gcd(b, a \bmod b)$, then $d \mid kb$

=> $d \mid (kb + a \bmod b)$

=> $d \mid a$.

Thus d is also a common divisor of a and b.

qed

Example (Calculation of GCD):

- gcd(12,18) = gcd(18,6) = gcd(6,0) = 6
- gcd(10,11) = gcd(11,1) = gcd(1,0) = 1

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Number Theory: Euclid's Extended Algorithm Finding the Multiplicative Inverse

If $\gcd(d,n) = 1$, then $(d^{-1} \bmod n)$ exists.

i.e., $dd^{-1} = 1 \bmod n$.

Complexity: The multiplicative inverse can be found in $O(\log^2 n)$ time.

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Number Theory: Discrete Logarithm

Definition: Let $Z_n^*=\{1,2,\ldots,(n-1)\}$, and g in Z_n^*. Then any integer x such that:

$$g^x = y \bmod n$$

is called a ***discrete logarithm*** *of y to base g*.

Example:

Z_7^*	1	2	3	4	5	6
	3^1	3^2	3^3	3^4	3^5	3^6
g=3	3	2	6	4	5	1

Z_7^*	1	2	3	4	5	6
$\log_3$	6	2	1	4	5	3

N.B. $g = 3$ is a generator of Z_7^*

Definition: If for g in Z_p^* $\{g^1,\ldots,g^{(p-1)}\} = Z_p^*$ holds, then g is a *generator* of Z_p^*.

Number Theory: Complexity of PRIMES, Discrete Log, FACTORIZE, etc.

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- Finding Primes (PRIMES is in P, AKS-Algorithm, August 2002)

After 1/115 tries success. Each try fastexp and some tests are executed => O(log n) time.

- Finding Safe Primes

It is unknown whether there exist infinitely many safe primes.

- Calculating the Discrete Logarithm

If the prime factors of (p-1) are small there exist efficient algorithms, otherwise roughly the same complexity as factorising.

- Factorising n (b-bits)

  Peter Shor(1994): $O(b^3)$ and $O(b)$ space on a quantum computer.

  Kleinjung et al. (2010) used general number field sieve GNFS- approach, $O(e^{\sqrt[3]{\frac{64}{9}b(\log b)^2}})$ time, for the factorization of a 768-bit RSA modulus n.

- Calculating Euler's Phi Function of n

It is unknown if this can be done without factorising n.

- Finding the multiplicative inverse mod n

$O(\log^2 n)$

Public Key Crypto Systems

Idea by Diffie and Hellman [1976]

- Encryption method made public.
- Decryption method kept secret.

Closely related to the idea of cryptographic one-way functions:

But there exists a trapdoor that makes it easy to calculate x given f(x).

Note: No known cryptographic one-way functions. Only likely to be intractable!

Public Key Crypto System: RSA

*Rivest, R.; Shamir, A.; Adleman, L. "A Method for Obtaining Digital Signatures and Public-Key Cryptosystems" . Communications of the ACM. **21** (2): 120–126. (Feb. 1978).*

- Key Generation
  - Select p, q, both primes.
  - Calculate $n = p * q$
  - Calculate $\varphi(n) = (p-1)(q-1)$
  - Select e such that $\gcd(\varphi(n), e) = 1$ and $1 < e < \varphi(n)$
  - Calculate d such that $d = e^{-1} \bmod \varphi(n)$
  - Public key is (e, n)
  - Secret key is (d, n)
- Encryption of plaintext $M < n$

  $C = M^e \bmod n$
- Decryption of ciphertext/crypto-text C

  $M = C^d \bmod n$

Public Key Crypto System: RSA Example

- Key Generation

  $p = 5, q = 11$

  => $n = p \times q = 5 \times 11 = 55$ and $\varphi(n) = (p-1)(q-1) = 4 \times 10 = 40$

  $e = 7$, is such that $\gcd(\varphi(n), e) = \gcd(40, 7) = 1$ and clearly $1 < 7 < \varphi(n)$

  then $d = 23 = e^{-1} \bmod \varphi(n)$ as $(7 \times 23 \bmod 24) = 161 \bmod 40 \equiv 1$

  Public key is $(7, 55)$

  Secret key is $(23, 55)$
- Encryption of plaintext $M = 2 < n = 55$

  $C = (M^e \bmod n) = (2^7 \bmod 55) = (128 \bmod 55) = 18 \bmod 55 \equiv 18$
- Decryption of ciphertext/crypto-text C

  now $C^d \bmod n = (18^{23} \bmod 55) = (2^{23} \times 3^{46} \bmod 55) = (4 \times 18 \times 18 \times 18 \times 3^6 \bmod 55) =$

  $(32 \times 3^{12} \bmod 55) = (32 \times 26^3 \bmod 55) = (2 \times 18 \times 13^3 \bmod 55) = (79092 \bmod 55) =$

  $(1438 \times 55 + 2 \bmod 55) = 2 \bmod 55 \equiv 2 = M$ (Note: you should use Fast Exponentiation)

RSA Chosen Ciphertext Attack

- Note for RSA: $E_e(M_1)xE_e(M_2) = E_e(M_1xM_2)$
- Assume (e,n) is the public keys and (d,n) the private key.
- Assume $C \equiv M^e \bmod n$ is intercepted.

Chosen Ciphertext attack:

- Compute $X \equiv (C \times 2^e) \bmod n$
- Submit X as a chosen ciphertext and receive back $Y \equiv X^d \bmod n$
- $X \equiv (C \times 2^e) \bmod n = (C \bmod n) \times (2^e \bmod n) = (M^e \bmod n) \times (2^e \bmod n) = (2M)^e \bmod n$
- Thus $Y \equiv X^d \bmod n = (2M)^{ed} \bmod n = 2M \bmod n$

Unsafe Modes of RSA

- Unsafe primes, etc.
- D. Boneh et al. Why Textbook ElGamal and RSA Encryption Are Insecure, T. Okamoto (Ed.): ASIACRYPT2000, LNCS 1976, pp. 30–43, 2000:
  - **without proper preprocessing of the plaintexts, both ElGamal and RSA encryption are fundamentally insecure.**
  - **when these systems encrypt a (short) secret key of a symmetric cipher it is often possible to recover the secret key from the ciphertext.**

  Conclusion: Preprocessing messages prior to encryption is an essential part of both systems.

Optimal Asymmetric Encryption Padding (**OAEP**)

Introduced by Bellare and Rogaway in 1994.

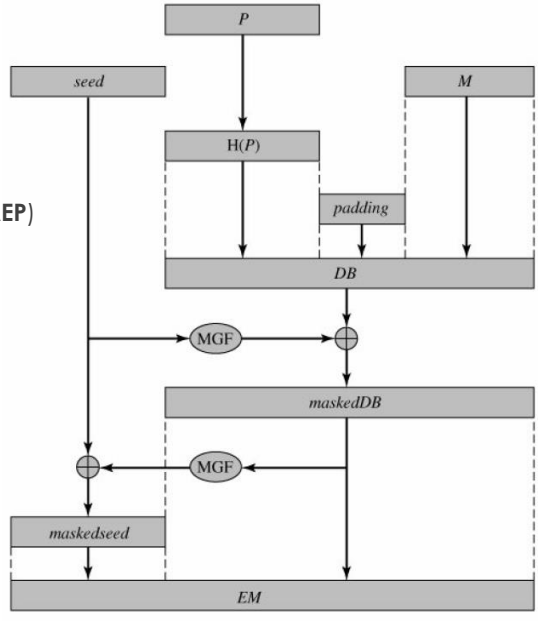
Safe Mode of RSA

Optimal Asymmetric Encryption Padding (**OAEP**)

Introduced by Bellare and Rogaway in 1994.

Note:
We can randomly pad the plaintext prior to encryption,
Then for this adapted version of RSA:
$\xi_e(M_1) \times \xi_e(M_2) \neq \xi_e(M_1 \times M_2)$, but this is not enough.

OAEP is a solution:
- P is a set of optional parameters.
- MGF is jus another hash function.
- EM is finally encoded using RSA.



Diffie-Hellman Key Exchange Algorithm

- Global and Public

  Prime **q**, and $\alpha < q$ a primitive root of **q**,

  i.e., α generates the multiplicative group of integers **mod q**
- Alice Key Generation

  Select private $X_A < q$

  Calculate public $Y_A = \alpha^{X_A} mod\ q$
- Bob Key Generation

  Select private $X_B < q$

  Calculate public $Y_B = \alpha^{X_B} mod\ q$
- Generation of the Exchanged Secret Key by Alice

  $K = (Y_B)^{X_A} mod\ q$
- Generation of the Exchanged Secret Key by Bob

  $K = (Y_A)^{X_B} mod\ q$

Diffie-Hellman Key-Exchange Example

- α generates the multiplicative group of integers mod q

For example:

If $\alpha = 2, q = 7$, then α is **no** generator of $\mathbb{Z}_7^* = \{1, 2, 3, 4, 5, 6\}$ as

$\{2^0 \bmod 7, 2^1 \bmod 7, 2^2 \bmod 7, 2^3 \bmod 7, 2^4 \bmod 7, 2^5 \bmod 7\} = \{1, 2, 4, 1, 2, 4, 1\} \neq \mathbb{Z}_7^*$

If $\alpha = 3, q = 7$, then

$\{3^0 \bmod 7, 3^1 \bmod 7, 3^2 \bmod 7, 3^3 \bmod 7, 3^4 \bmod 7, 3^5 \bmod\} = \{1, 3, 2, 6, 4, 5\} = \mathbb{Z}_7^*$

Note: Assuming the generalized Riemann hypothesis, the least primitive root $\alpha_p = O(\log^6 p)$ (Shoup,1990, 1992).

- Alice Key Generation:

  Select private $X_A < q$

  Calculate public $Y_A = \alpha^{X_A} mod\ q$
- From Y_A it should be difficult to calculate X_A.
- Calculating X_A can be done taking the discrete log of Y_A to the base α modulo q

Diffie-Hellman Key Exchange Complexity of Discrete Log

- Calculating X_A by taking the discrete log of Y_A to the base α modulo q is **assumed** to be intractable.
- *Shor, Peter (1997). "**Polynomial-Time Algorithms for Prime Factorization and Discrete Logarithms on a Quantum Computer**". SIAM Journal on Computing.* ***26*** *(5): 1484–1509.*
- *Adrian, David et al. (October 2015). "**Imperfect Forward Secrecy: How Diffie-Hellman Fails in Practice**":*
  - ***Logjam a flaw in TLS*** *that lets man-in-the middle downgrade connections to 'export-grade'-DH*
  - *Precomputations of a week for a 512-bit group => calculate discrete log for that group in ~ 1 minute.*
  - They found that 82% of vulnerable servers use a single 512-bit group, allowing us to compromise connections to 7% of Alexa Top Million HTTPS sites.
- **WeakDH.org: use a 2048-bit Diffie-Hellman group**

Digital Signatures

Using a public key crypto system.

(M,S)

Alice

Bob

Message M and signature $S = D_{Alice}(M)$

Checks with public E_{Alice}
if $E_{Alice}(S) = E_{Alice}(D_{Alice}(M)) = M$

Public Key Crypto System ElGamal

- Public Key
  - p a prime
  - $g < p$
  - $y = g^x mod\ p$
- Private Key
  - $x < p$
- Encryption of message M
  - random k, with $\gcd(k, p-1)=1$
  - $C = (a, b),\ where\ \ a = g^k mod\ p\ \ and\ \ b = y^k M\ mod\ p$
- Decryption
  - $M = b/a^x \bmod p$

ElGamal Signatures

- Public Key
  - p a prime
  - $g < p$
  - $y = g^x mod\ p$
- Private Key
  - $x < p$
- Signing of message M
  - random k, with $\gcd(k, p-1)=1$
  - Signature $S = (a, b),\ where$
  - $a = g^k mod\ p \quad and \quad b\ such\ that\ M = (xa + kb)\ mod\ p\text{-}1$
- Verification
  - Accept as valid if $y^a a^b\ mod\ p = g^M mod\ p$