# Concepts and

# Techniques

– Chapter 2 –

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# Chapter 2: Getting to Know Your Data

Data Objects and Attribute Types



- Basic Statistical Descriptions of Data
- Data Visualization
- Measuring Data Similarity and Dissimilarity
- Summary

## Types of Data Sets

#### Record

- Relational records
- Data matrix, e.g., numerical matrix, crosstabs
- Document data: text documents: termfrequency vector
- Transaction data
- Graph and network
  - World Wide Web
  - Social or information networks
  - Molecular Structures
- Ordered
  - Video data: sequence of images
  - Temporal data: time-series
  - Sequential Data: transaction sequences
  - Genetic sequence data
- Spatial, image and multimedia
  - Spatial data: maps
  - Image data
  - Video data

)-	team	coach	pla y	ball	score	game	ח <u>א</u> ו	lost	timeout	season
Document 1	3	0	5	0	2	6	0	2	0	2
Document 2	0	7	0	2	1	0	0	3	0	0
Document 3	0	1	0	0	1	2	2	0	3	0

TID	Items
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Coke, Diaper, Milk

#### Important Characteristics of Structured Data

- Dimensionality
  - Curse of dimensionality
- Sparsity
  - Only presence counts
- Resolution
  - Patterns depend on the scale
- Distribution
  - Centrality and dispersion

### Data Objects

- Data sets are made up of data objects.
- A data object represents an entity.
- Examples:
  - sales database: customers, store items, sales
  - medical database: patients, treatments
  - university database: students, professors, courses
- Also called samples , examples, instances, data points, objects, tuples.
- Data objects are described by attributes.
- Database rows -> data objects; columns ->attributes.

## Attributes

- Attribute (or dimensions, features, variables): a data field, representing a characteristic or feature of a data object.
  - E.g., customer\_ID, name, address
- Types:
  - Nominal: "red", "black", "blue", ...
  - Binary: 1/0, TRUE/FALSE
  - Numeric: quantitative
    - Interval-scaled
    - Ratio-scaled

# Attribute Types

- **Nominal:** categories, states, or "names of things"
  - Hair\_color = {auburn, black, blond, brown, grey, red, white}
  - marital status, occupation, ID numbers, zip codes
- Binary
  - Nominal attribute with only 2 states (0 and 1)
  - Symmetric binary: both outcomes equally important
    - e.g., gender
  - Asymmetric binary: outcomes not equally important.
    - e.g., medical test (positive vs. negative)
    - Convention: assign 1 to most important outcome (e.g., HIV positive)
- Ordinal
  - Values have a meaningful order (ranking) but magnitude between successive values is not known.
  - Size = {small, medium, large}, grades, army rankings

# Numeric Attribute Types

- Quantity (integer or real-valued)
- Interval
  - Measured on a scale of equal-sized units
  - Values have order
    - E.g., temperature in C°or F°, calendar dates
  - No true zero-point
- Ratio
  - Inherent zero-point
  - We can speak of values as being an order of magnitude larger than the unit of measurement (10 K° is twice as high as 5 K°).
    - e.g., temperature in Kelvin, length, counts, monetary quantities

# Attributes

#### Discrete Attribute

- Has only a finite or countably infinite set of values
  - E.g., zip codes, profession, or the set of words in a collection of documents
- Sometimes, represented as integer variables
- Note: Binary attributes are a special case of discrete attributes

#### Continuous Attribute

- Has real numbers as attribute values
  - E.g., temperature, height, or weight
- Practically, real values can only be measured and represented using a finite number of digits
- Continuous attributes are typically represented as floating-point variables

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# Basic Statistical Descriptions of Data

#### Motivation

- To better understand the data: central tendency, variation and spread
- Data dispersion characteristics
  - median, max, min, quantiles, outliers, variance, etc.
- Numerical dimensions correspond to sorted intervals
  - Data dispersion: analyzed with multiple granularities of precision
  - Boxplot or quantile analysis on sorted intervals
- Dispersion analysis on computed measures
  - Folding measures into numerical dimensions
  - Boxplot or quantile analysis on the transformed cube

# Measuring the Central Tendency

- <u>Mean (algebraic measure) (sample vs. population):</u>  $\overline{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$   $\mu = \frac{\sum x_i}{N}$ Note: *n* is sample size and *N* is population size.
  - Weighted arithmetic mean:
  - Trimmed mean: chopping extreme values
  - Median:
    - Middle value if odd number of values, or average of the middle two values otherwise
    - Estimated by interpolation (for grouped data):

$$median = L_1 + \left(\frac{n/2 - (\sum freq)l}{freq_{median}}\right) width \begin{array}{c} 16-20 & 300\\ 21-50 & 1500\\ 51-80 & 700 \end{array}$$

 $\sum_{i=1}^{n} w_i x_i$ 

 $\mathcal{W}_i$ 

age

1 - 5

6 - 15

81 - 110

 $\overline{x} = \frac{\overline{i=1}}{n}$ 

i=1

Mode

- Value that occurs most frequently in the data
- Unimodal, bimodal, trimodal
- Empirical formula:  $mean mode = 3 \times (mean median)$

12

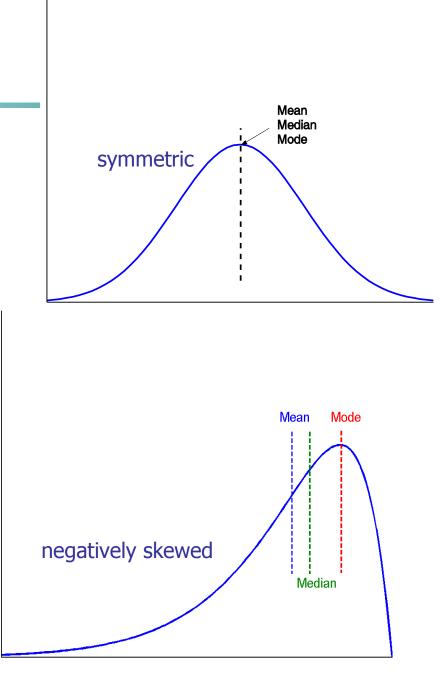
frequency

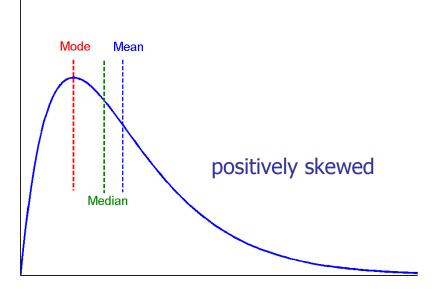
200

450

# Symmetric vs.Skewed Data

 Median, mean and mode of symmetric, positively and negatively skewed data





# Measuring the Dispersion of Data

- Quartiles, outliers and boxplots
  - Quartiles: Q<sub>1</sub> (25<sup>th</sup> percentile), Q<sub>3</sub> (75<sup>th</sup> percentile)
  - **Inter-quartile range**:  $IQR = Q_3 Q_1$
  - **Five number summary**: min, Q<sub>1</sub>, median, Q<sub>3</sub>, max
  - Boxplot: ends of the box are the quartiles; median is marked; add whiskers, and plot outliers individually
  - **Outlier**: usually, a value higher/lower than 1.5 x IQR
- Variance and standard deviation (*sample: s, population: \sigma*)

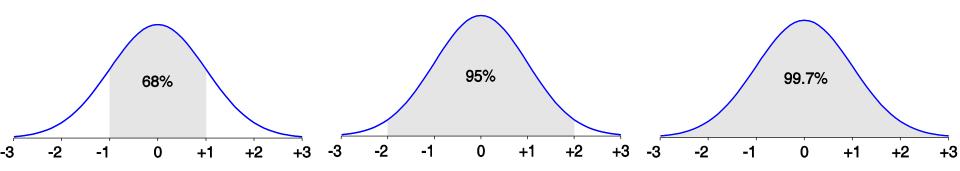
**Variance:** (algebraic, scalable computation)  

$$s^{2} = \frac{1}{n-1} \sum_{i=1}^{n} (x_{i} - \bar{x})^{2} = \frac{1}{n-1} \left[ \sum_{i=1}^{n} x_{i}^{2} - \frac{1}{n} (\sum_{i=1}^{n} x_{i})^{2} \right] \qquad \sigma^{2} = \frac{1}{N} \sum_{i=1}^{n} (x_{i} - \mu)^{2} = \frac{1}{N} \sum_{i=1}^{n} x_{i}^{2} - \mu^{2}$$

**Standard deviation** *s* (or  $\sigma$ ) is the square root of variance  $s^{2}(or \sigma^{2})$ 

### **Properties of Normal Distribution Curve**

- The normal (distribution) curve
  - From μ–σ to μ+σ: contains about 68% of the measurements (μ: mean, σ: standard deviation)
  - From  $\mu$ -2 $\sigma$  to  $\mu$ +2 $\sigma$ : contains about 95% of it
  - From  $\mu$ -3 $\sigma$  to  $\mu$ +3 $\sigma$ : contains about 99.7% of it



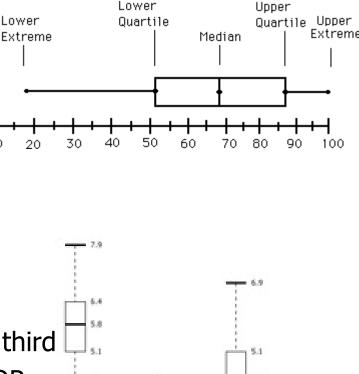
# Boxplot Analysis

Five-number summary of a distribution

Minimum, Q1, Median, Q3, Maximum

Boxplot

- Data is represented with a box
- The ends of the box are at the first and third quartiles, i.e., the height of the box is IQR
- The median is marked by a line within the box
- Whiskers: two lines outside the box extended to Minimum and Maximum
- Outliers: points beyond a specified outlier threshold, plotted individually



14140

+2.0

3.3

4.3

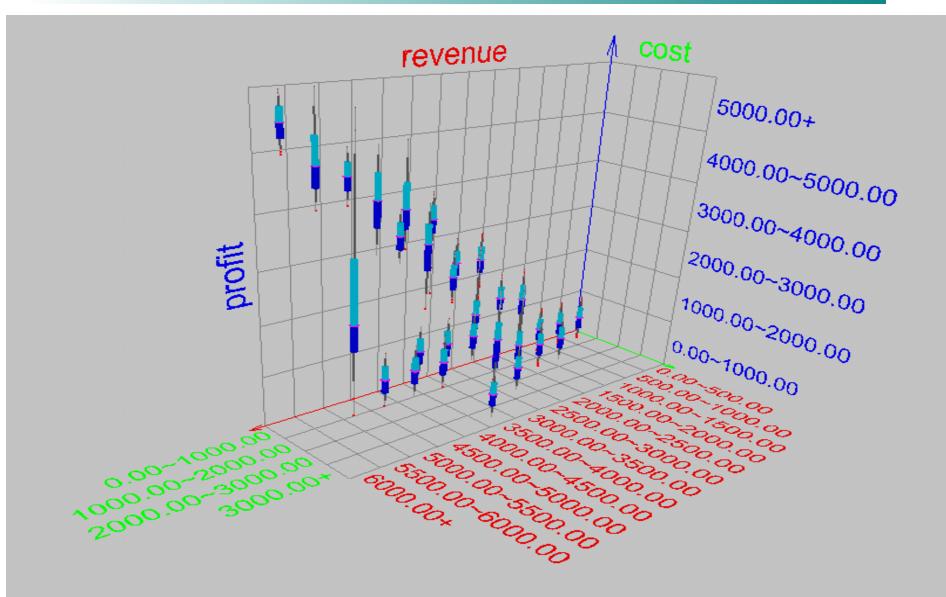
1.535

1.0

10

1.31

#### Visualization of Data Dispersion: 3-D Boxplots

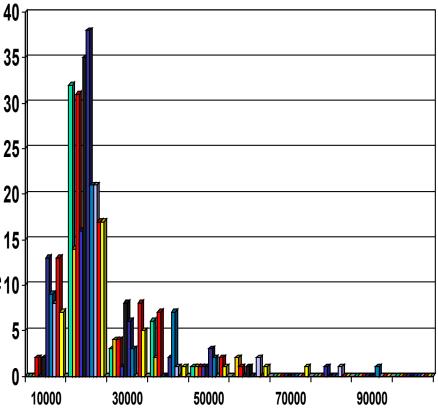


#### Graphic Displays of Basic Statistical Descriptions

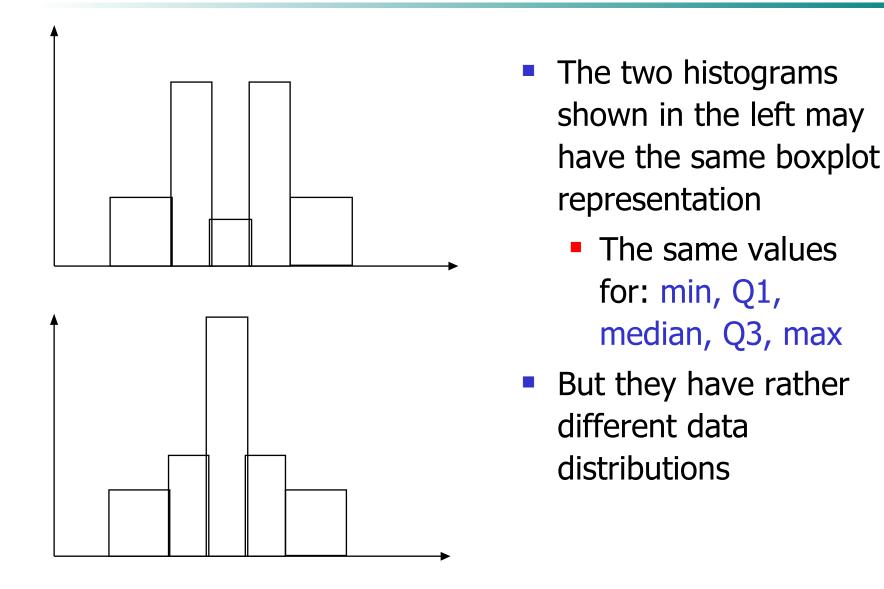
- Boxplot: graphic display of five-number summary
- Histogram: x-axis are values, y-axis repres. frequencies
- **Quantile plot**: each value  $x_i$  is paired with  $f_i$  indicating that approximately 100  $f_i$ % of data are  $\leq x_i$
- Quantile-quantile (q-q) plot: graphs the quantiles of one univariant distribution against the corresponding quantiles of another
- Scatter plot: each pair of values is a pair of coordinates and plotted as points in the plane

# Histogram Analysis

- Histogram: Graph display of tabulated frequencies, shown as bars
- It shows what proportion of cases fall into each of several categories
- Differs from a bar chart in that it is the area of the bar that denotes the value, not the height as in bar 15 charts, a crucial distinction when the 10 categories are not of uniform width
- The categories are usually specified as non-overlapping intervals of some variable. The categories (bars) must be adjacent

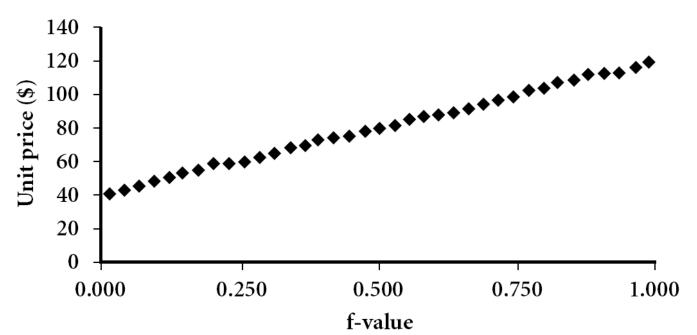


#### Histograms Often Tell More than Boxplots



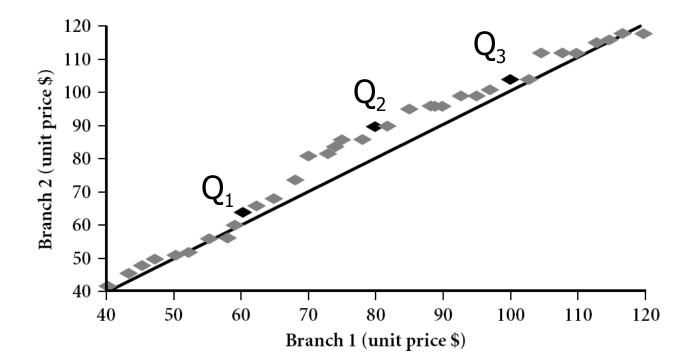
# Quantile Plot

- Displays all of the data (allowing the user to assess both the overall behavior and unusual occurrences)
- Plots quantile information
  - Let data x<sub>i</sub> data sorted in increasing order
  - $f_i$  indicates that approximately 100  $f_i^{\%}$  of the data  $\leq$  to the value  $x_i$



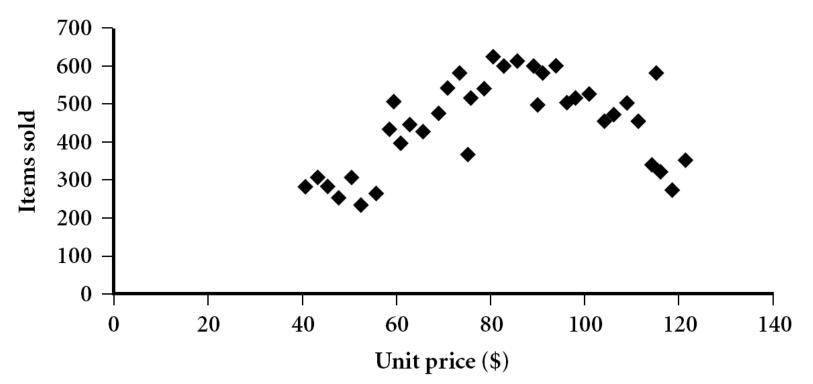
# Quantile-Quantile (Q-Q) Plot

- Graphs the quantiles of one univariate distribution against the corresponding quantiles of another
- View: Is there is a shift in going from one distribution to another?
- Example shows unit price of items sold at Branch 1 vs. Branch 2 for each quantile. Unit prices of items sold at Branch 1 tend to be lower than those at Branch 2.

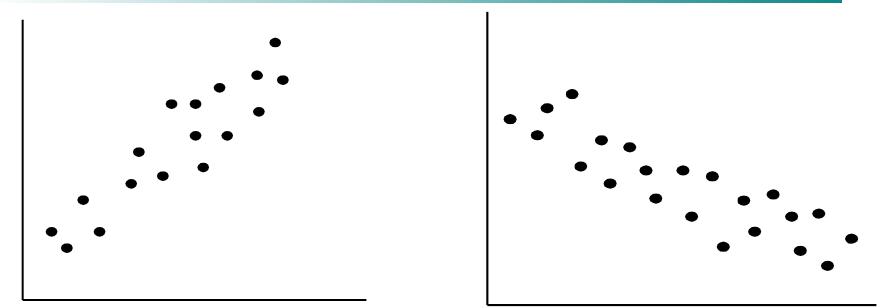


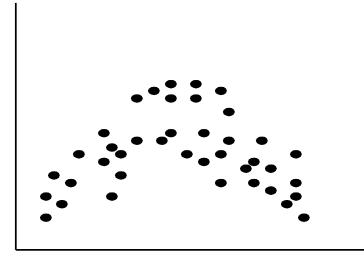
# Scatter plot

- Provides a first look at bivariate data to see clusters of points, outliers, etc
- Each pair of values is treated as a pair of coordinates and plotted as points in the plane



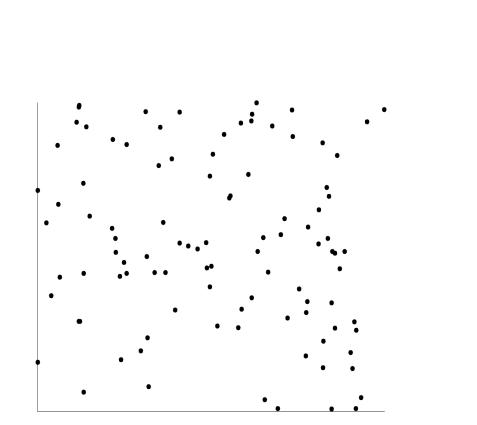
#### Positively and Negatively Correlated Data

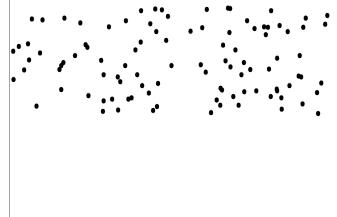


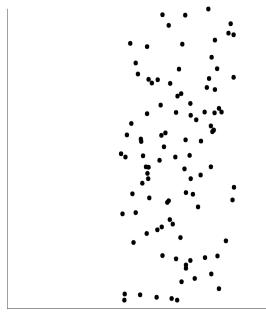


- The left half fragment is positively correlated
- The right half is negative correlated

## Uncorrelated Data







# Chapter 2: Getting to Know Your Data

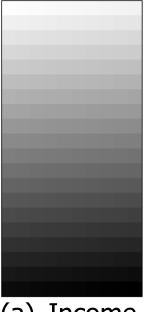
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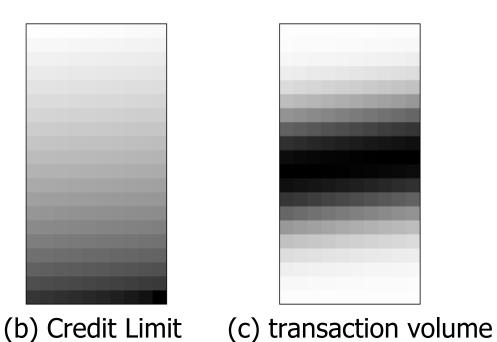
# Data Visualization

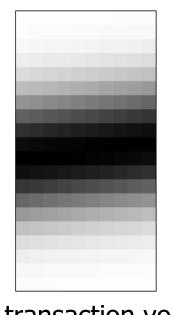
- Why data visualization?
  - Gain insight into an information space by mapping data onto graphical primitives
  - Provide qualitative overview of large data sets
  - Search for patterns, trends, structure, irregularities, relationships among data
  - Help find interesting regions and suitable parameters for further quantitative analysis
  - Provide a visual proof of computer representations derived
- Categorization of visualization methods:
  - Pixel-oriented visualization techniques
  - Geometric projection visualization techniques
  - Icon-based visualization techniques
  - Hierarchical visualization techniques
  - Visualizing complex data and relations

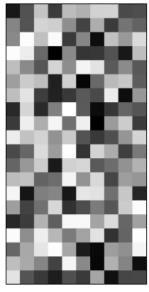
### Pixel-Oriented Visualization Techniques

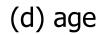
- For a data set of m dimensions, create m windows on the screen, one for each dimension
- The m dimension values of a record are mapped to m pixels at the corresponding positions in the windows
- The colors of the pixels reflect the corresponding values





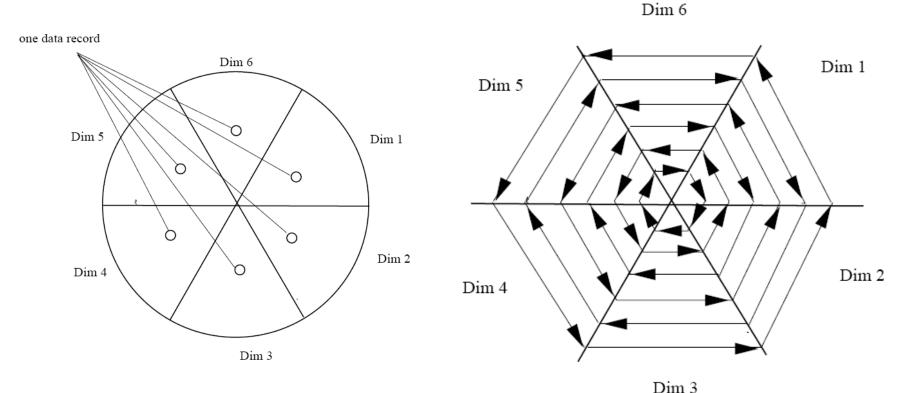






# Segments

 To save space and show the connections among multiple dimensions, space filling is often done in a circle segment



(a) Representing a data record in circle segment

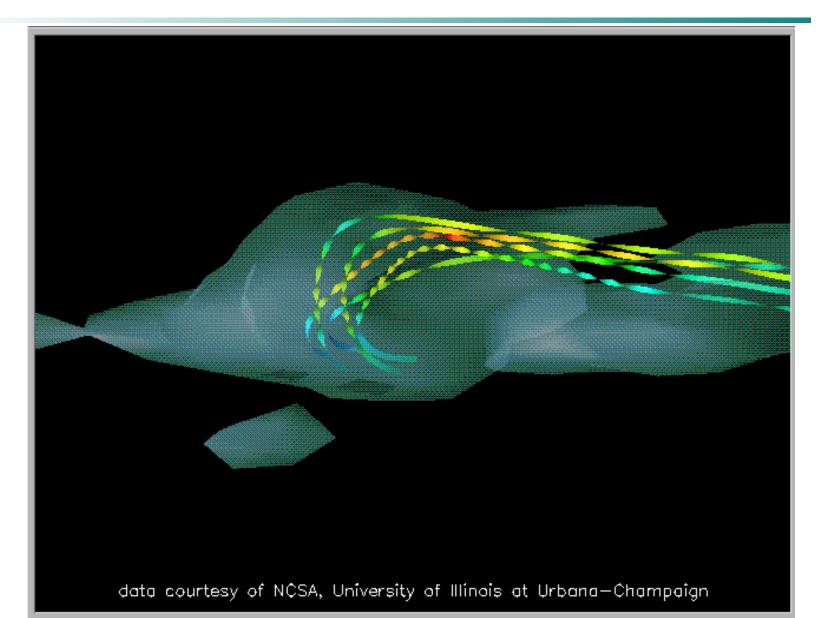
(b) Laying out pixels in circle segment

### Geometric Projection Visualization Techniques

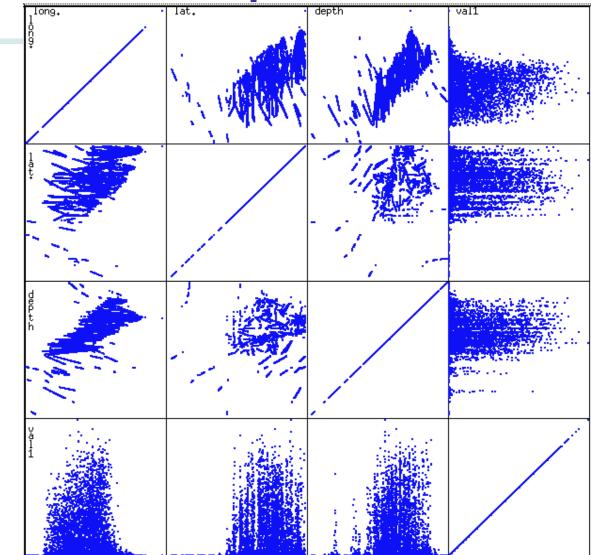
- Visualization of geometric transformations and projections of the data
- Methods
  - Direct visualization
  - Scatterplot and scatterplot matrices
  - Landscapes
  - Projection pursuit technique: Help users find meaningful projections of multidimensional data
  - Prosection views
  - Hyperslice
  - Parallel coordinates

## **Direct Data Visualization**

Ribbons with Twists Based on Vorticity



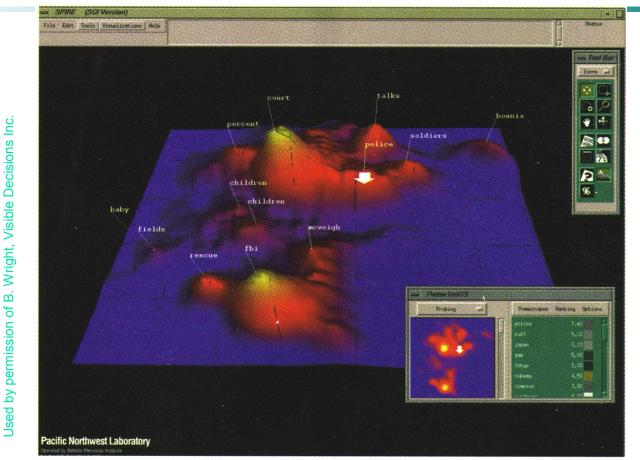
#### **Scatterplot Matrices**



Used by\_ermission of M. Ward, Worcester Polytechnic Institute

Matrix of scatterplots (x-y-diagrams) of the k-dim. data [total of  $(k^2/2 - k)$  scatterplots]

### Landscapes



news articles visualized as a landscape

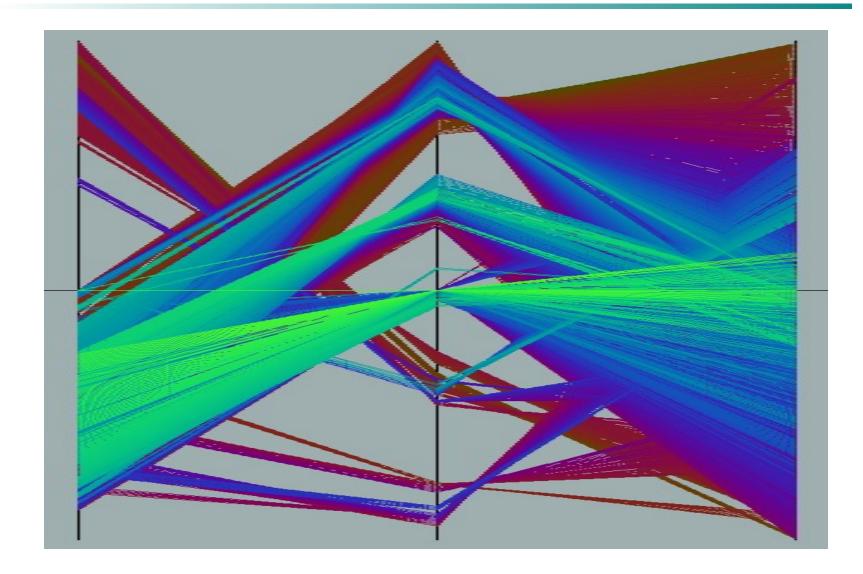
- Visualization of the data as perspective landscape
- The data needs to be transformed into a (possibly artificial) 2D spatial representation which preserves the characteristics of the data

# Parallel Coordinates

- n equidistant axes which are parallel to one of the screen axes and correspond to the attributes
- The axes are scaled to the [minimum, maximum]: range of the corresponding attribute
- Every data item corresponds to a polygonal line which intersects each of the axes at the point which corresponds to the value for the attribute

. . .

## Parallel Coordinates of a Data Set

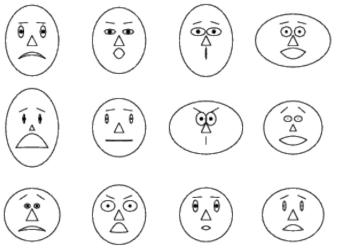


# Techniques

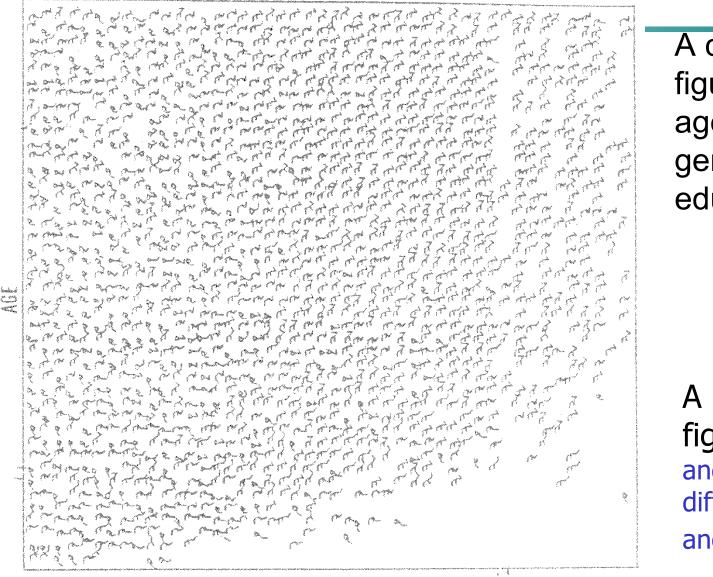
- Visualization of the data values as features of icons
- Typical visualization methods
  - Chernoff Faces
  - Stick Figures
- General techniques
  - Shape coding: Use shape to represent certain information encoding
  - Color icons: Use color icons to encode more information
  - Tile bars: Use small icons to represent the relevant feature vectors in document retrieval

# Chernoff Faces

- A way to display variables on a two-dimensional surface, e.g., let x be eyebrow slant, y be eye size, z be nose length, etc.
- The figure shows faces produced using 10 characteristics--head eccentricity, eye size, eye spacing, eye eccentricity, pupil size, eyebrow slant, nose size, mouth shape, mouth size, and mouth opening): Each assigned one of 10 possible values, generated using *Mathematica* (S. Dickson)
- REFERENCE: Gonick, L. and Smith, W.
   *The Cartoon Guide to Statistics.* New York: Harper Perennial, p. 212, 1993
- Weisstein, Eric W. "Chernoff Face." From MathWorld--A Wolfram Web Resource. mathworld.wolfram.com/ChernoffFace.html



# Stick Figure



A census data figure showing age, income, gender, education, etc.

A 5-piece stick figure (1 body and 4 limbs w. different angle/length)

Two attributes mapped to axes, remaining attributes mapped to angle or length of limbs". Look at texture pattern

INCOME

# Hierarchical Visualization Techniques

- Visualization of the data using a hierarchical partitioning into subspaces
- Methods
  - Dimensional Stacking
  - Worlds-within-Worlds
  - Tree-Map
  - Cone Trees
  - InfoCube

### Dimensional Stacking

attribute 4

attribute 2

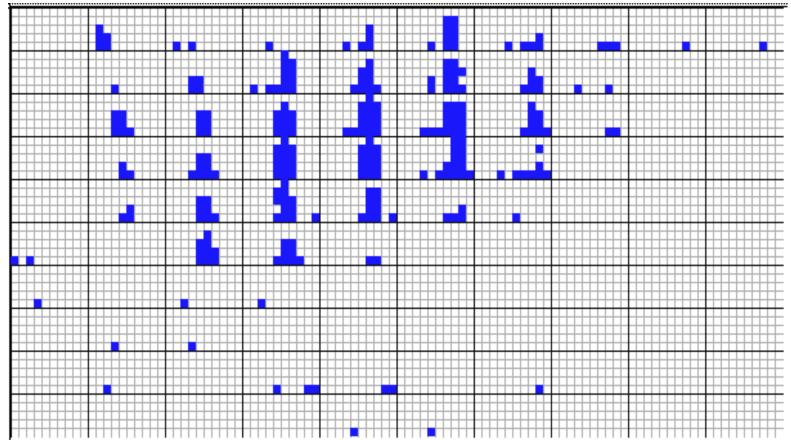
attribute 3

#### attribute 1

- Partitioning of the n-dimensional attribute space in 2-D subspaces, which are 'stacked' into each other
- Partitioning of the attribute value ranges into classes. The important attributes should be used on the outer levels.
- Adequate for data with ordinal attributes of low cardinality
- But, difficult to display more than nine dimensions
- Important to map dimensions appropriately

# Dimensional Stacking

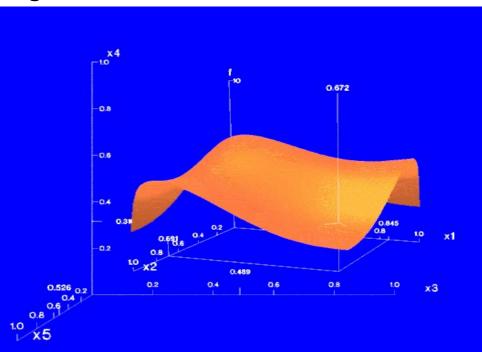
#### Used by permission of M. Ward, Worcester Polytechnic Institute



Visualization of oil mining data with longitude and latitude mapped to the outer x-, y-axes and ore grade and depth mapped to the inner x-, y-axes

# Worlds-within-Worlds

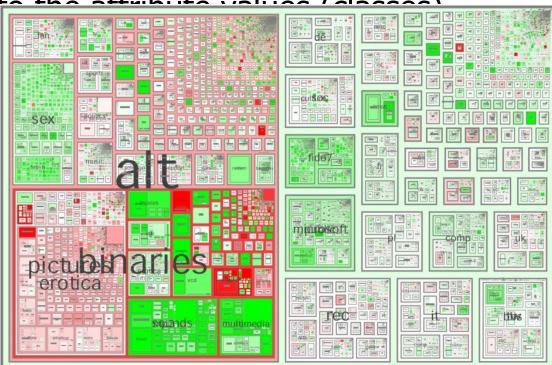
- Assign the function and two most important parameters to innermost world
- Fix all other parameters at constant values draw other (1 or 2 or 3 dimensional worlds choosing these as the axes)
- Software that uses this paradigm
  - N-vision: Dynamic interaction through data glove and stereo displays, including rotation, scaling (inner) and translation (inner/outer)
  - Auto Visual: Static interaction by means of queries



# Tree-Map

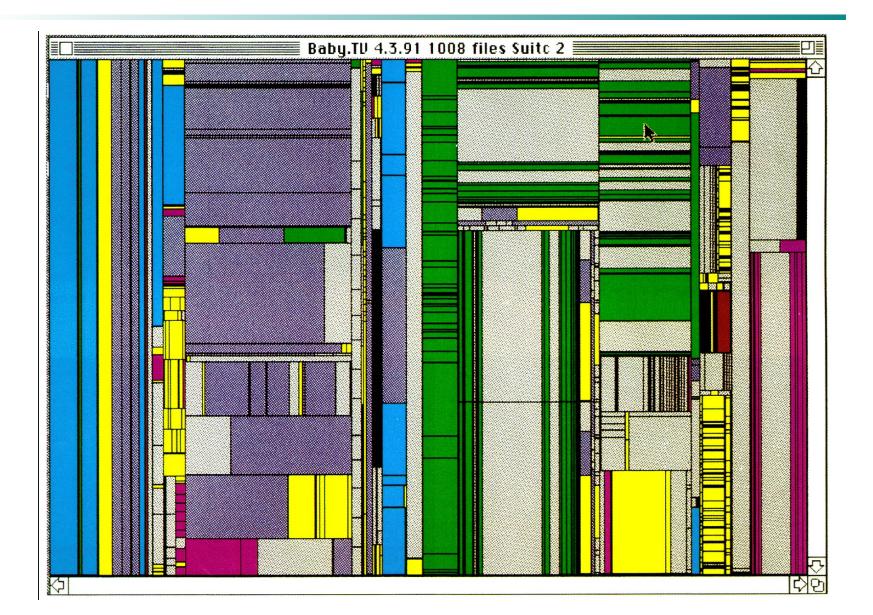
- Screen-filling method which uses a hierarchical partitioning of the screen into regions depending on the attribute values
- The x- and y-dimension of the screen are partitioned the attribute values (alassa) alternately according

MSR Netscan Image



Ack.: http://www.cs.umd.edu/hcil/treemap-history/all102001.jpg

### Tree-Map of a File System (Schneiderman)



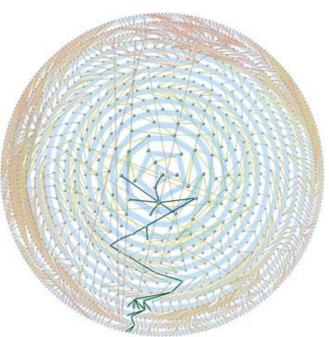
# InfoCube

- A 3-D visualization technique where hierarchical information is displayed as nested semi-transparent cubes
- The outermost cubes correspond to the top level data, while the subnodes or the lower level data are represented as smaller cubes inside the outermost cubes, and so on



# Three-D Cone Trees

- 3D cone tree visualization technique works well for up to a thousand nodes or so
- First build a 2D circle tree that arranges its nodes in concentric circles centered on the root node
- Cannot avoid overlaps when projected to 2D
- G. Robertson, J. Mackinlay, S. Card. "Cone Trees: Animated 3D Visualizations of Hierarchical Information", ACM SIGCHI'91
- Graph from Nadeau Software Consulting website: Visualize a social network data set that models the way an infection spreads from one person to the next



Ack.: http://nadeausoftware.com/articles/visualization

### Visualizing Complex Data and Relations

- Visualizing non-numerical data: text and social networks
- Tag cloud: visualizing user-generated tags
  - The importance of tag is represented by font size/color
- Besides text data, there are also methods to visualize relationships, such as visualizing social networks

Penta on Denio	g 'Mo All Sm	Mother Of Called Kind All to Iraqis at Smokescree Sentencing		Ruling wine shipme could u new ma	nts ncork	Khodorkov sky awaits 'guilty' verdict		Fever pitch		Missed cut not all bad for Tiger		
Desecr		in US Office		Stormy Morgan Stanley Unite Grim over Legal Costs deal e season		United Air inks labour deal with mechanics	Daily Playoff Essentials		Derby Winner Giacomo Has Plenty to Prove			
ation	Harvard Preside Pledge Million Women Prograf	?s	nate Passe ay Bill, ds Bush's Update2)		predicte d	Toyota to Make Hybrids in US	Prospective Ar merger dominates annual mer	etino Alerts on Wireless Phones	The Golden Bear	Temple tootball to join MAC	Federer gamers another award Gana and Cana	Rran eyei Reds recu
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#### Newsmap: Google News Stories in 2005

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### Summary

# Similarity and Dissimilarity

### Similarity

- Numerical measure of how alike two data objects are
- Value is higher when objects are more alike
- Often falls in the range [0,1]
- Dissimilarity (e.g., distance)
  - Numerical measure of how different two data objects are
  - Lower when objects are more alike
  - Minimum dissimilarity is often 0
  - Upper limit varies
- Proximity refers to a similarity or dissimilarity

# Matrix and Dissimilarity

- Data matrix
  - n data points with p dimensions
  - Two modes

- Dissimilarity matrix
  - n data points, but registers only the distance
  - A triangular matrix
  - Single mode

$$n \left\{ \begin{bmatrix} x_{11} & \cdots & x_{1f} & \cdots & x_{1p} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{i1} & \cdots & x_{if} & \cdots & x_{ip} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{n1} & \cdots & x_{nf} & \cdots & x_{np} \end{bmatrix} \right\}$$

$$\begin{bmatrix} 0 & & & \\ d(2,1) & 0 & & \\ d(3,1) & d(3,2) & 0 & \\ \vdots & \vdots & \vdots & \\ d(n,1) & d(n,2) & \dots & 0 \end{bmatrix}$$

### Proximity Measure for Nominal Attributes

- Attribute an take 2 or more states, e.g., red, yellow, blue, green (generalization of a binary attribute)
- Method 1: Simple matching
  - m = # of matches, p = total # of variables

$$d(i,j) = \frac{p-m}{p}$$

- Method 2: Use a large number of binary attributes
  - creating a new binary attribute for each of the M nominal states (see next slide)

### Proximity Measure for Binary Attributes

- A contingency table for binary data Ob<sup>1</sup>
- Distance measure for symmetric binary variables:
- Distance measure for asymmetric binary variables (q = both 1's):
- Jaccard coefficient (*similarity* measure for *asymmetric* binary variables):

	0	bject <i>j</i>	
	1	0	sum
bject $i = \frac{1}{0}$	q	r	q+r
0	8	t	s+t
sum	q+s	r+t	p
d(i, j) =		$\frac{+s}{+s+t}$	
d(i, j)	$=\frac{r}{q+q}$	$\frac{+s}{r+s}$	
$sim_{Jaccar}$	d(i, j)	$=\frac{q}{q+r}$	$\frac{l}{r+s}$

Note: Jaccard coefficient is the same as "coherence":

$$coherence(i, j) = \frac{sup(i, j)}{sup(i) + sup(j) - sup(i, j)} = \frac{q}{(q+r) + (q+s) - q}$$

# Dissimilarity between Binary Variables

#### Example

Name	Gender	Fe	ver	Cou	ıgh	Tes	st-1	Te	st-2	Tes	st-3	Tes	st-4
Jack	Μ	Y	1	Ν	0	Р	1	N	0	Ν	0	N	0
Mary	F	Y	1	Ν	0	Р	1	Ν	0	Р	1	Ν	0
Jim	M	Y	0	P	0	N	0	N	0	N	0	N	0

Gender is a symmetric attribute

$$d(i, j) = \frac{r+s}{q+r+s}$$

4

jack \_

- The remaining attributes are asymmetric binary
- Let the values Y and P be 1, and the value N 0

$$d(jack, mary) = \frac{0+1}{2+0+1} = 0.33$$
$$d(jack, jim) = \frac{1+1}{1+1+1} = 0.67$$
$$d(jim, mary) = \frac{1+2}{1+1+2} = 0.75$$

### Standardizing Numeric Data

- Z-score:  $Z = \frac{X \mu}{\sigma}$ 
  - X: raw score to be standardized,  $\mu$ : mean of the population,  $\sigma$ : standard deviation
  - the distance between the raw score and the population mean in units of the standard deviation
  - negative when the raw score is below the mean, "+" when above
  - An alternative way: Calculate the mean absolute deviation

$$S_{f} = \frac{1}{n} (|x_{1f} - m_{f}| + |x_{2f} - m_{f}| + \dots + |x_{nf} - m_{f}|)$$

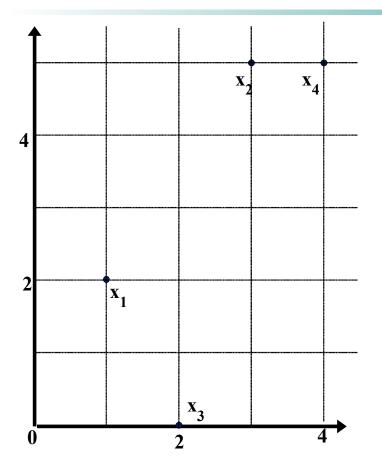
where  $m_f = \frac{1}{n} (x_{1f} + x_{2f} + ... + x_{nf})$ 

standardized measure (*z-score*):

$$Z_{if} = \frac{X_{if} - M_f}{S_f}$$

Using mean absolute deviation is more robust than using standard deviation

### Example: Data Matrix and Dissimilarity Matrix



#### **Data Matrix**

point	attribute1	attribute2
<i>x1</i>	1	2
<i>x2</i>	3	5
<i>x3</i>	2	0
<i>x4</i>	4	5

#### **Dissimilarity Matrix**

#### (with Euclidean Distance)

	<i>x1</i>	<i>x2</i>	x3	<i>x4</i>
<i>x1</i>	0			
<i>x2</i>	3.61	0		
<i>x3</i>	5.1	5.1	0	
<i>x4</i>	4.24	1	5.39	0

### Distance on Numeric Data: Minkowski Distance

Minkowski distance: A popular distance measure

$$d(i, j) = \sqrt[h]{|x_{i1} - x_{j1}|^h} + |x_{i2} - x_{j2}|^h + \dots + |x_{ip} - x_{jp}|^h$$

where  $i = (x_{i1}, x_{i2}, ..., x_{ip})$  and  $j = (x_{j1}, x_{j2}, ..., x_{jp})$  are two *p*-dimensional data objects, and *h* is the order (the distance so defined is also called L-*h* norm)

- Properties
  - d(i, j) > 0 if i ≠ j, and d(i, i) = 0 (Positive definiteness)
  - d(i, j) = d(j, i) (Symmetry)
  - $d(i, j) \le d(i, k) + d(k, j)$  (Triangle Inequality)
- A distance that satisfies these properties is a metric

### Special Cases of Minkowski Distance

- h = 1: Manhattan (city block, L<sub>1</sub> norm) distance
  - E.g., the Hamming distance: the number of bits that are different between two binary vectors

$$d(i,j) = |x_{i_1} - x_{j_1}| + |x_{i_2} - x_{j_2}| + \dots + |x_{i_p} - x_{j_p}|$$

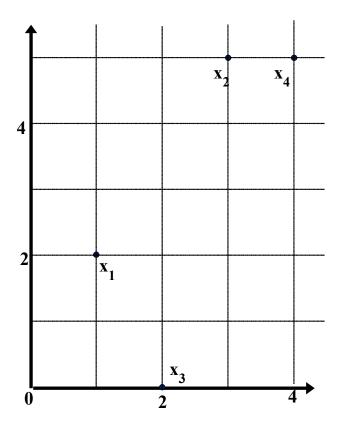
• 
$$h = 2$$
: (L<sub>2</sub> norm) Euclidean distance  
 $d(i,j) = \sqrt{(|x_{i_1} - x_{j_1}|^2 + |x_{i_2} - x_{j_2}|^2 + ... + |x_{i_p} - x_{j_p}|^2)}$ 

- $h \rightarrow \infty$ . "supremum" (L<sub>max</sub> norm, L<sub>∞</sub> norm) distance.
  - This is the maximum difference between any component (attribute) of the vectors

$$d(i,j) = \lim_{h \to \infty} \left( \sum_{f=1}^{p} |x_{if} - x_{jf}|^h \right)^{\frac{1}{h}} = \max_{f}^{p} |x_{if} - x_{jf}|$$

# Example: Minkowski Distance

point	attribute 1	attribute 2
x1	1	2
x2	3	5
x3	2	0
x4	4	5



#### Dissimilarity Matrices Manhattan (L<sub>1</sub>)

L	x1	x2	x3	x4
x1	0			
x2	5	0		
x3	3	6	0	
x4	6	1	7	0

### Euclidean (L<sub>2</sub>)

L2	x1	x2	x3	x4
x1	0			
x2	3.61	0		
x3	2.24	5.1	0	
x4	4.24	1	5.39	0

#### Supremum

L∞	x1	x2	x3	x4
x1	0			
x2	3	0		
x3	2	5	0	
x4	3	1	5	0

### **Ordinal Variables**

- An ordinal variable can be discrete or continuous
- Order is important, e.g., rank
- Can be treated like interval-scaled
  - replace  $X_{if}$  by their rank  $r_{if} \in \{1, ..., M_f\}$
  - map the range of each variable onto [0, 1] by replacing *i*-th object in the *f*-th variable by

$$z_{if} = \frac{r_{if} - 1}{M_f - 1}$$

 compute the dissimilarity using methods for intervalscaled variables

# Attributes of Mixed Type

- A database may contain all attribute types
  - Nominal, symmetric binary, asymmetric binary, numeric, ordinal
- One may use a weighted formula to combine their effects

$$d(i,j) = \frac{\sum_{f=1}^{p} \delta_{ij}^{(f)} d_{ij}^{(f)}}{\sum_{f=1}^{p} \delta_{ij}^{(f)}}$$

- *f* is binary or nominal:
  - $d_{_{ij}}{}^{_{(f)}}$  = 0  $\,$  if  $x_{_{if}}$  =  $x_{_{jf}}$  , or  $d_{_{ij}}{}^{_{(f)}}$  = 1 otherwise
- f is numeric: use the normalized distance
- *f* is ordinal
  - Compute ranks r<sub>if</sub> and
  - Treat z<sub>if</sub> as interval-scaled

$$Z_{if} = \frac{r_{if} - 1}{M_f - 1}$$

# Cosine Similarity

A document can be represented by thousands of attributes, each recording the *frequency* of a particular word (such as keywords) or phrase in the document.

Document	team	coach	hockey	base ball	soccer	penalty	score	win	loss	season
Document1	5	0	3	0	2	0	0	2	0	0
Document2	3	0	2	0	1	1	0	1	0	1
Document3	0	7	0	2	1	0	0	3	0	0
Document4	0	1	0	0	1	2	2	0	3	0

- Other vector objects: gene features in micro-arrays, ...
- Applications: information retrieval, biologic taxonomy, gene feature mapping, ...
- Cosine measure: If d<sub>1</sub> and d<sub>2</sub> are two vectors (e.g., term-frequency vectors), then

 $\cos(d_{1'} d_2) = (d_1 \bullet d_2) / (||d_1|| ||d_2||),$ 

where  $\bullet$  indicates vector dot product, ||d||: the length of vector d

# Example: Cosine Similarity

- $\cos(d_1, d_2) = (d_1 \cdot d_2) / (||d_1|| ||d_2||)$ , where • indicates vector dot product, ||d|: the length of vector d
- Ex: Find the **similarity** between documents 1 and 2.

 $d_1 = (5, 0, 3, 0, 2, 0, 0, 2, 0, 0)$  $d_2 = (3, 0, 2, 0, 1, 1, 0, 1, 0, 1)$ 

- $\begin{aligned} &d_1 \bullet d_2 = 5^* 3 + 0^* 0 + 3^* 2 + 0^* 0 + 2^* 1 + 0^* 1 + 2^* 1 + 0^* 0 + 0^* 1 = 25 \\ &||d_1|| = (5^* 5 + 0^* 0 + 3^* 3 + 0^* 0 + 2^* 2 + 0^* 0 + 0^* 0 + 2^* 2 + 0^* 0 + 0^* 0)^{\mathbf{0.5}} = (42)^{\mathbf{0.5}} \\ &= 6.481 \end{aligned}$
- $||d_2|| = (3*3+0*0+2*2+0*0+1*1+1*1+0*0+1*1+0*0+1*1)^{0.5} = (17)^{0.5}$ = 4.12

 $\cos(d_{1'} d_2) = 0.94$ 

# Chapter 2: Getting to Know Your Data

- Data Objects and Attribute Types
- Basic Statistical Descriptions of Data
- Data Visualization
- Measuring Data Similarity and Dissimilarity



Summary

# Summary

- Data attribute types: nominal, binary, ordinal, interval-scaled, ratioscaled
- Many types of data sets, e.g., numerical, text, graph, Web, image.
- Gain insight into the data by:
  - Basic statistical data description: central tendency, dispersion, graphical displays
  - Data visualization: map data onto graphical primitives
  - Measure data similarity
- Above steps are the beginning of data preprocessing.
- Many methods have been developed but still an active area of research.

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