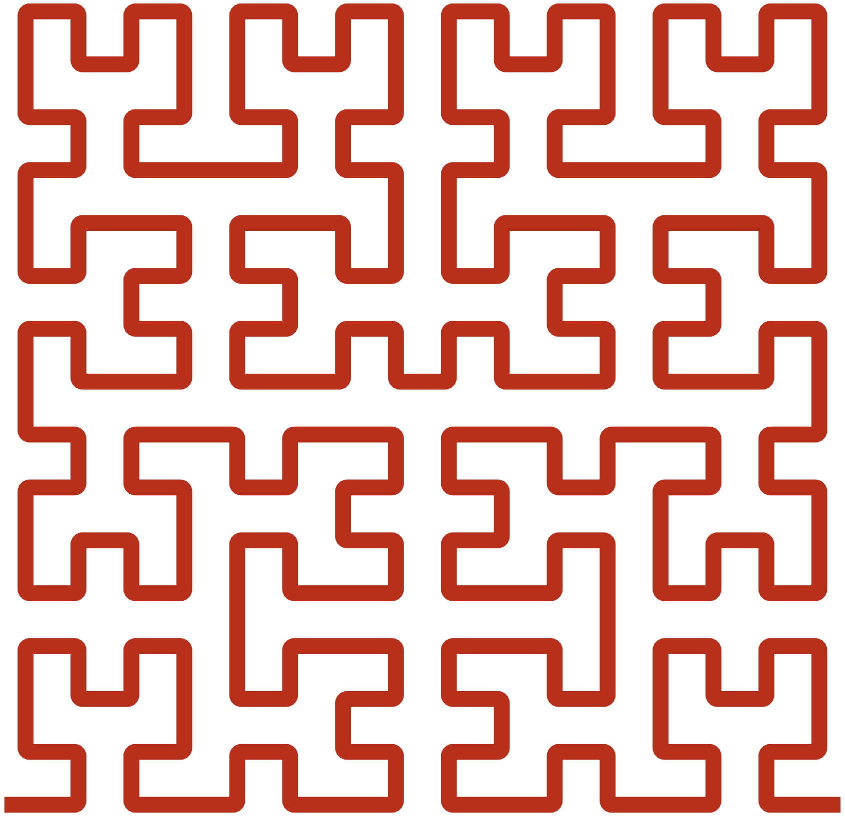
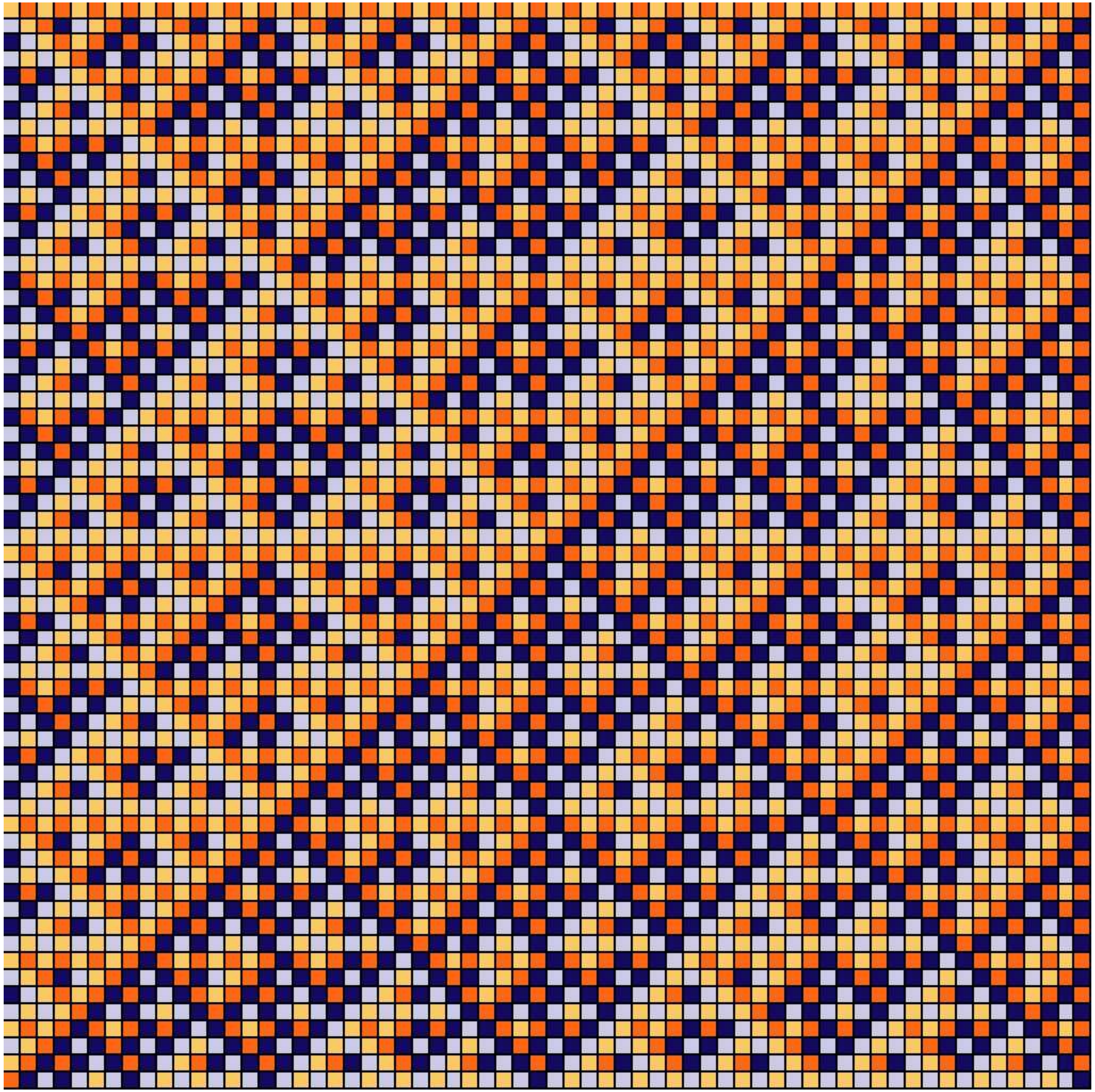
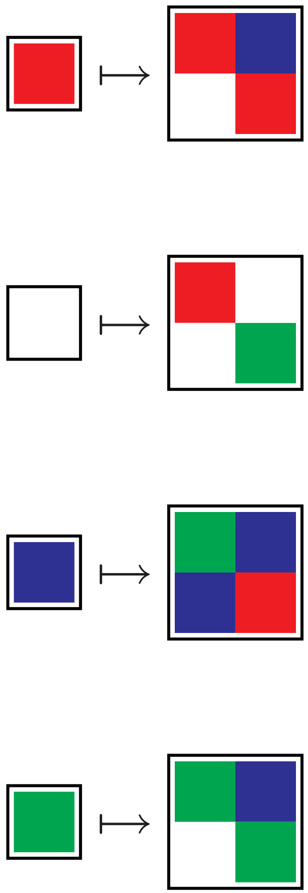
TILES

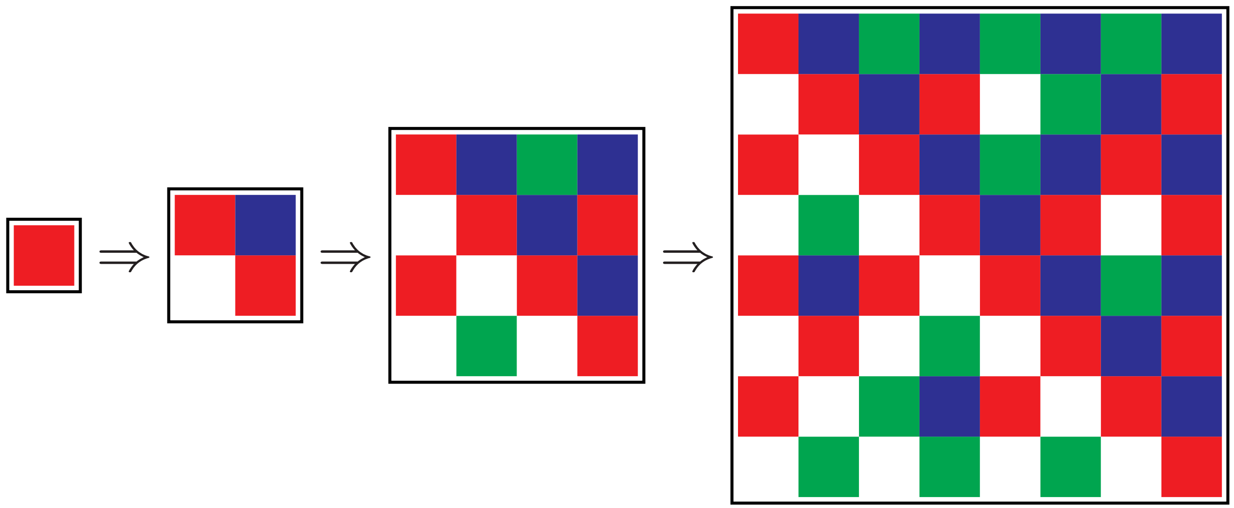
from pattern to computation

Hendrik Jan Hoogeboom
Universiteit Leiden, Computer Science
www.liacs.nl/~hoogeboo/praatjes/tegels/









square chair tiling

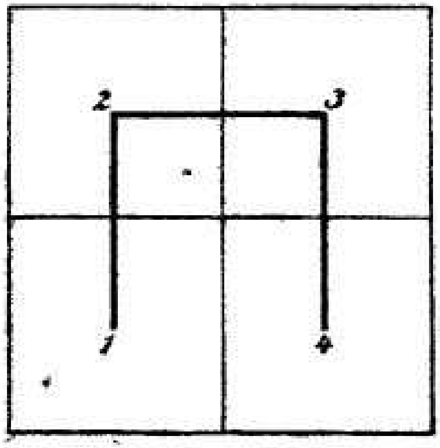


Fig. 1.

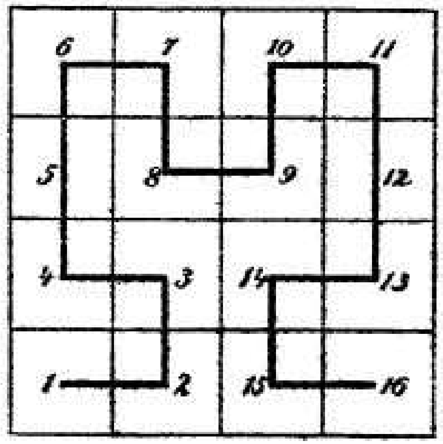


Fig. 2.

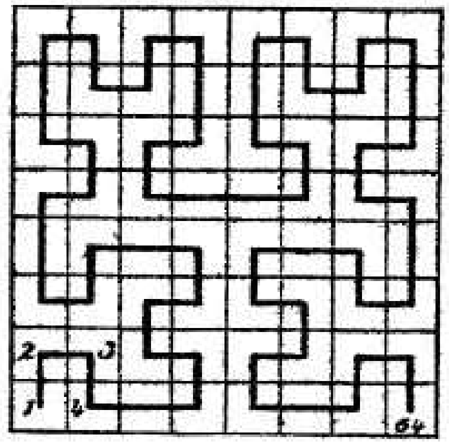
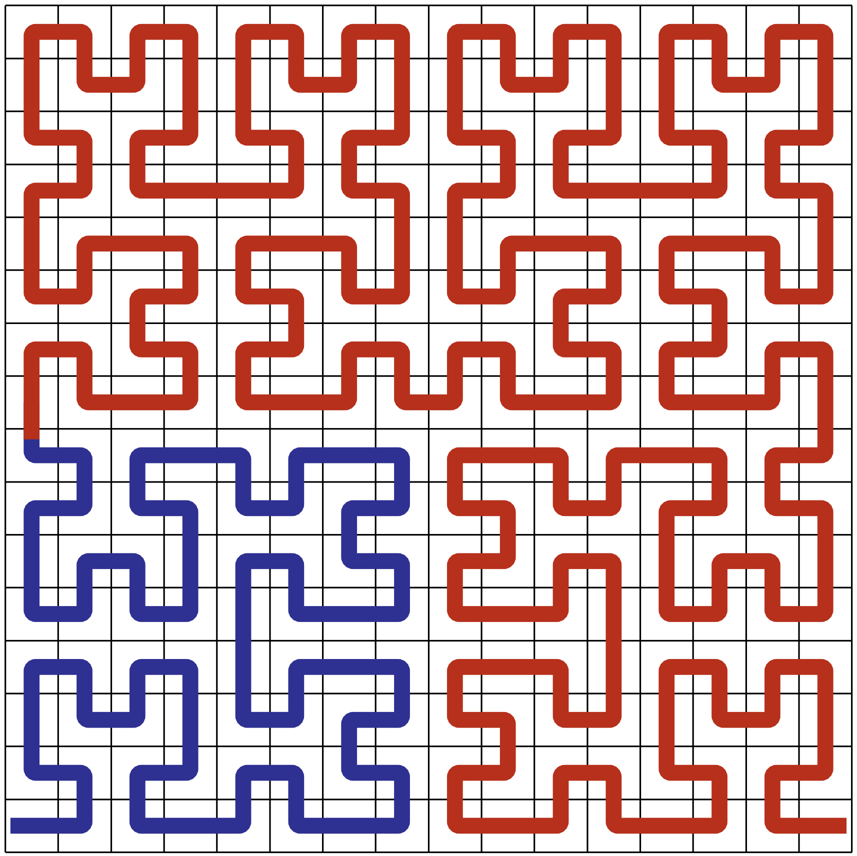
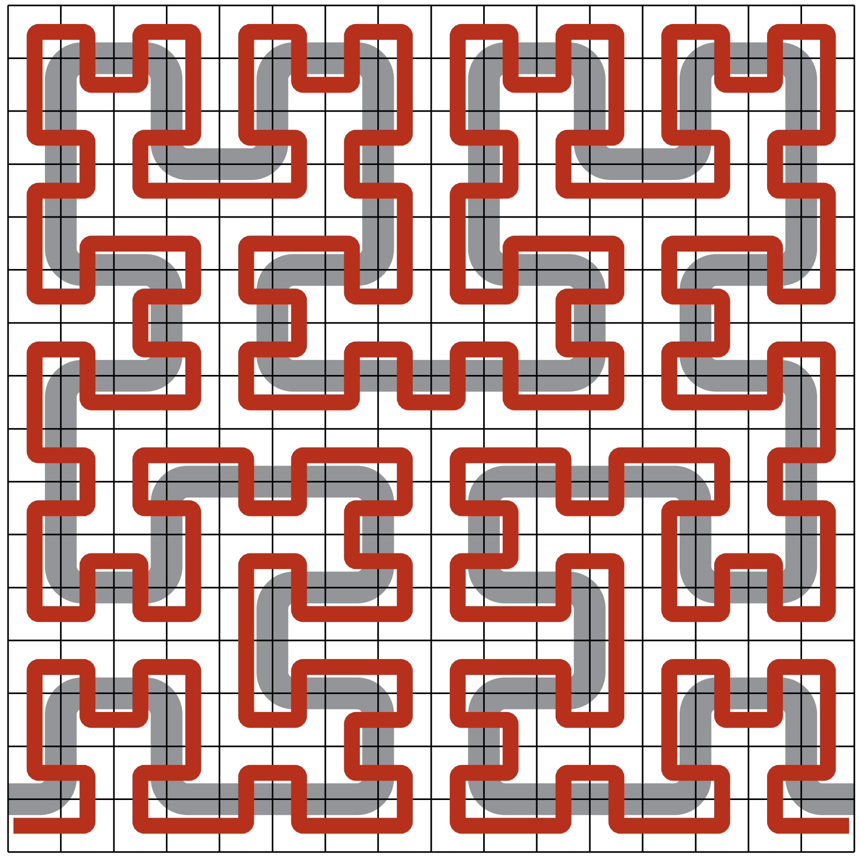


Fig. 3.

Ueber die stetige Abbildung einer Linie auf ein Flächenstück (1891)

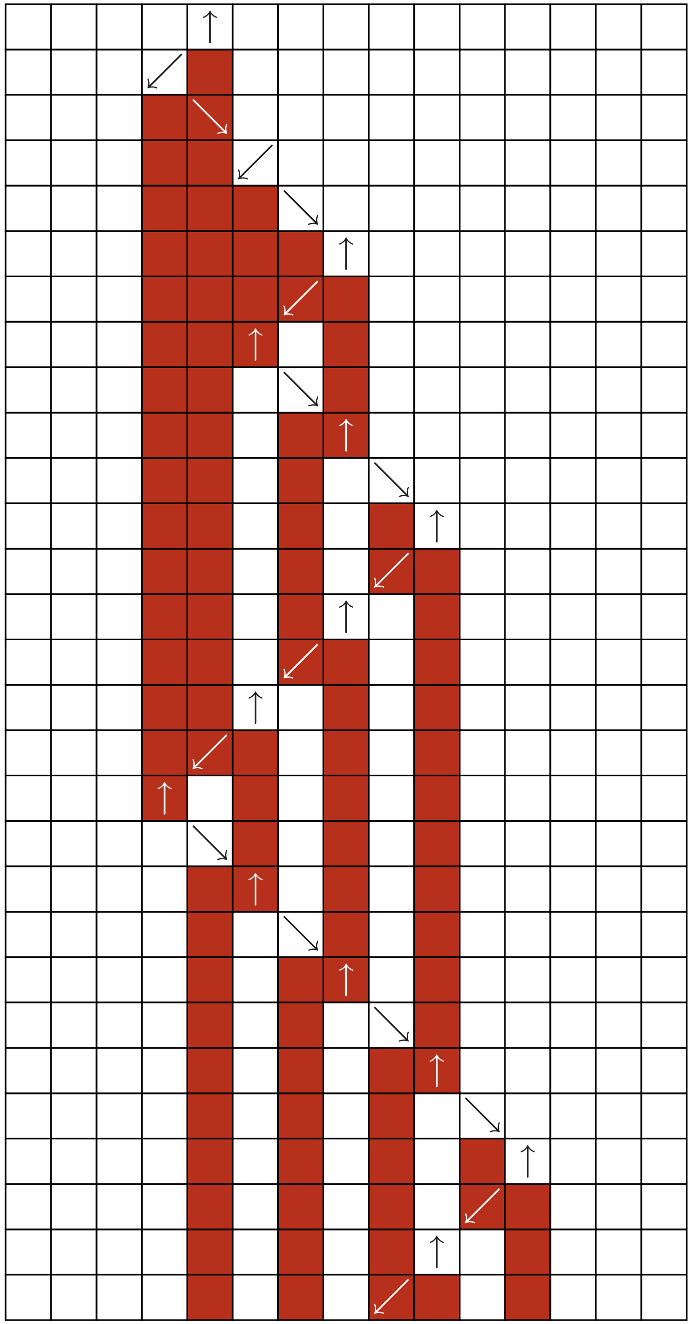
David Hilbert in Königsberg i. Pr.

recursion, iteration

“ . . . the formal analysis of efficient computation and computational processes. [. . .]

TCS covers a wide variety of topics including algorithms, data structures, computational complexity, parallel and distributed computation, probabilistic computation, quantum computation, automata theory, information theory, cryptography, program semantics and verification, machine learning, computational biology, computational economics, computational geometry, and computational number theory and algebra. Work in this field is often distinguished by its emphasis on mathematical technique and rigor. ” [sigact]



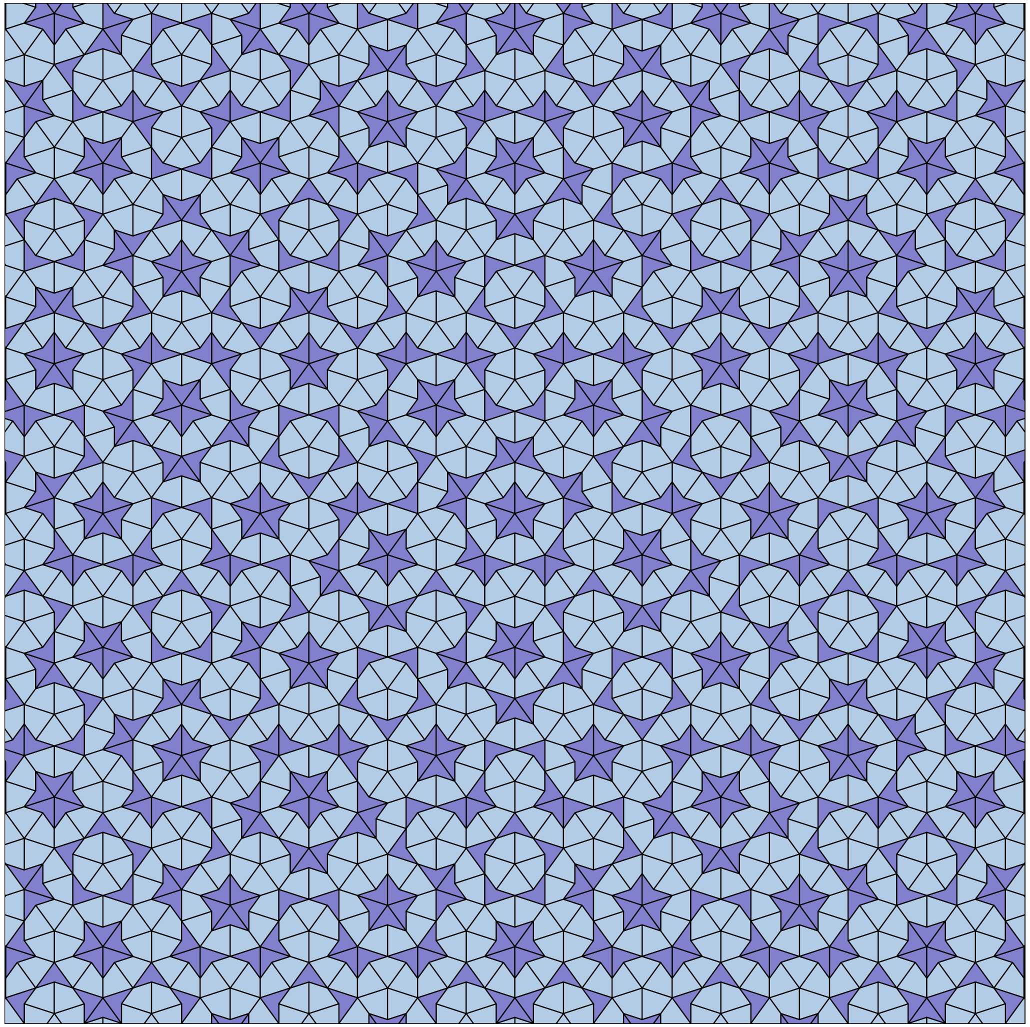
math: regularities, patterns

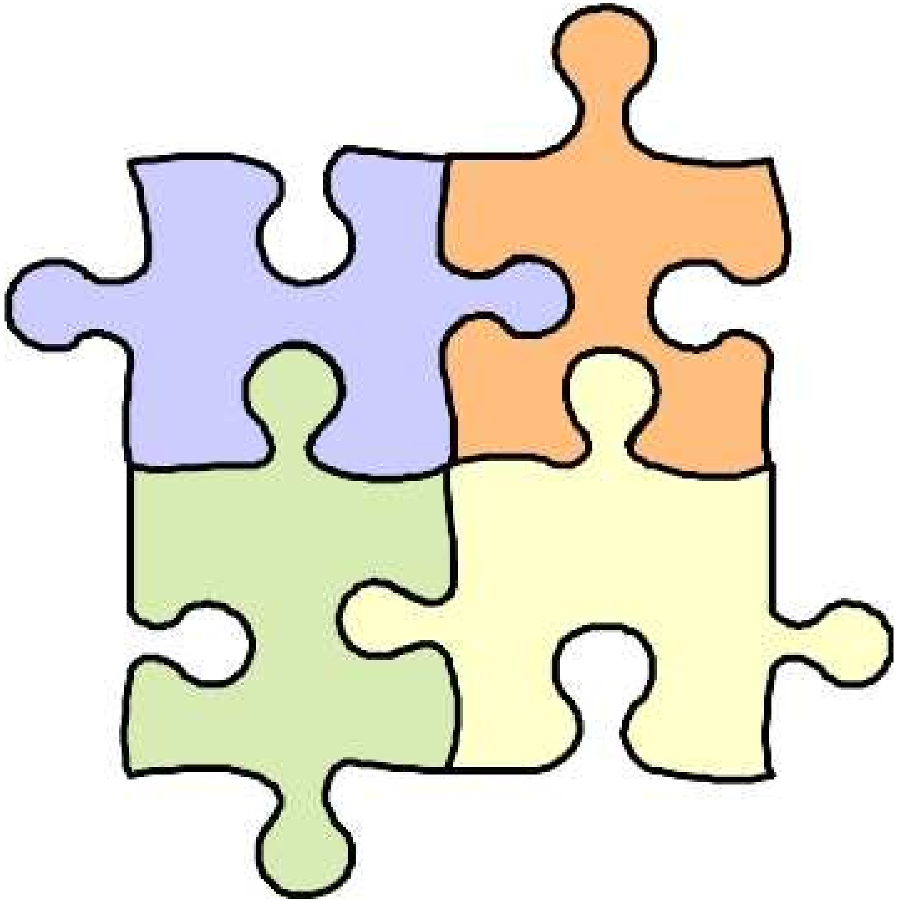
computer science: computations

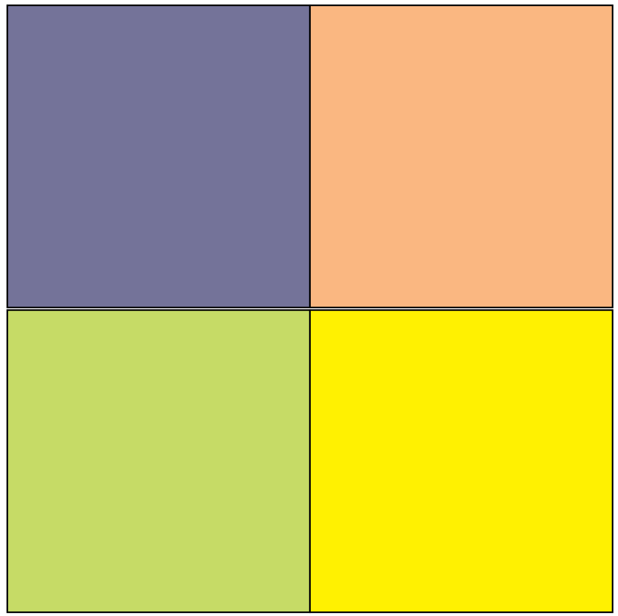
models:

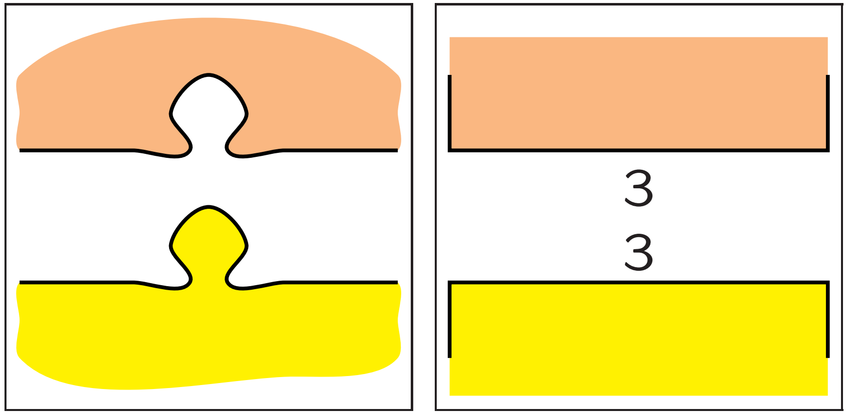
- Wang tiles
- Cellular automata
- Turing machines

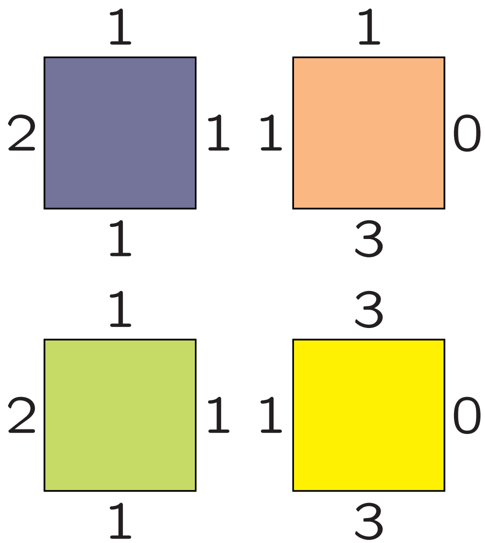
■ Rules of the game

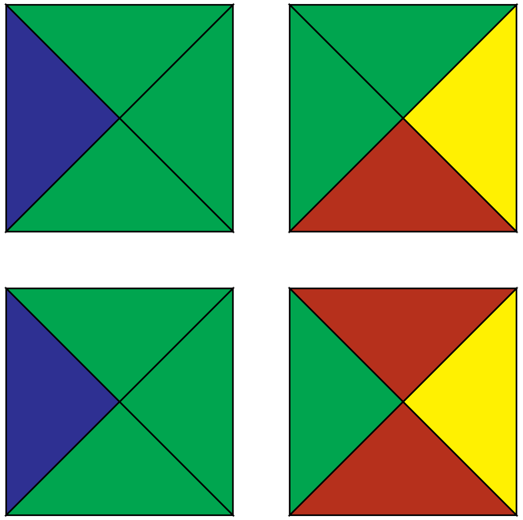


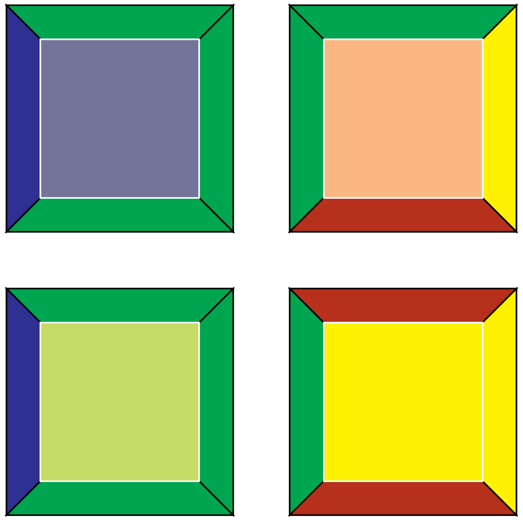


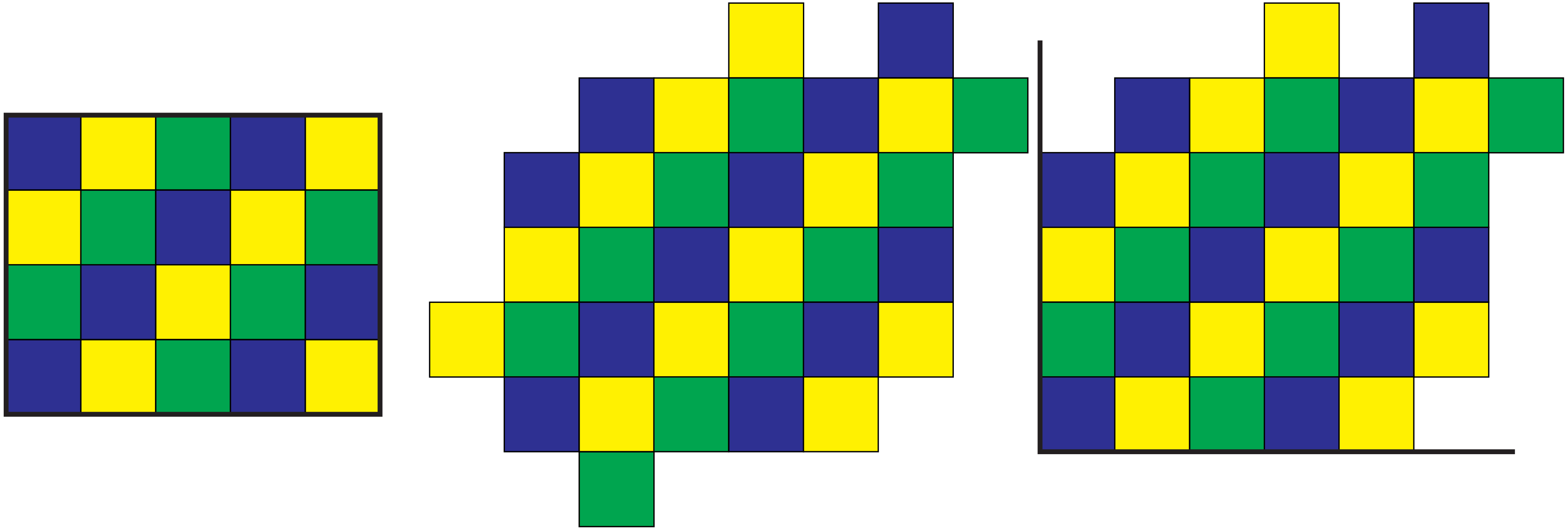



3
3


1 1
2 1 1 0
1 3
1 3
2 1 1 0
1 3

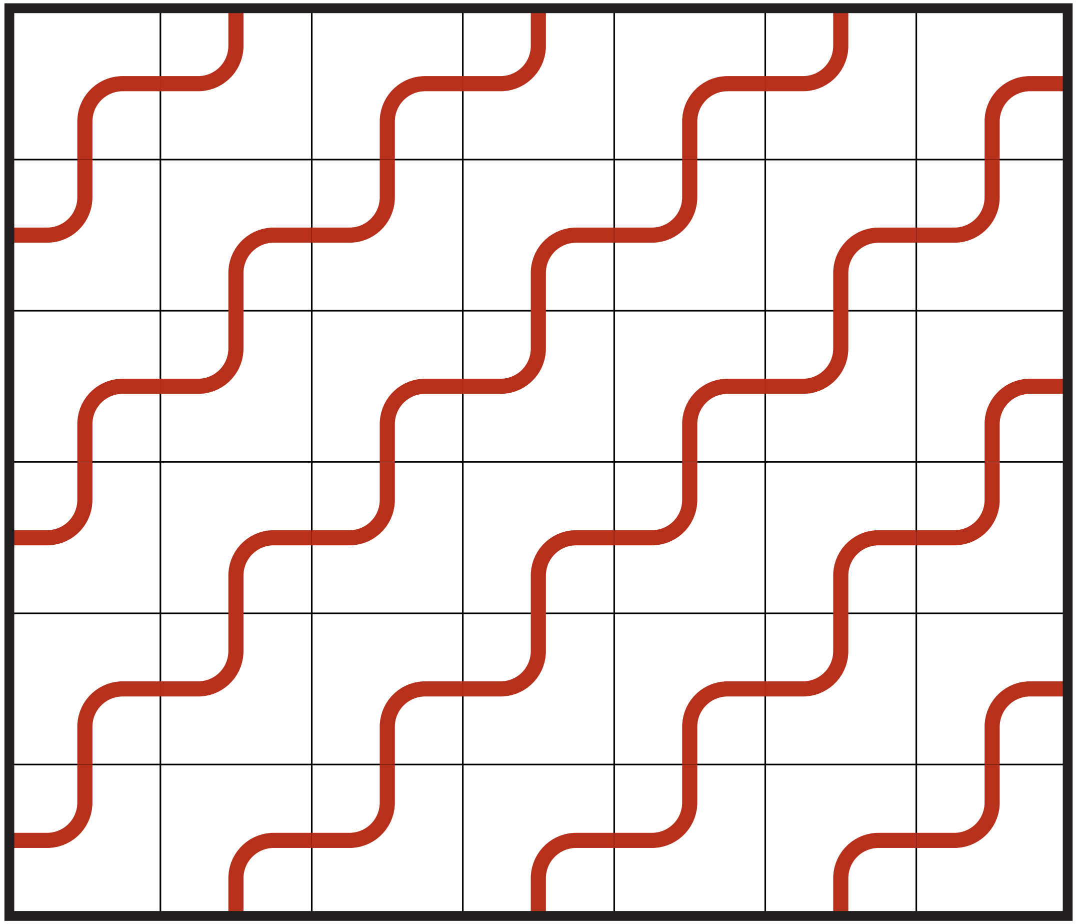


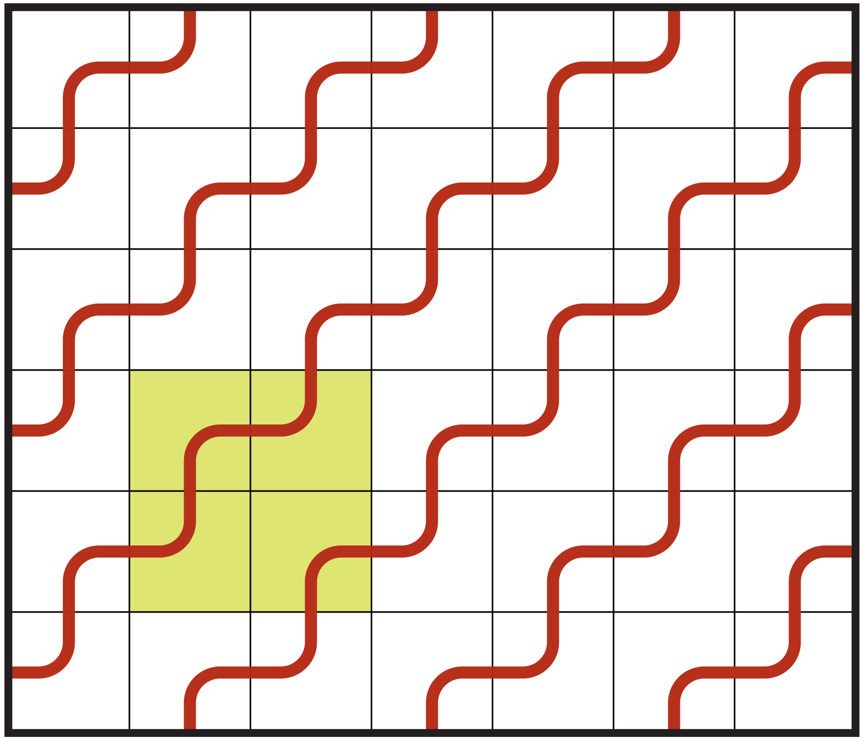


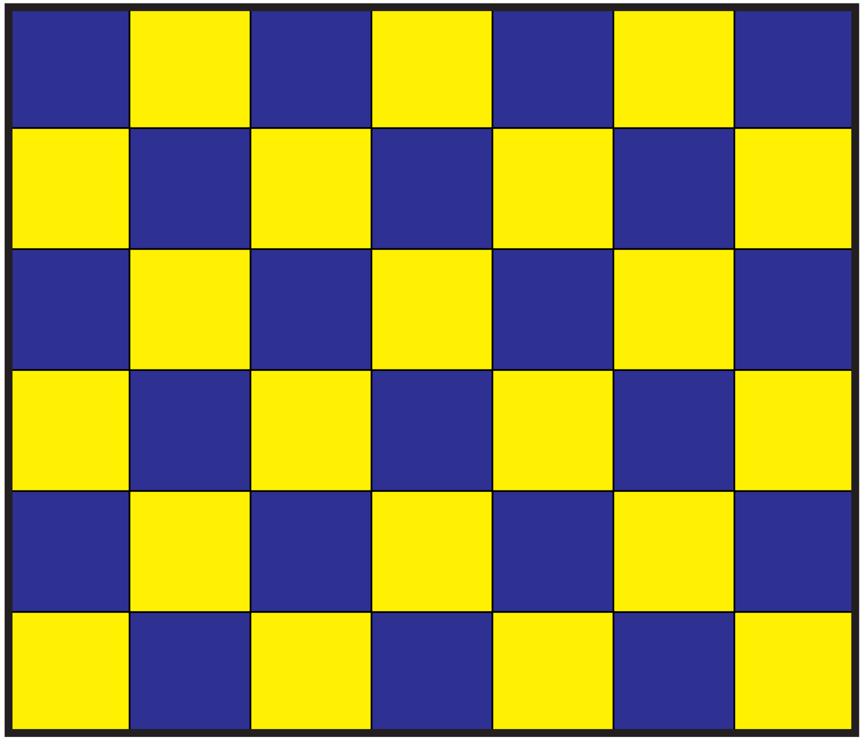


- rectangle
- (half) plane
- quadrant

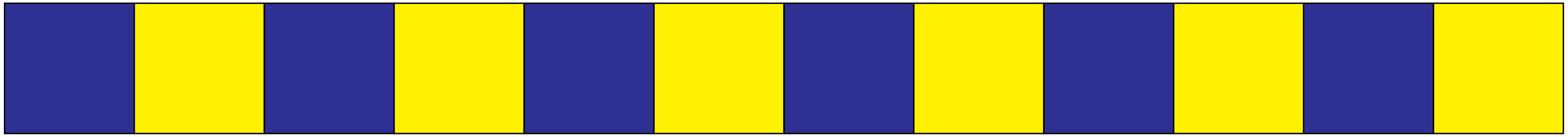
Patterns

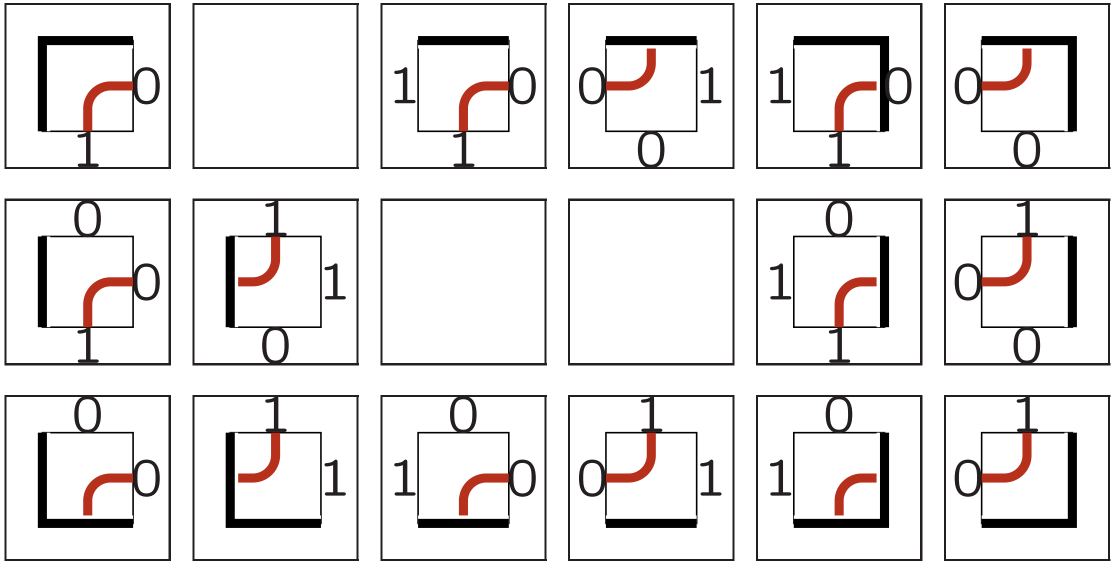


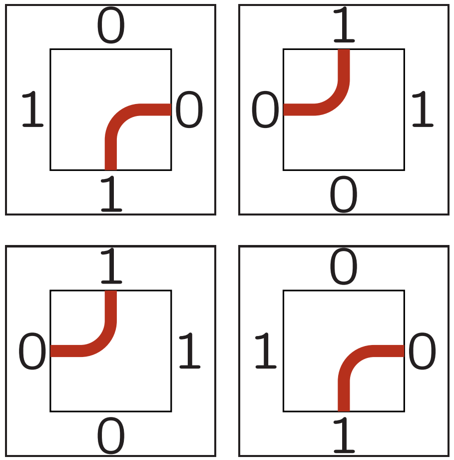




in two directions:

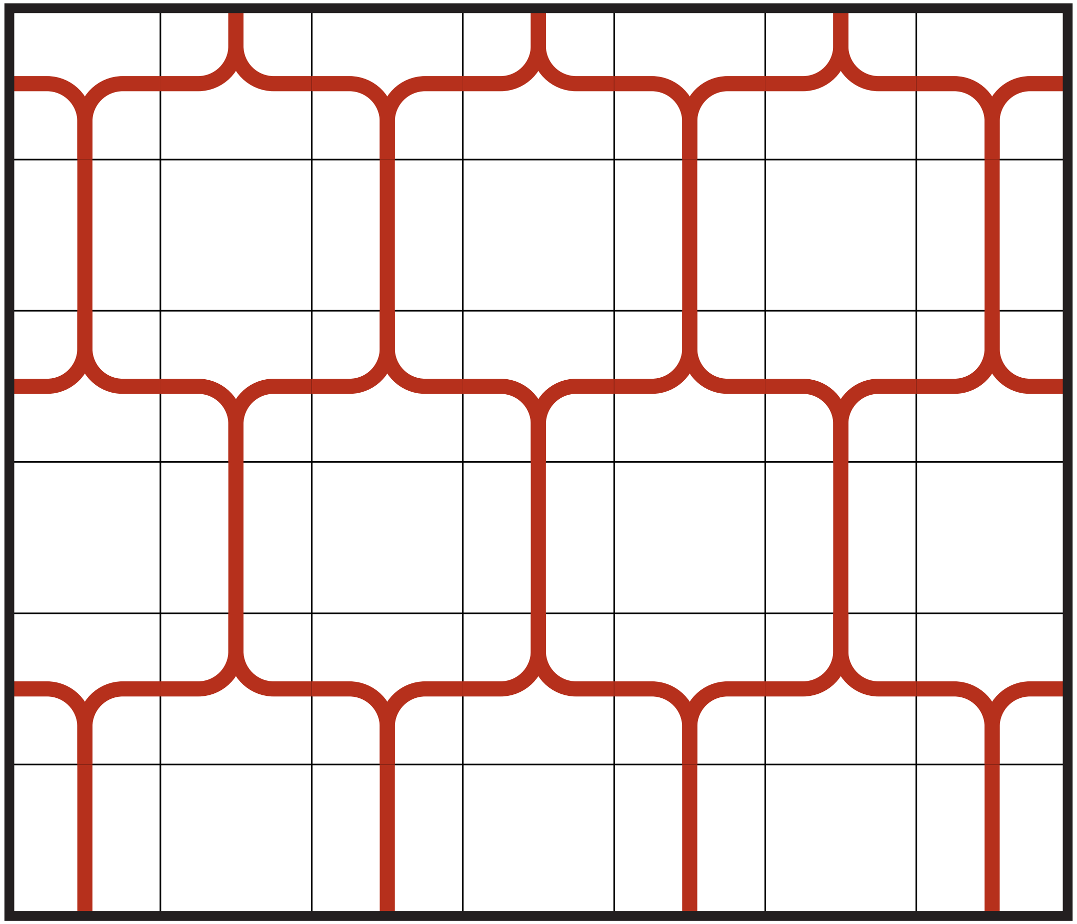


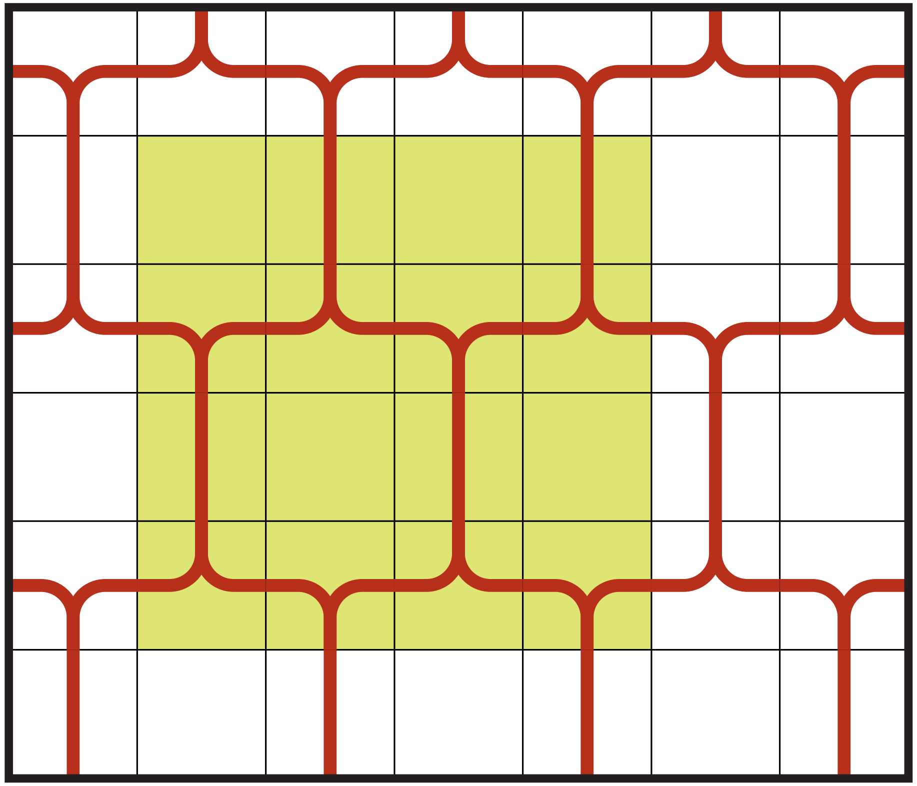


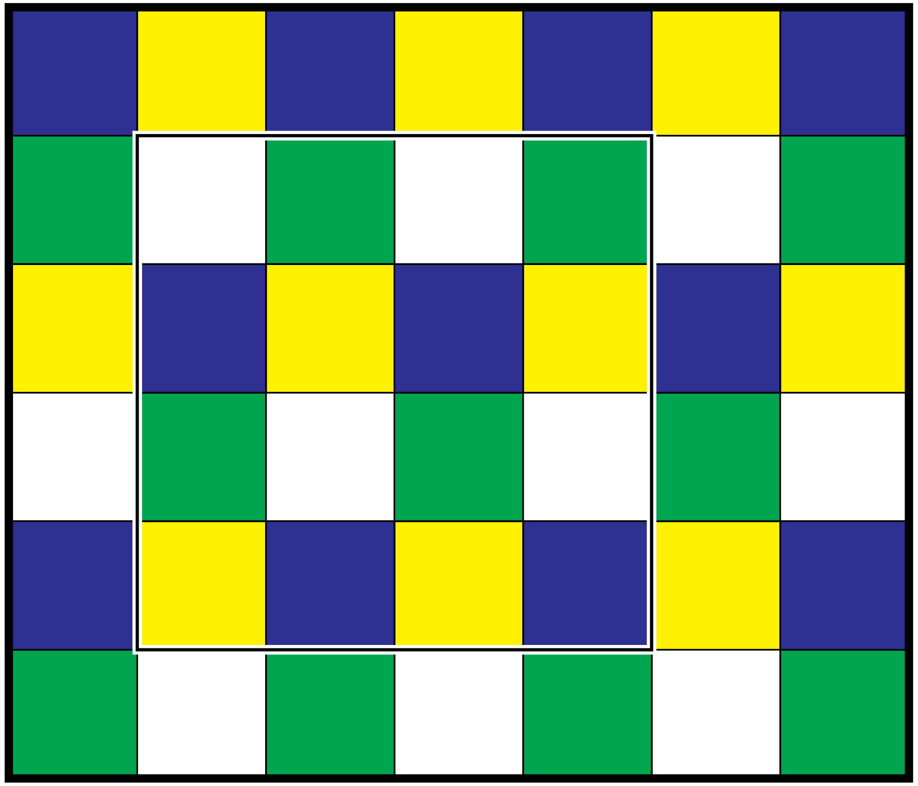


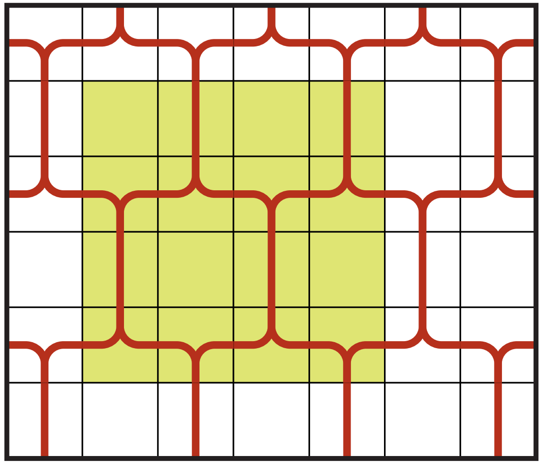
two tiles

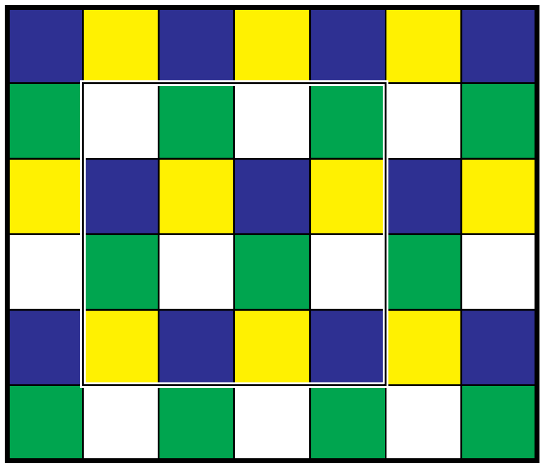
and some borders

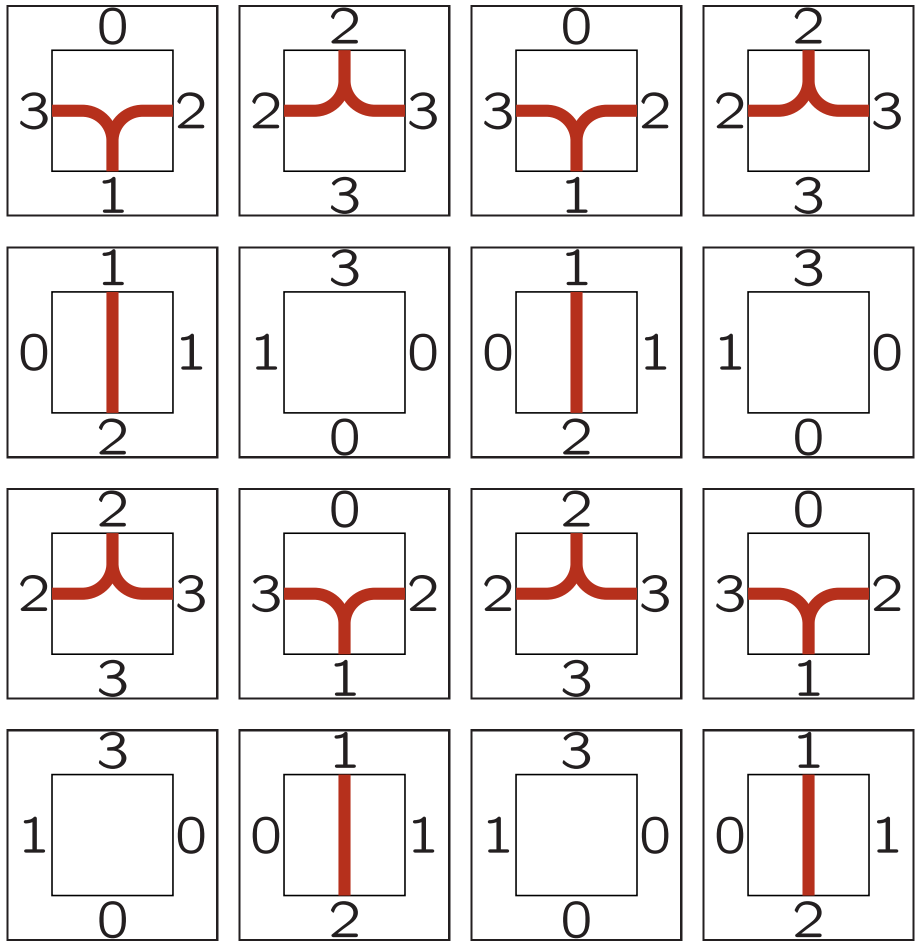








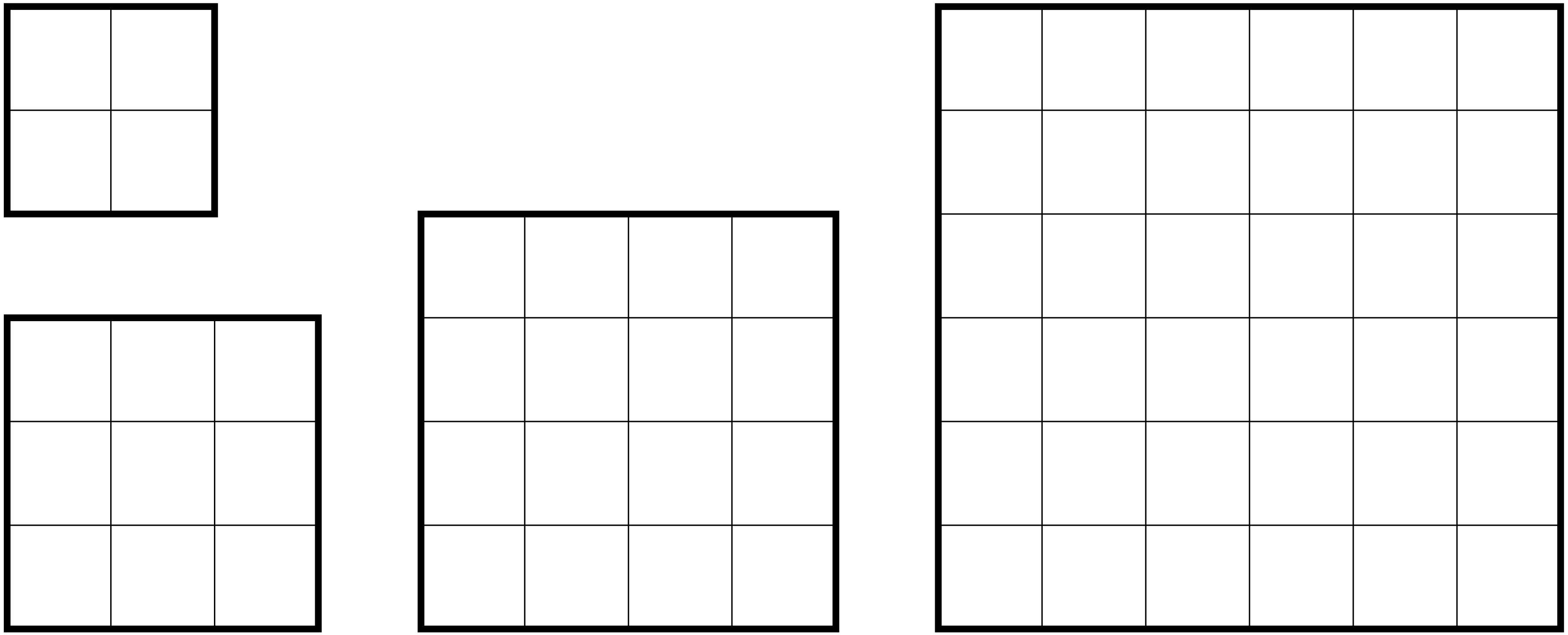


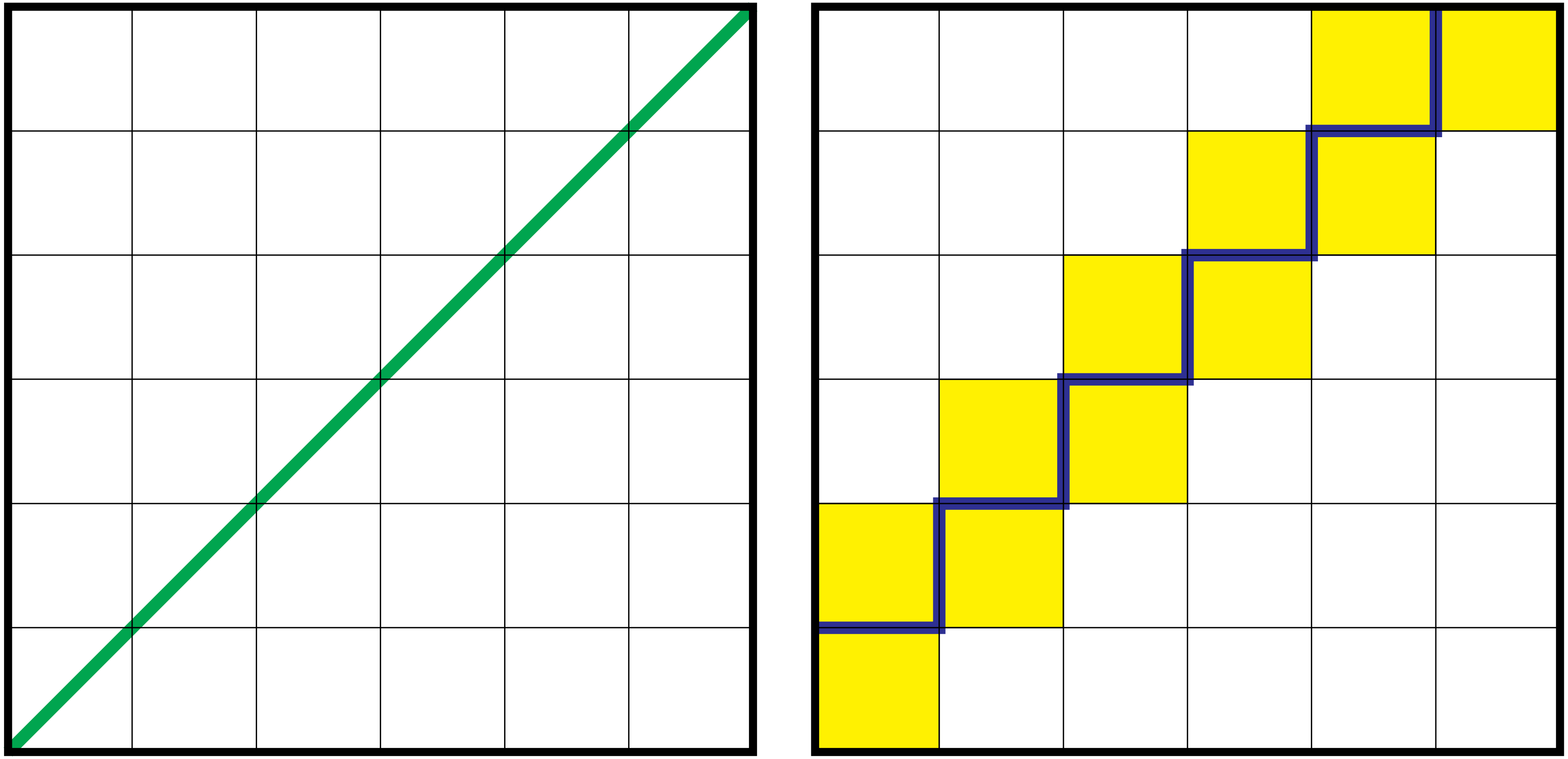


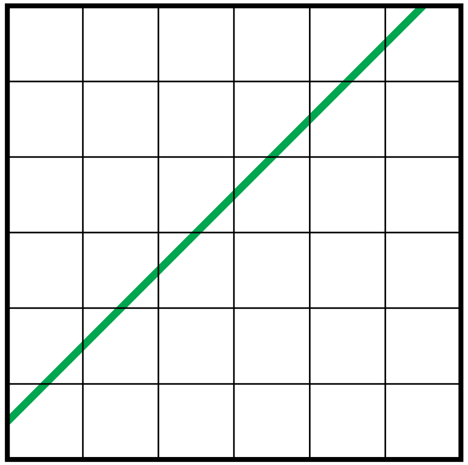
four tiles

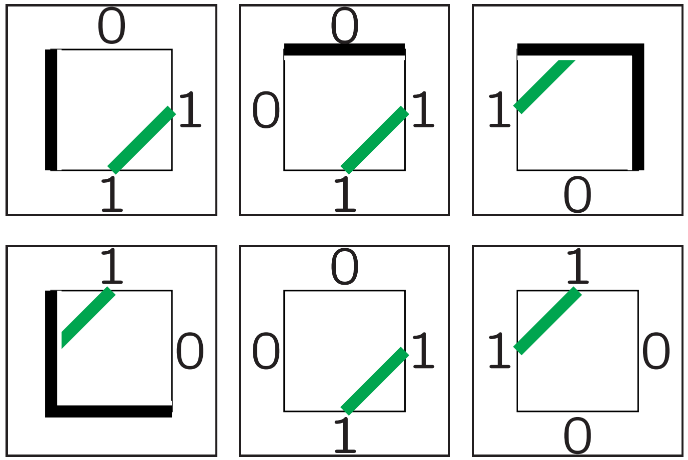
(and some borders)

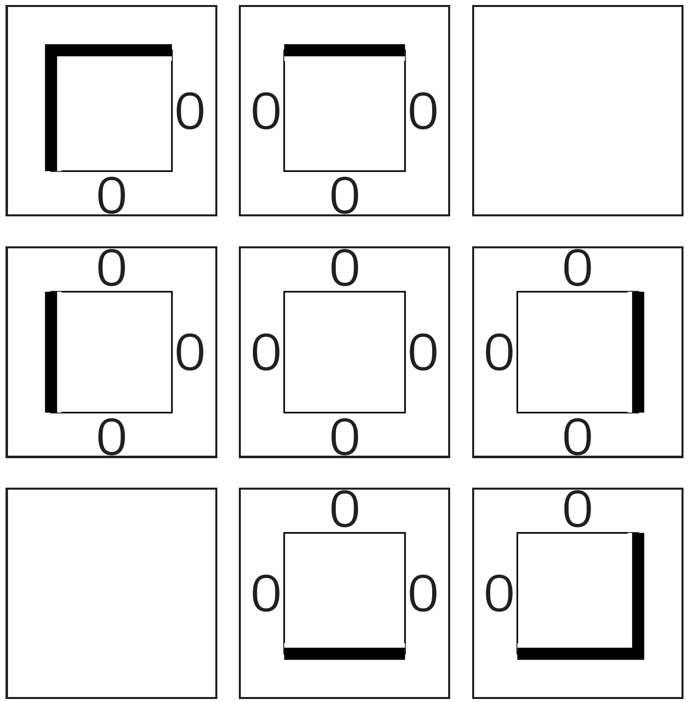
■ Form

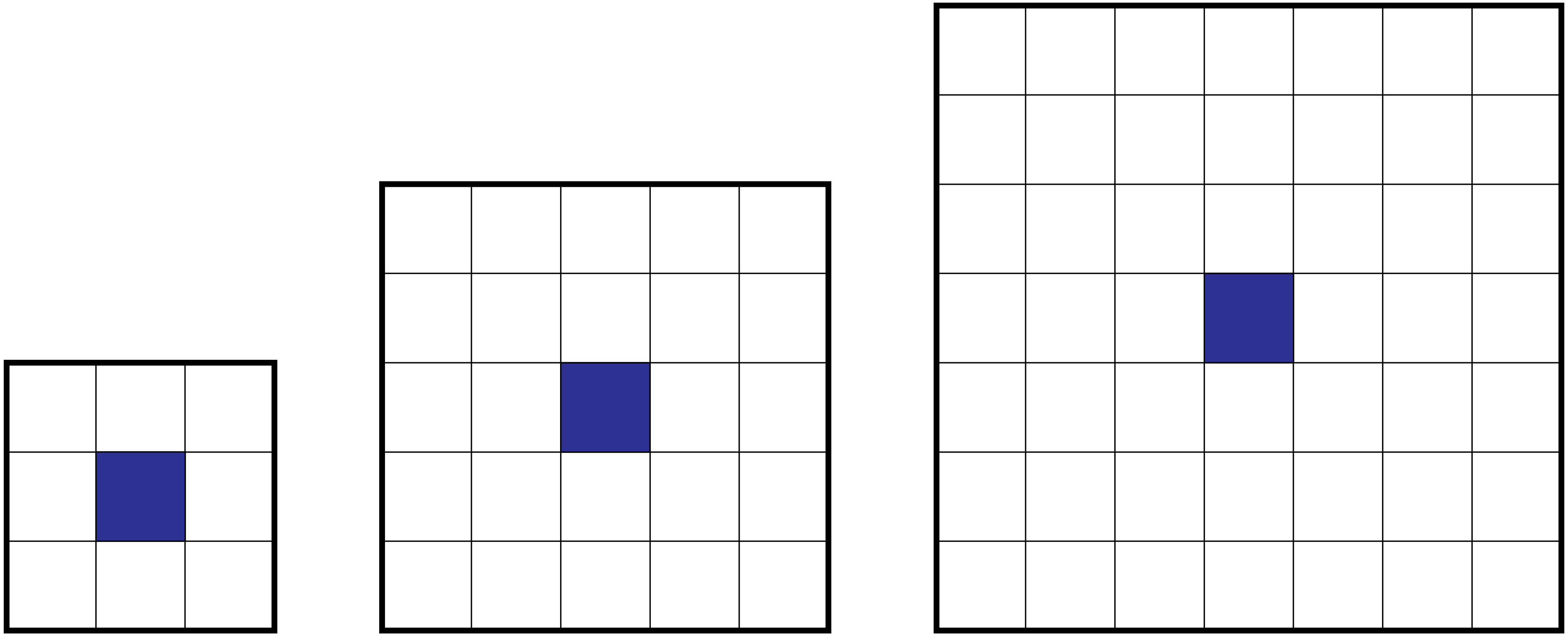


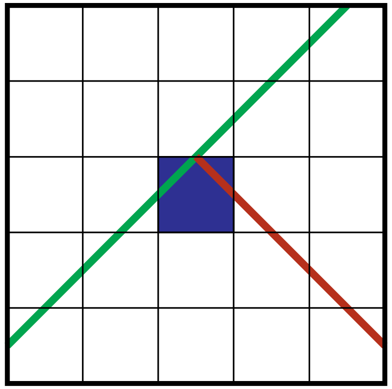


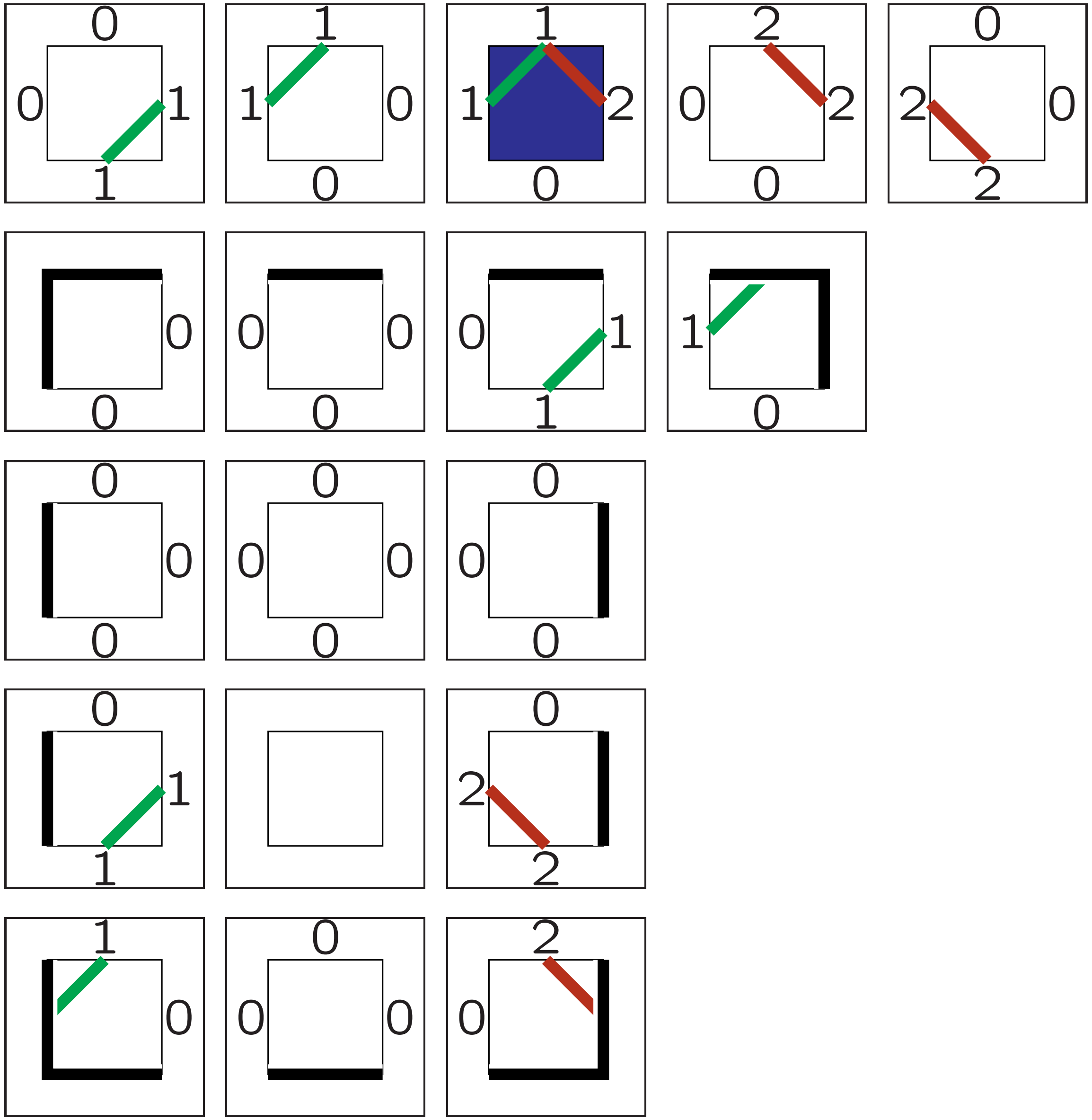


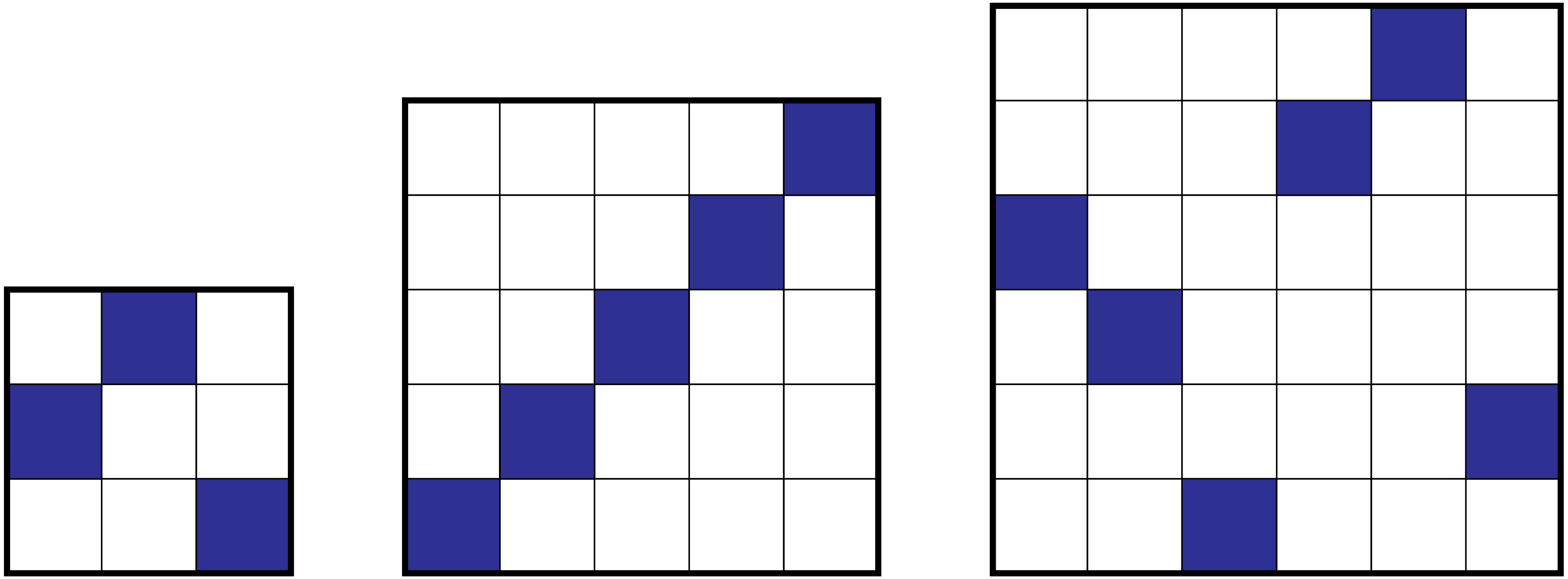

0 1 1
0 0 1 1
1 0
1 0
0 0 1 1
1 1 0 0

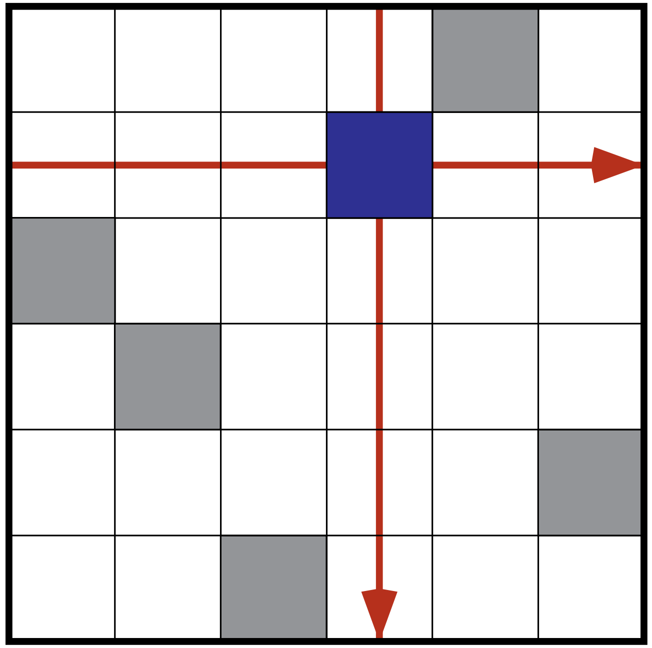

0 0
0 0 0
0 0 0
0 0 0 0
0 0 0
0 0 0
0 0

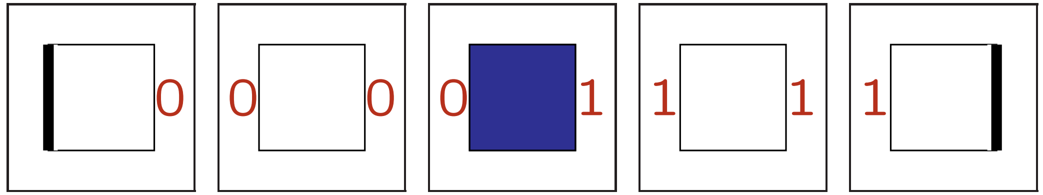


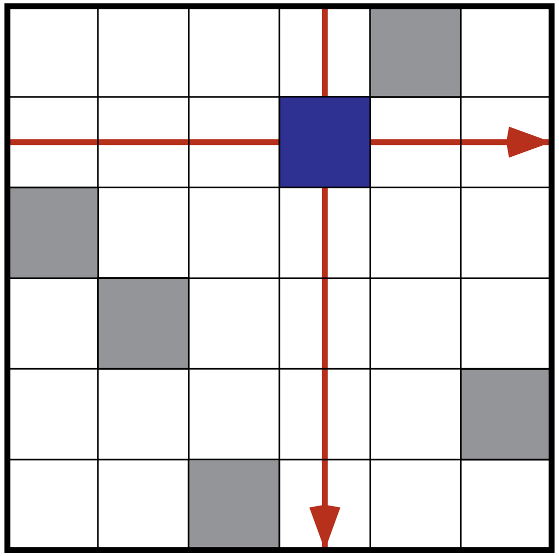


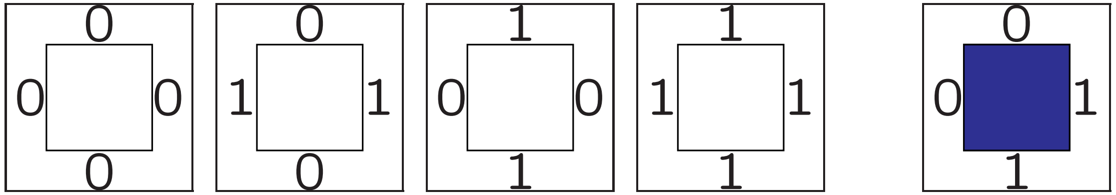

0 0 1 1
1 1 0 0
1 1 2 0
2 0 2 0
0 2 0 2
0 0
0 0 0
0 1 1
1 0
0 0 0
0 0 0 0
0 0 0
0 1 1
0 2 2
1 0
0 0 0
2 0



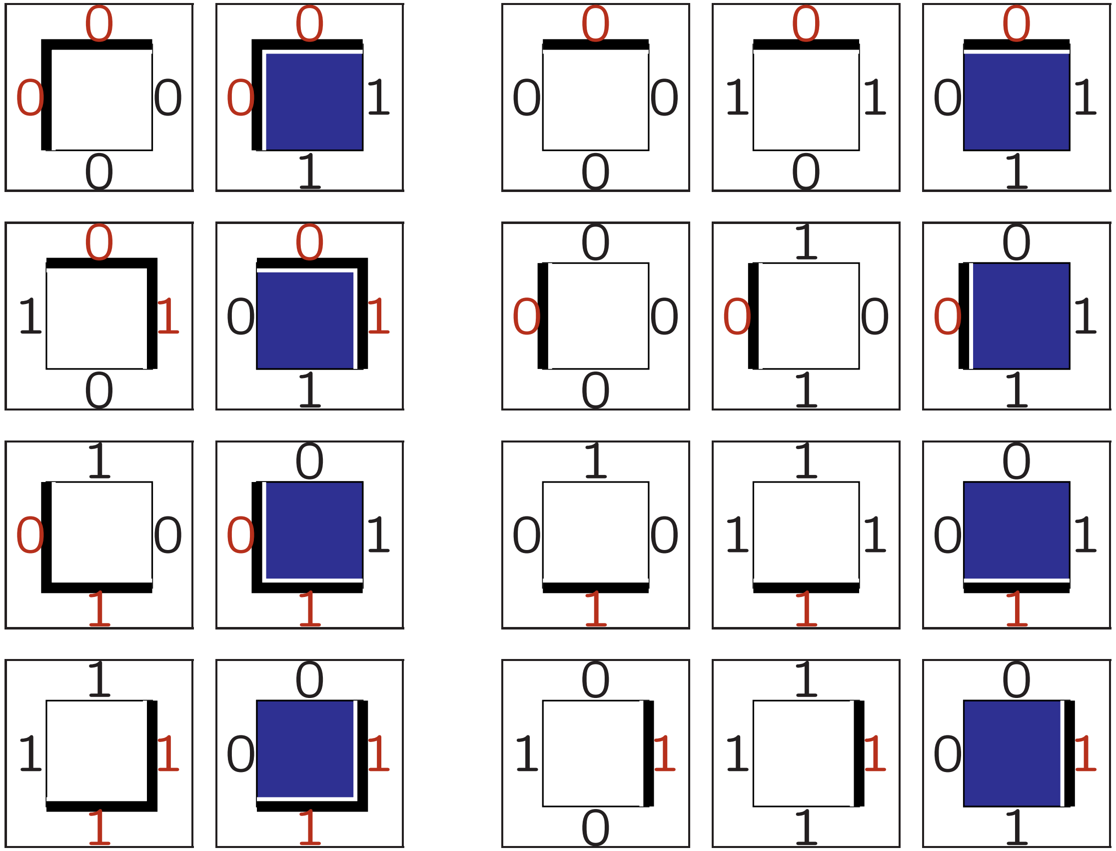



0
0
0
0
1
1
1
1

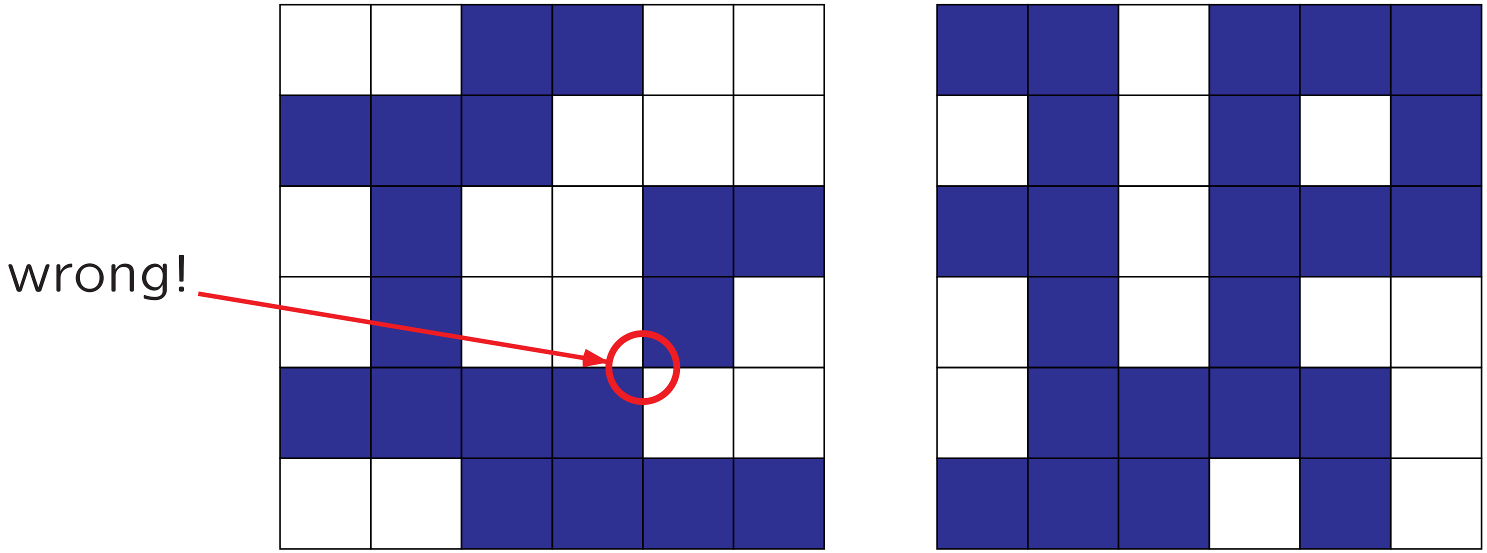




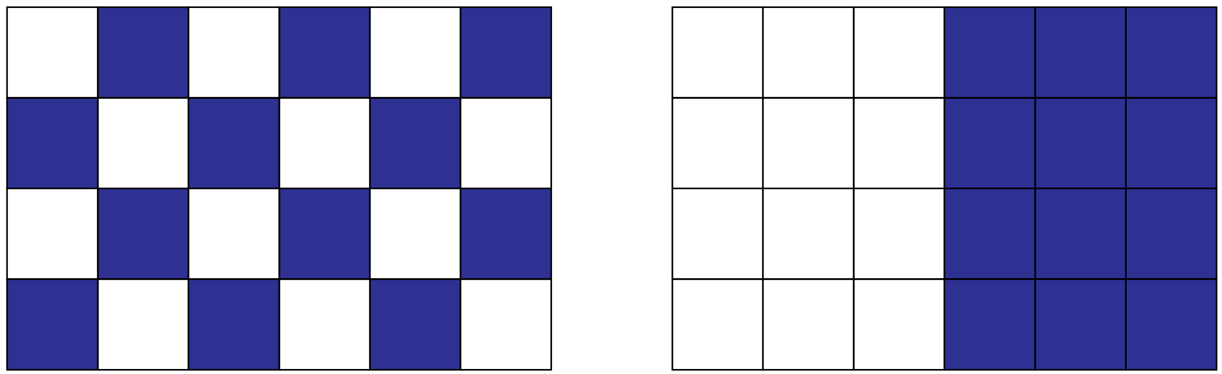
left/top 0 right/bottom 1

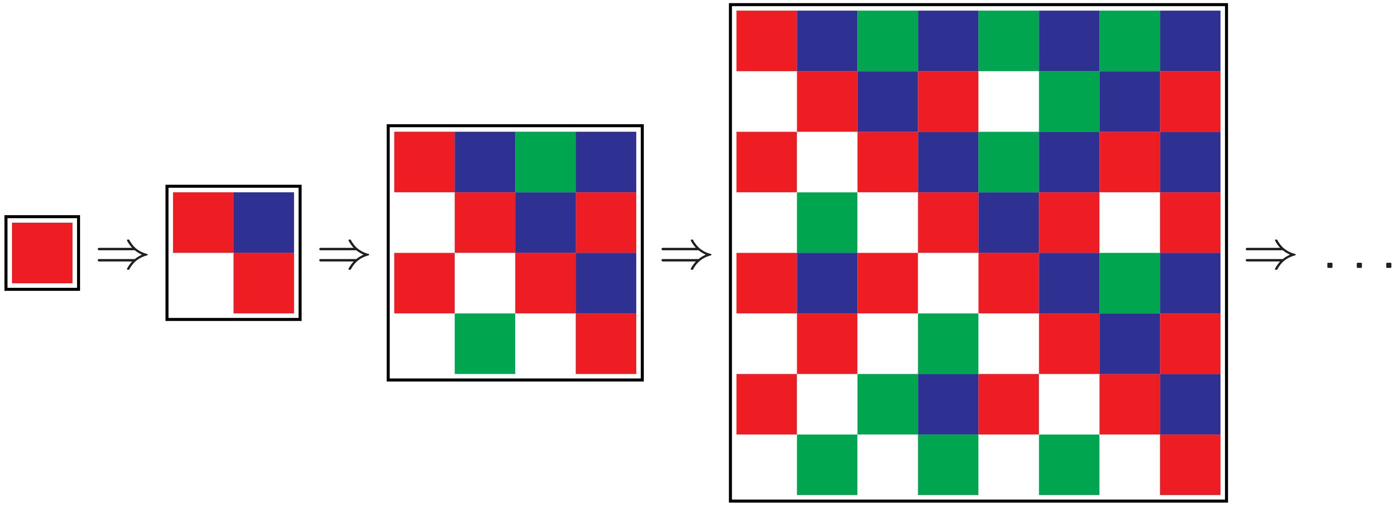


- blue tiles are connected (*difficult*)



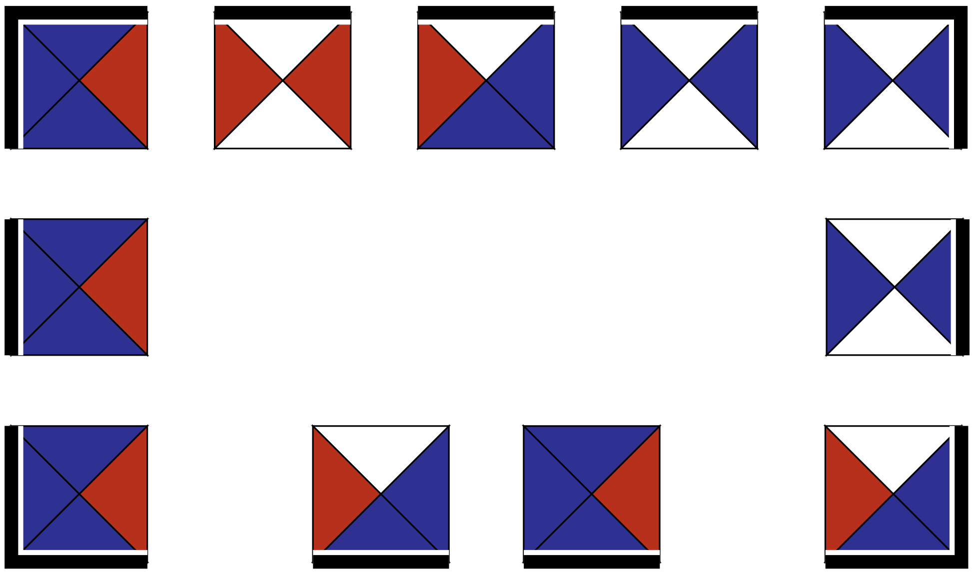
- equal numbers (*very difficult*)

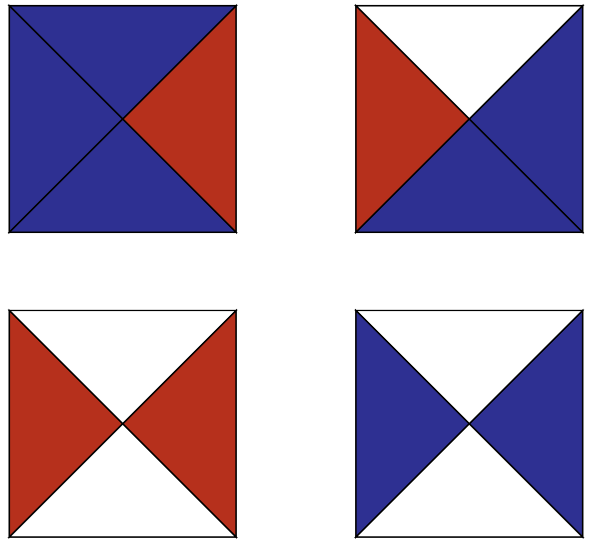




iterated substitutions form patterns that can be tiled

■ Computing

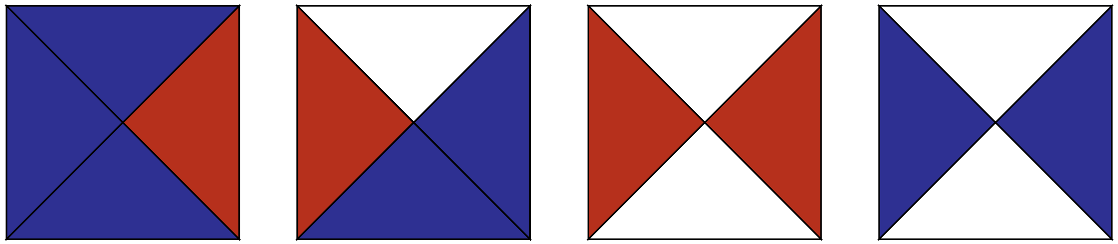


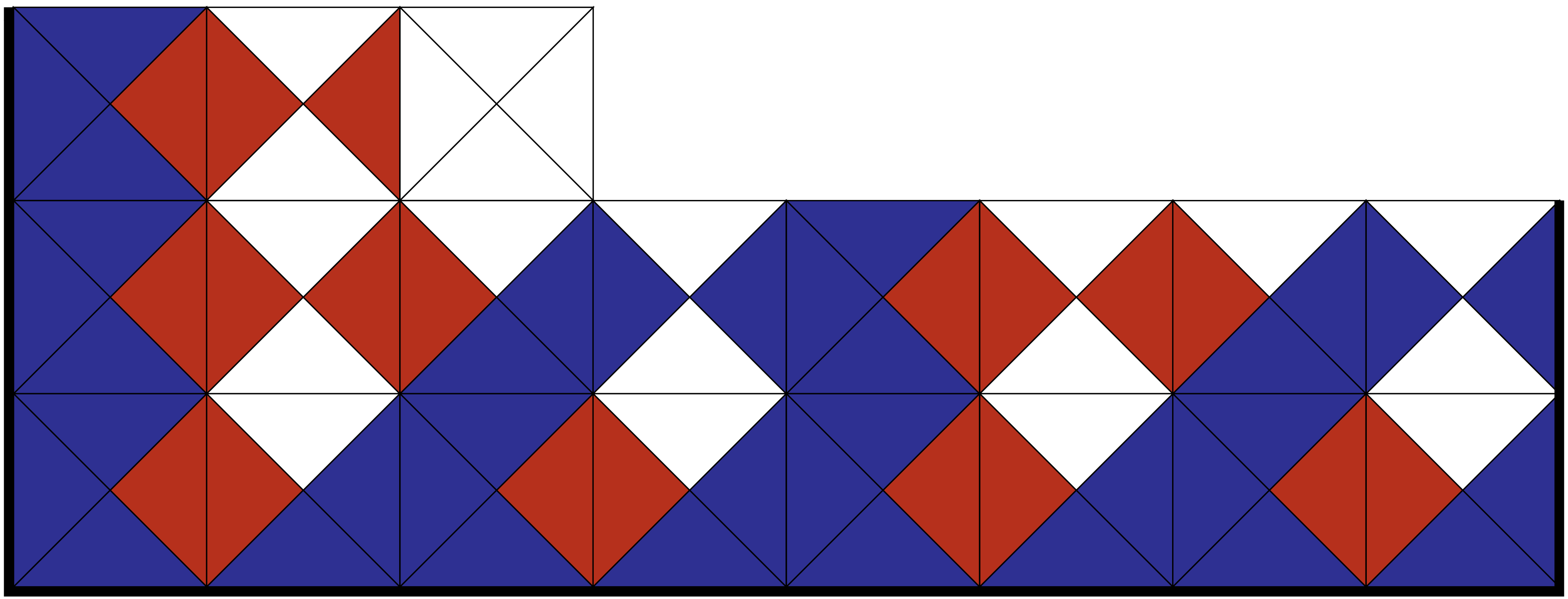


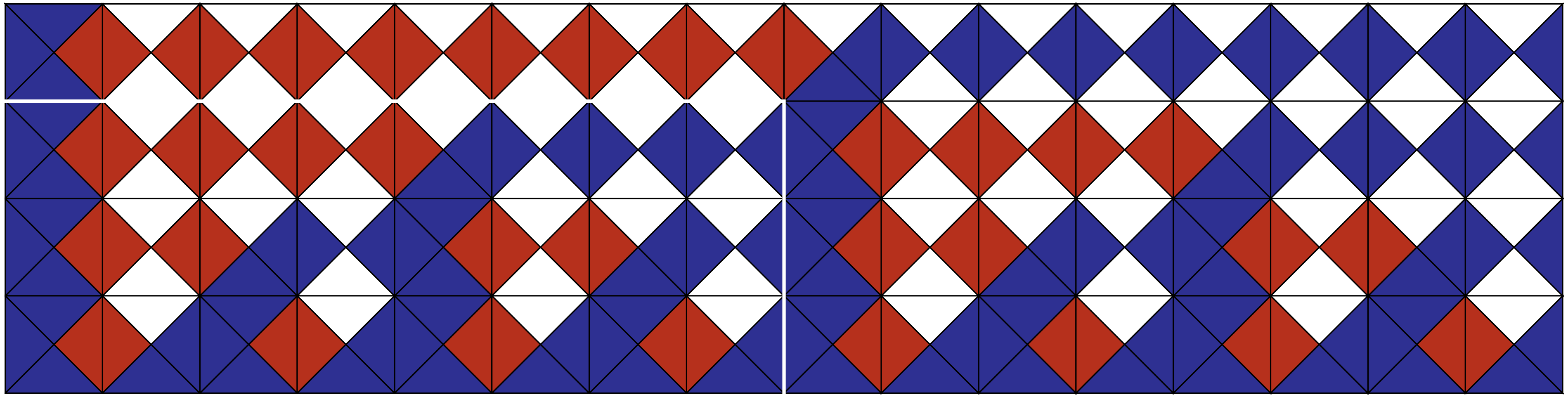
rectangles:

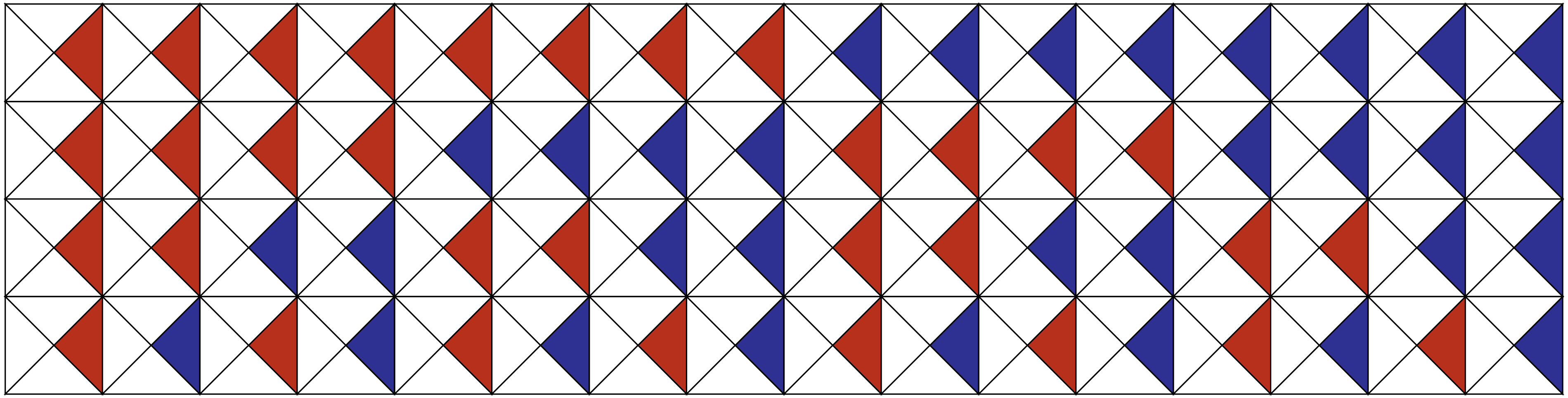
8×3 & 16×4

start at bottom

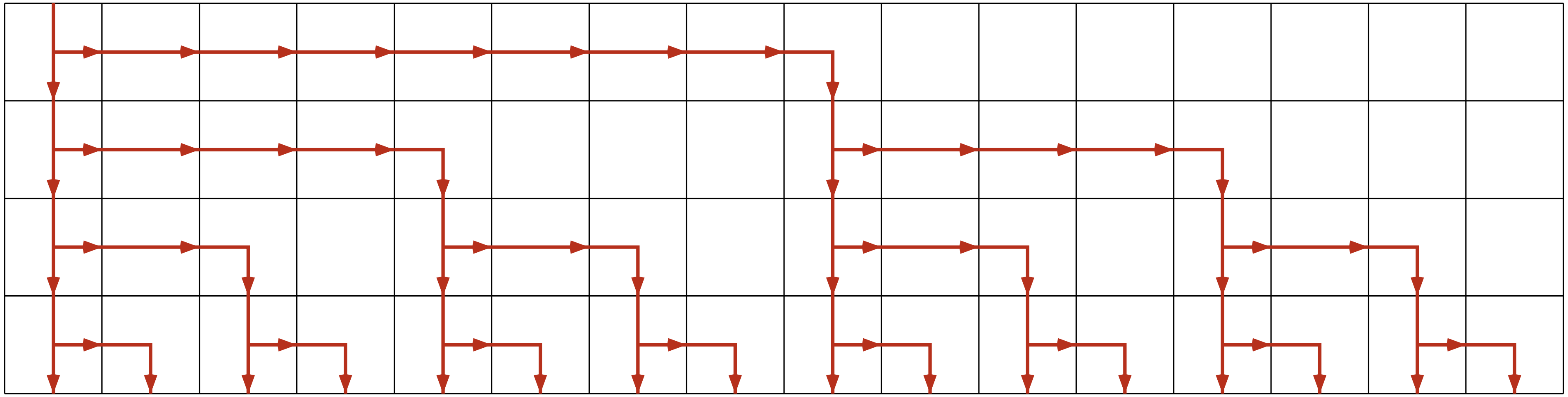






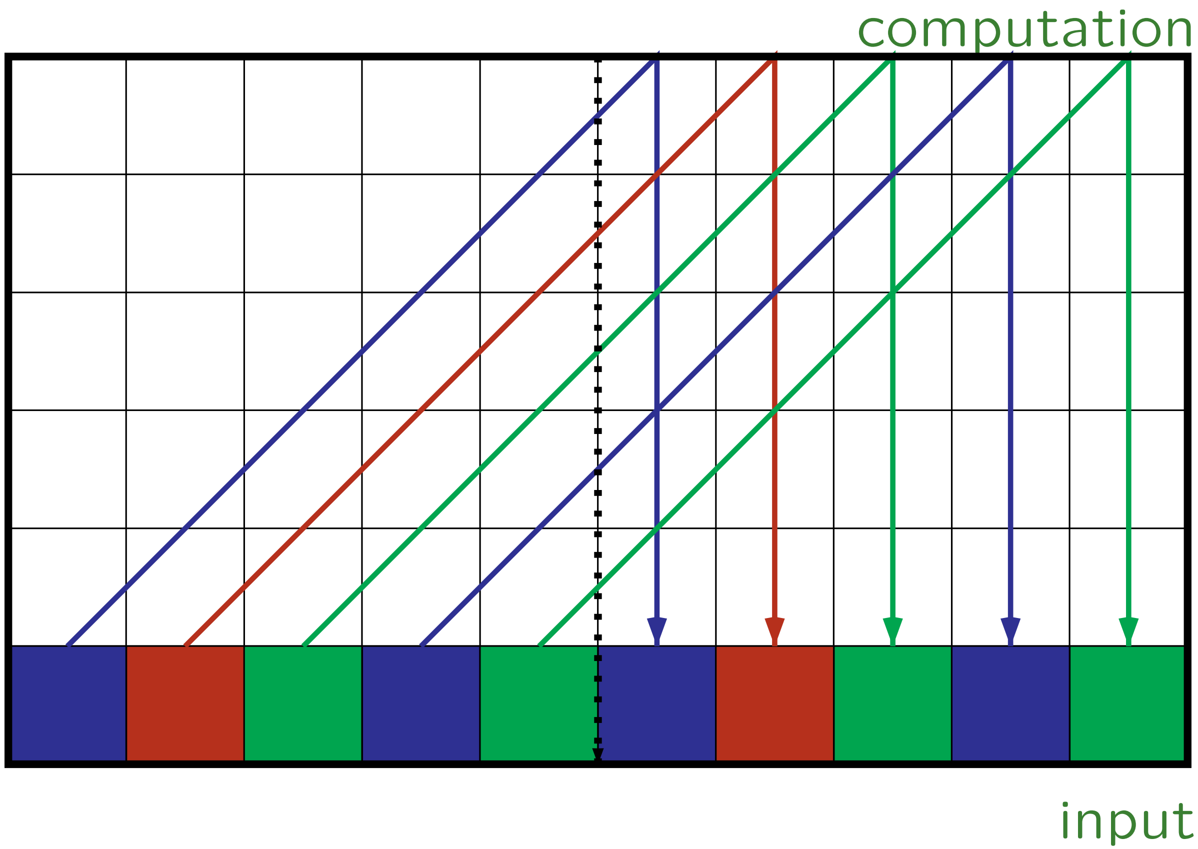


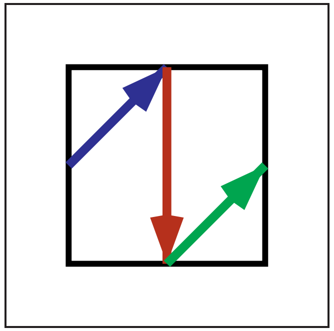
output



input
(number of squares)

powers of two



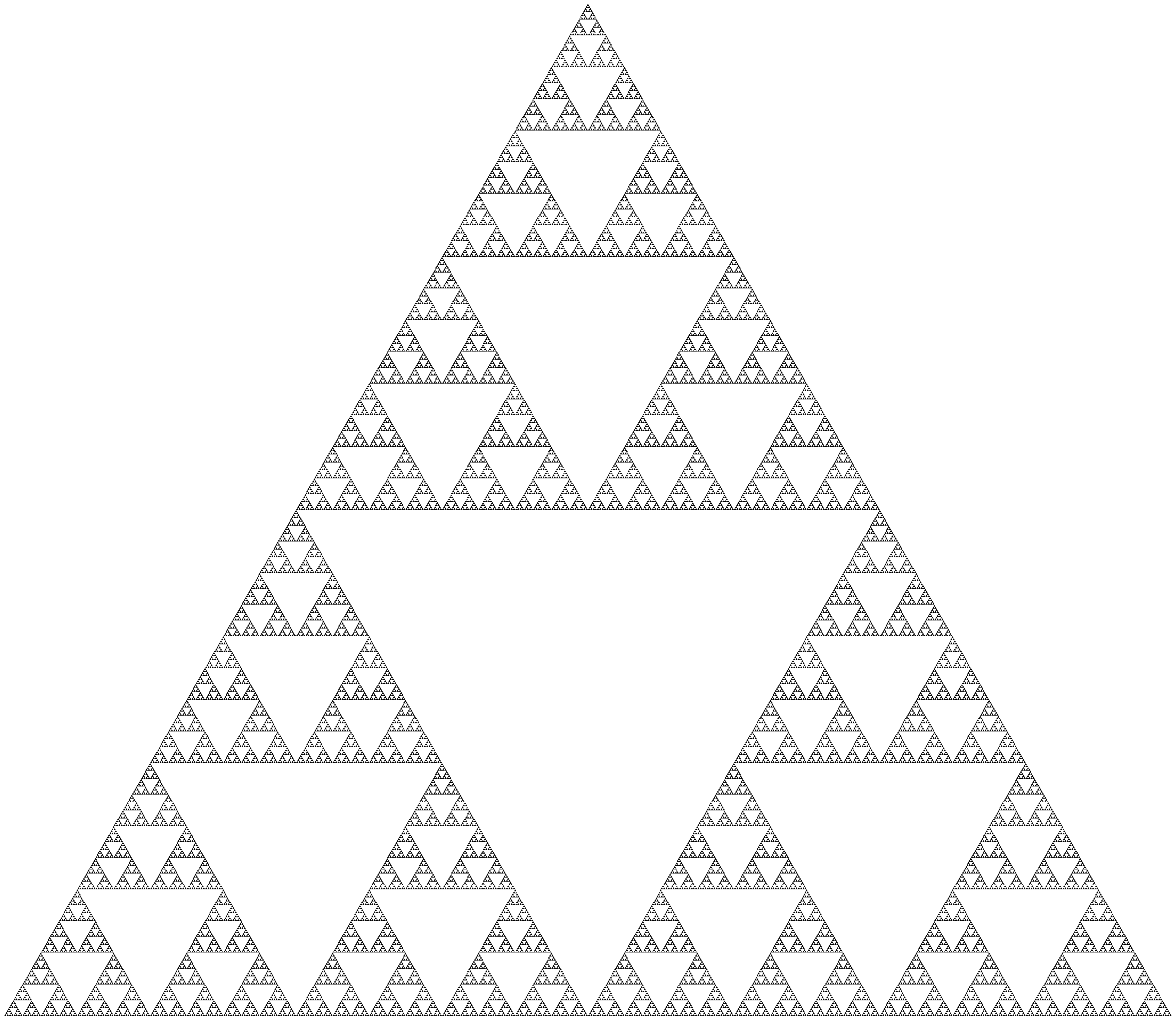


(at most) three
colours per tile

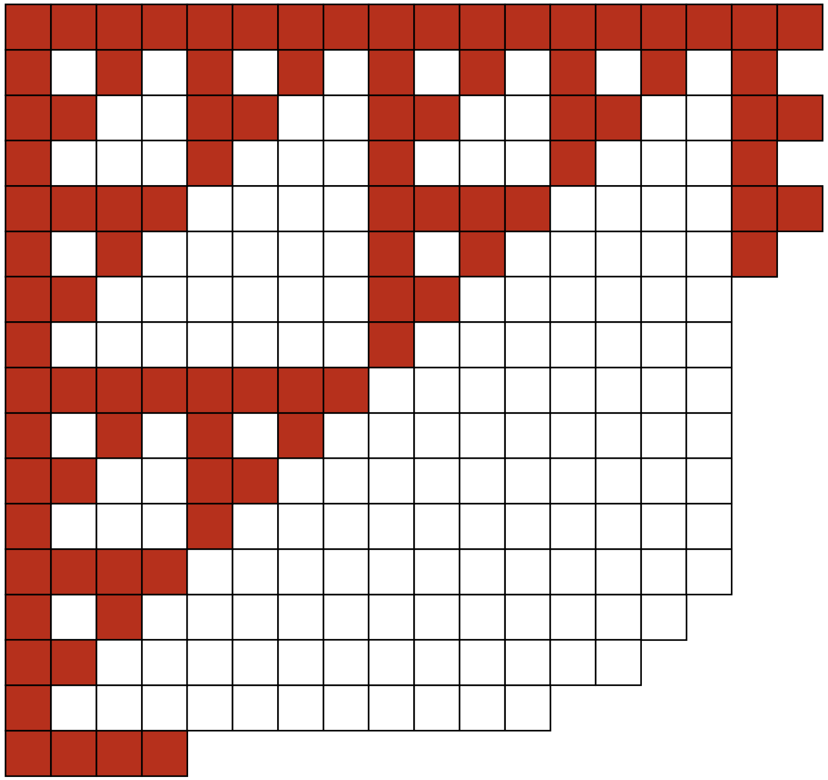

't Leven is
een
krentenbol

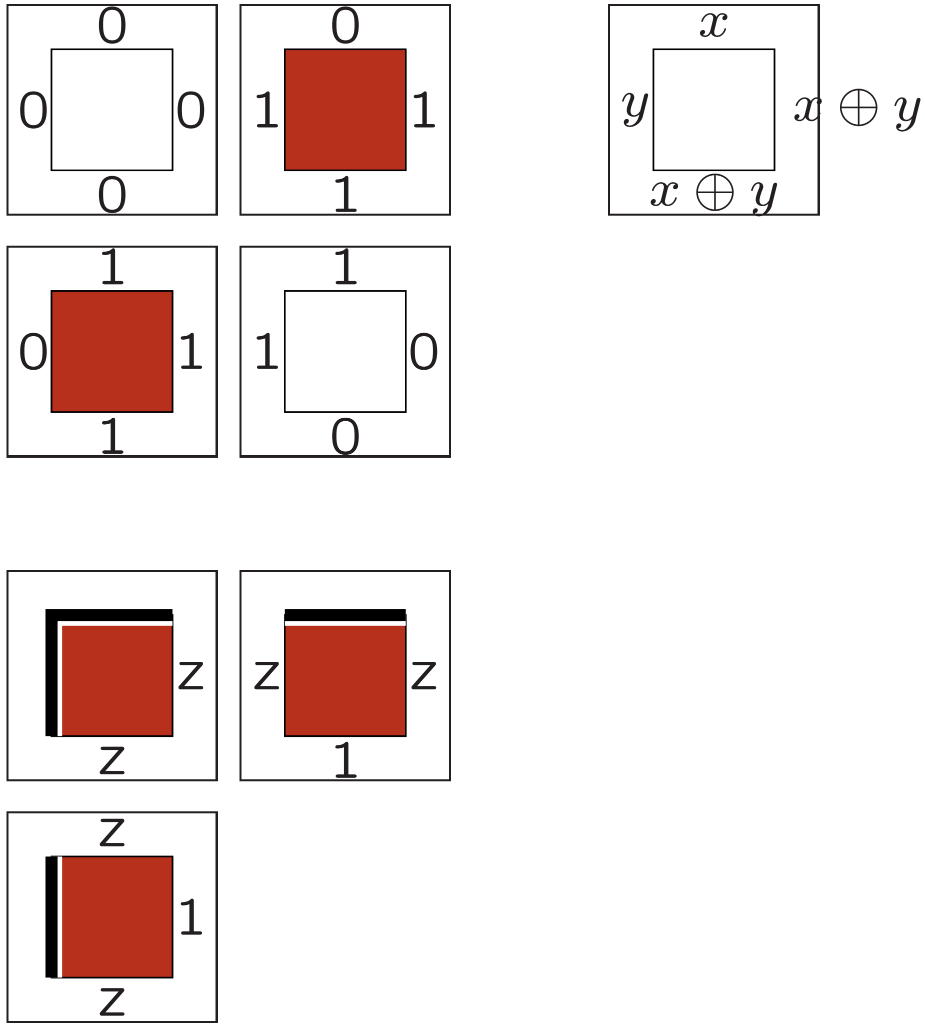
with tiles you can count

■ Self-Assembly

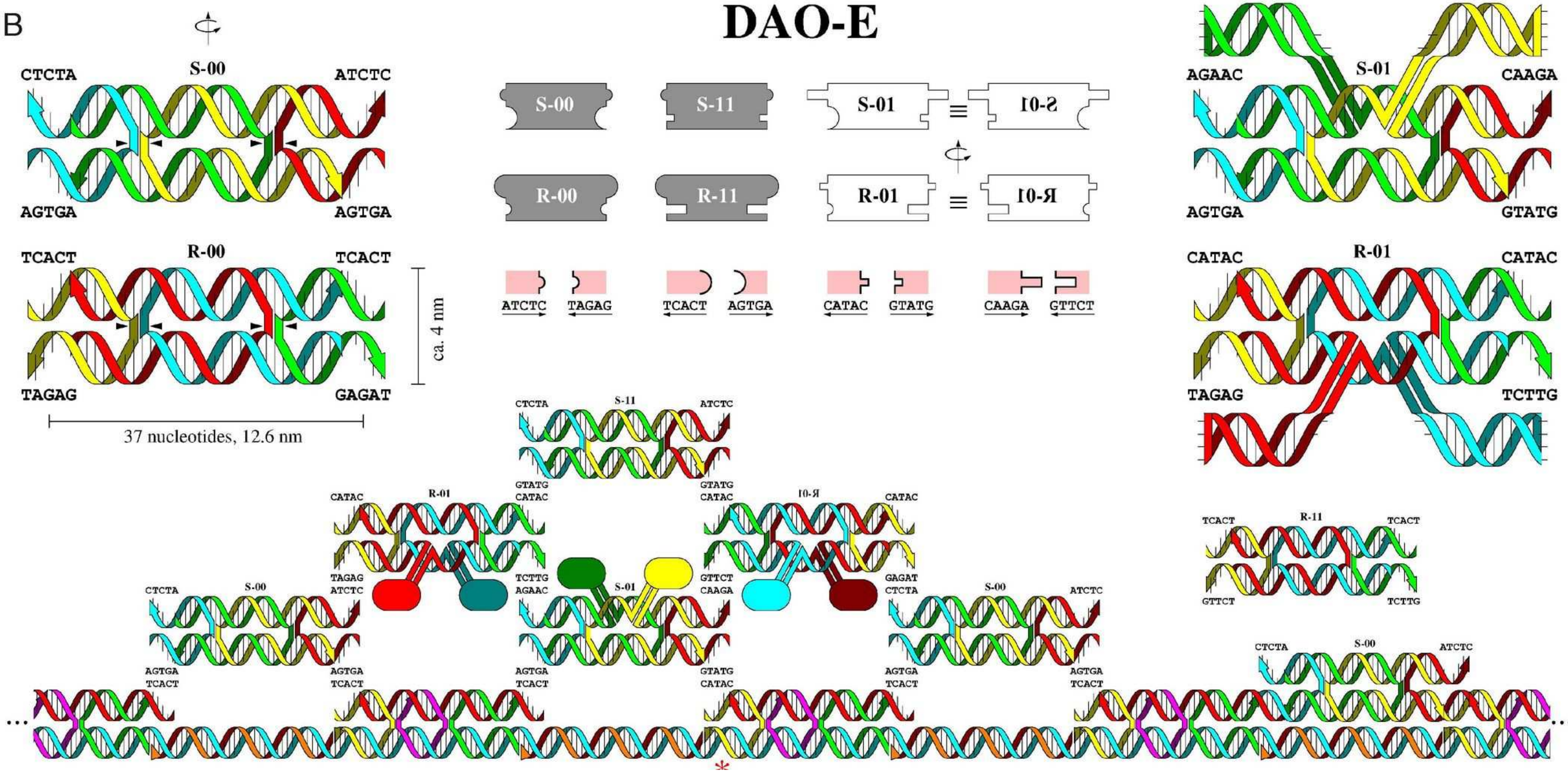


Qef's Website
wikipedia



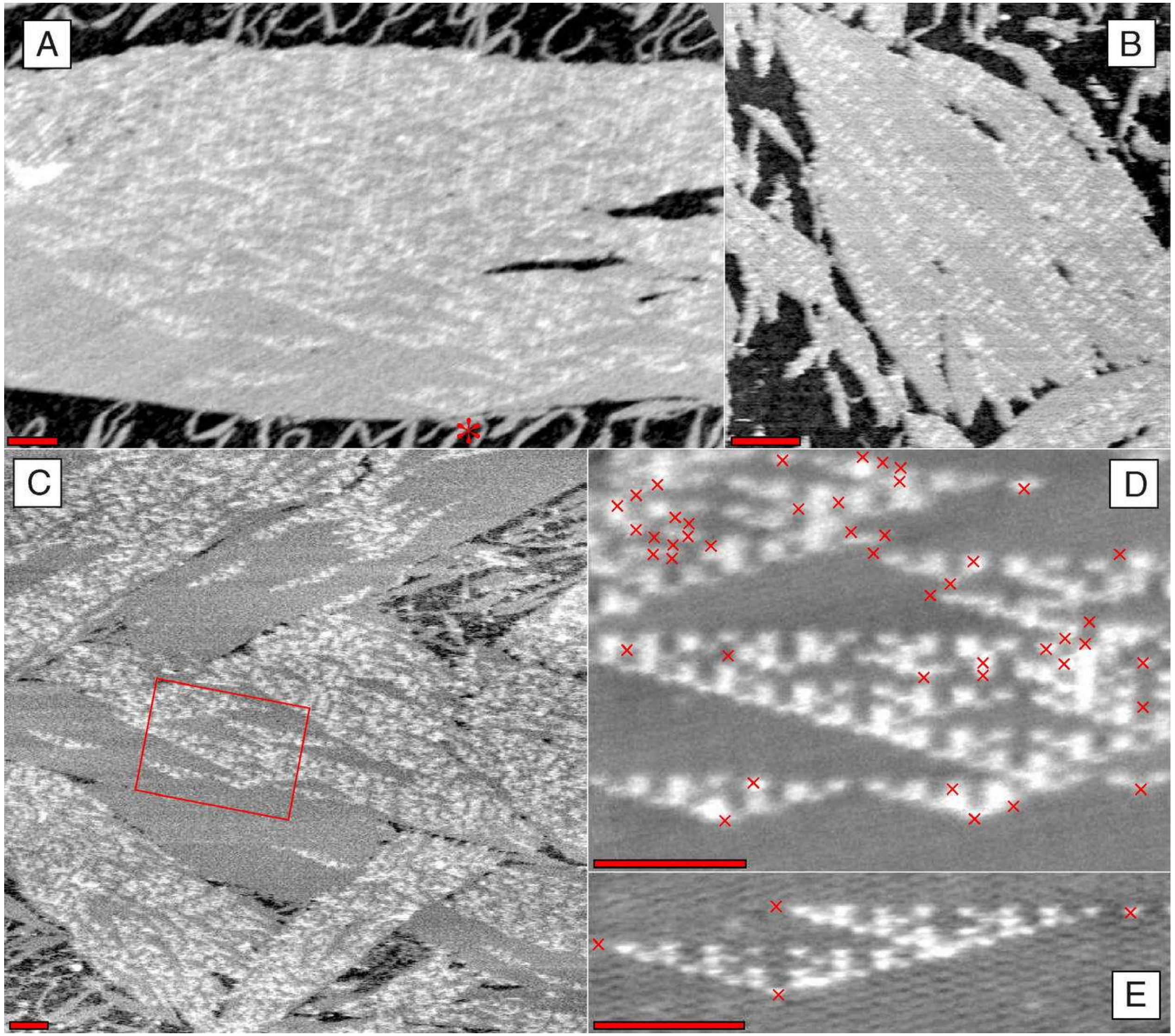


x	y	$x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0



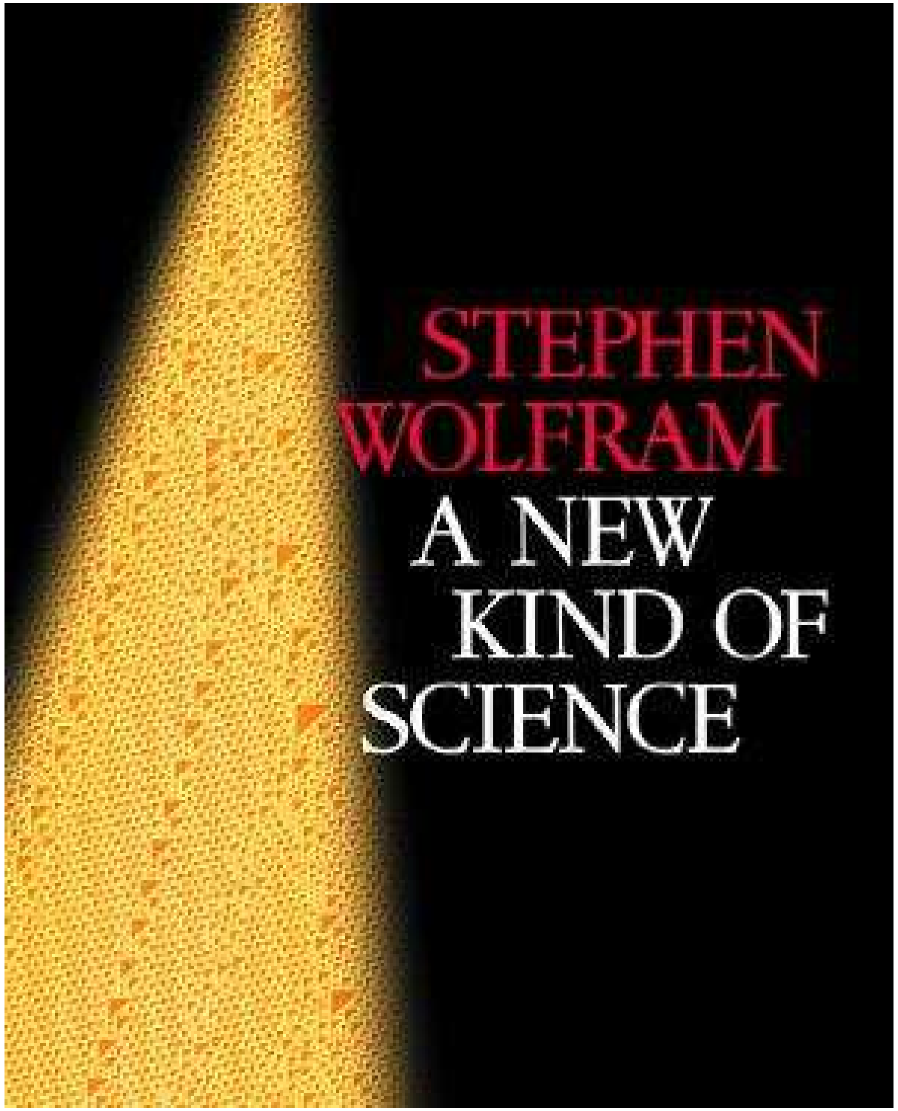
Algorithmic Self-Assembly of DNA Sierpinski Triangles (2004)

Rothemund, Papadakis, Winfree; PLoS Biology

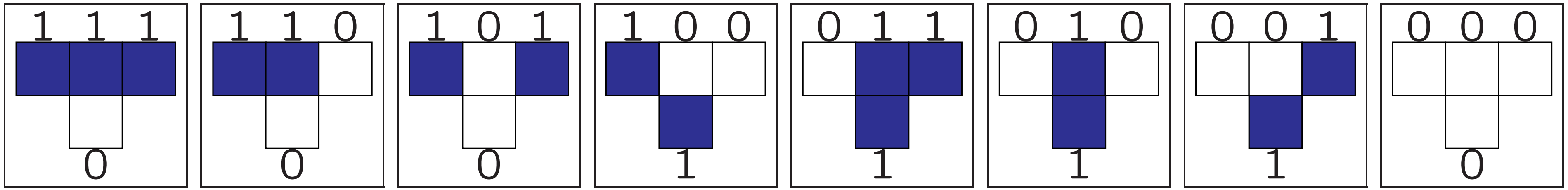

A
B
C
D
E

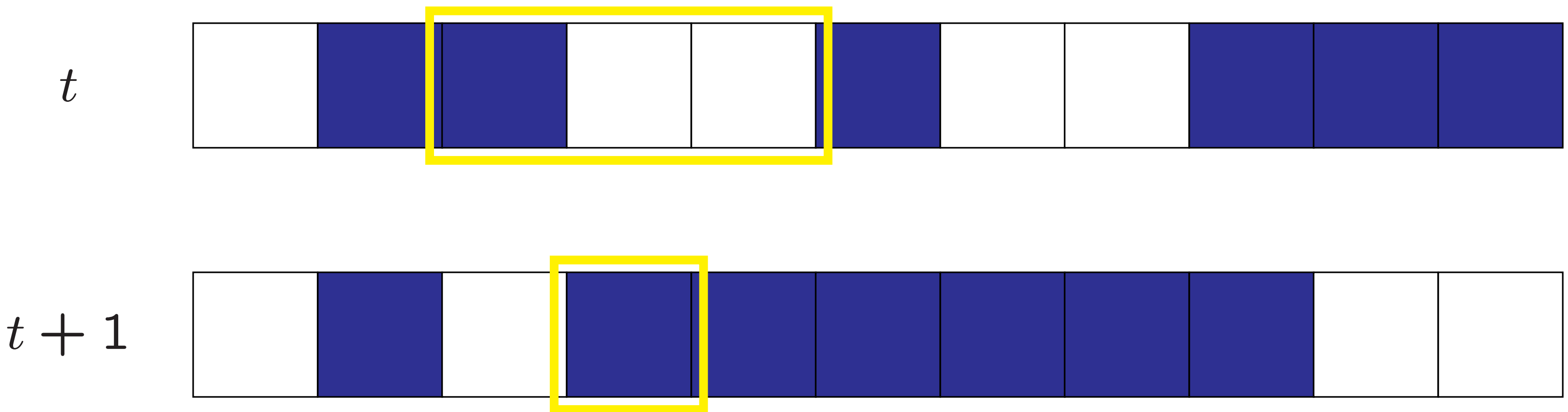
■ Cellular Automata

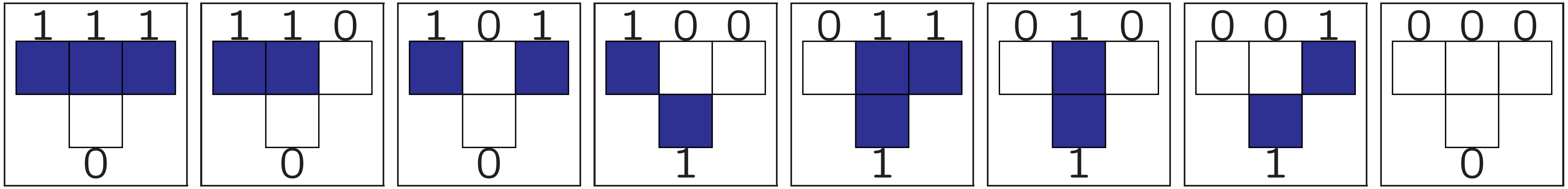
see: *collection of reviews*

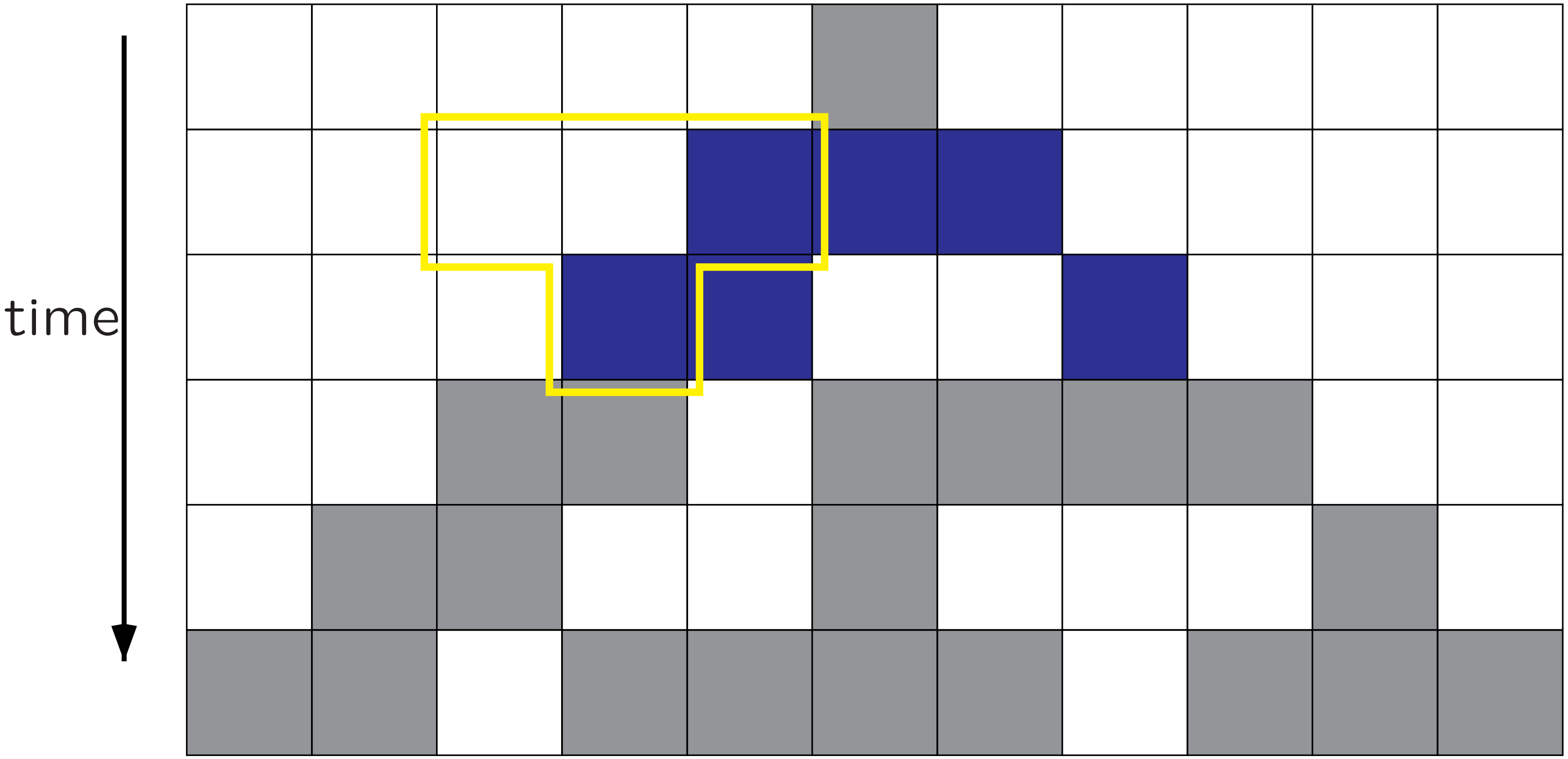


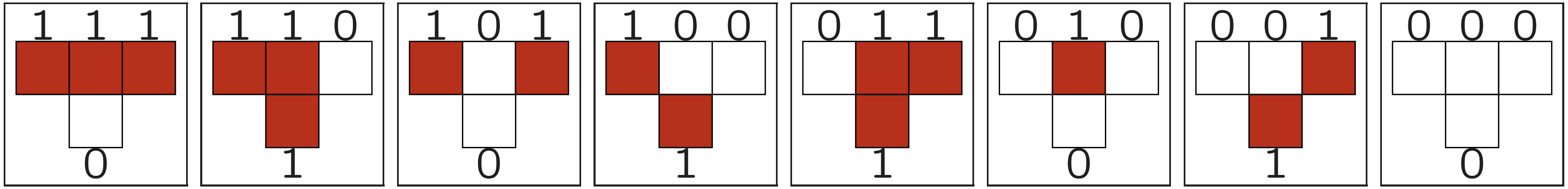
Stephen Wolfram

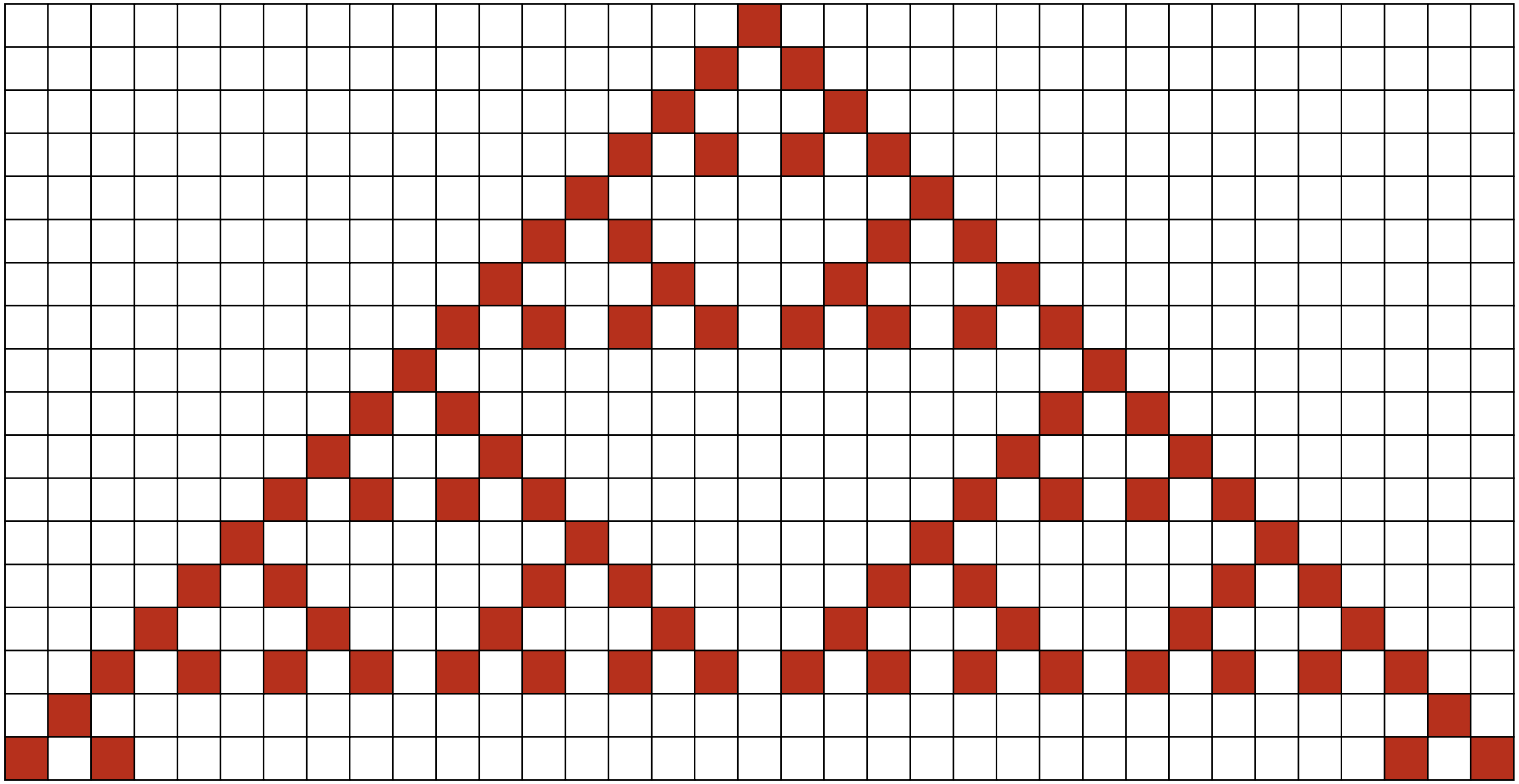

1 1 1
0
1 1 0
0
1 0 1
0
1 0 0
1
0 1 1
1
0 1 0
1
0 0 1
1
0 0 0
0

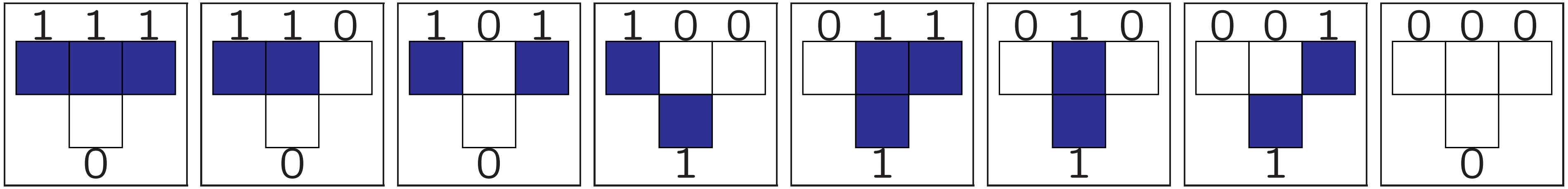

t
$t + 1$

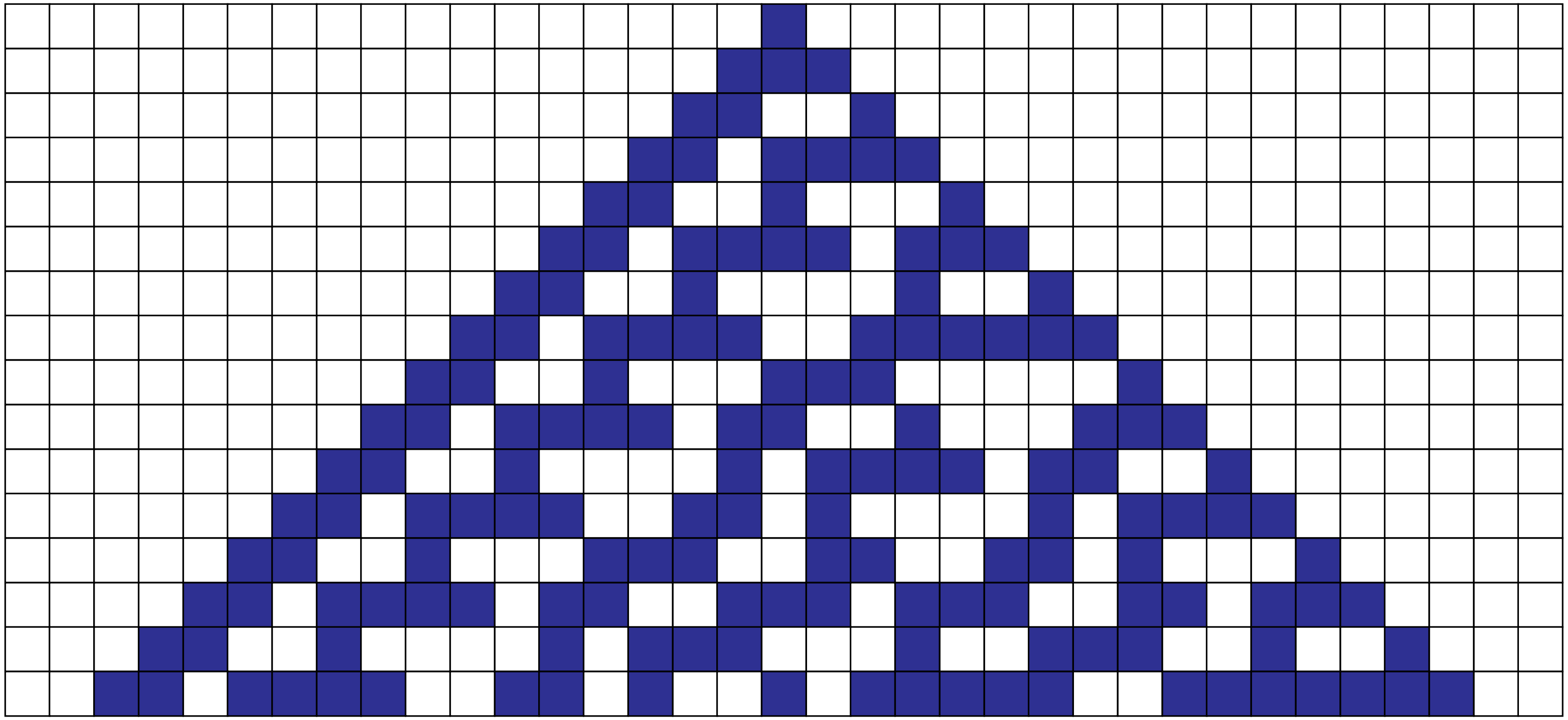

1 1 1
0
1 1 0
0
1 0 1
0
1 0 0
1
0 1 1
1
0 1 0
1
0 0 1
1
0 0 0
0

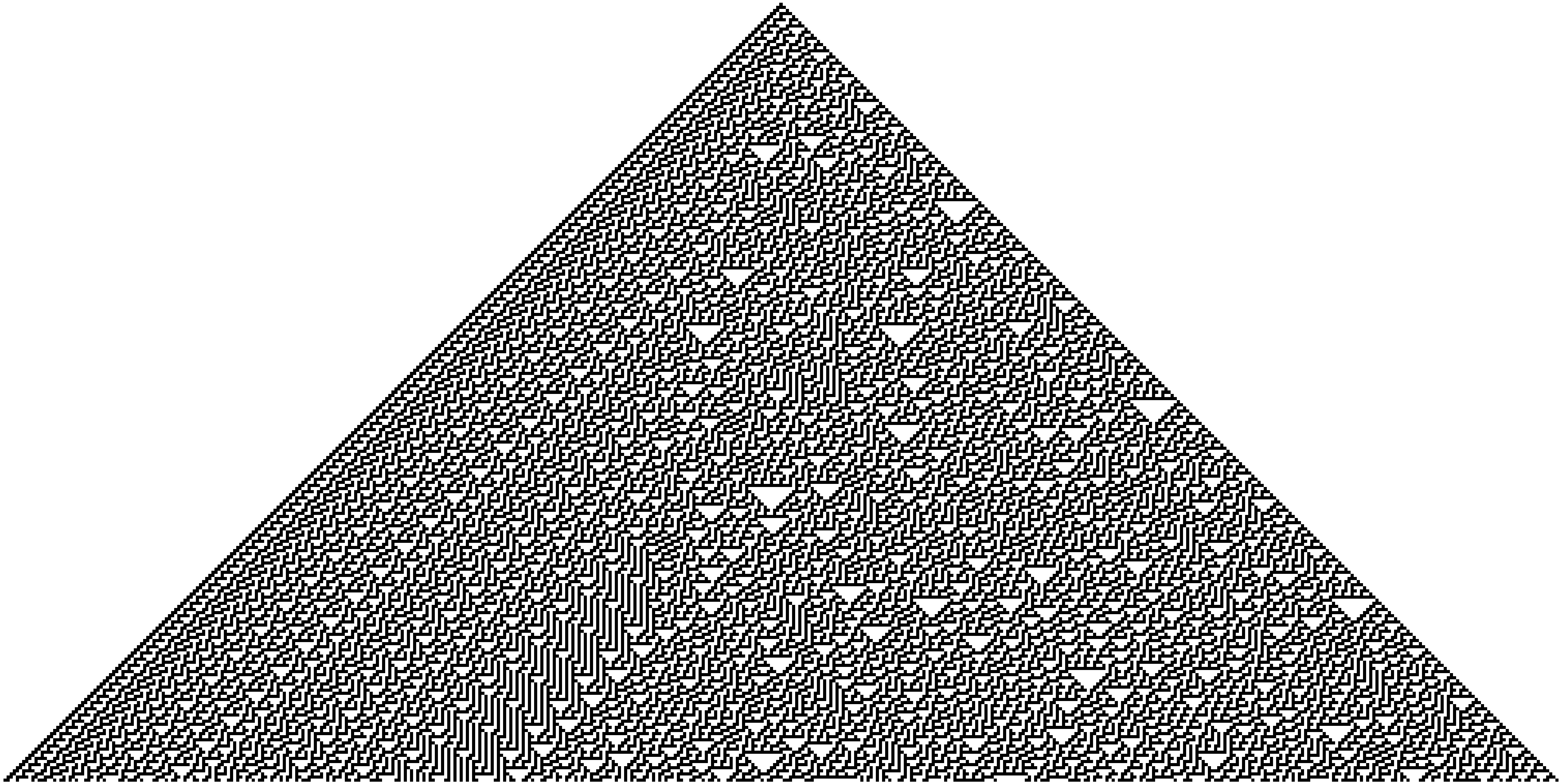

time

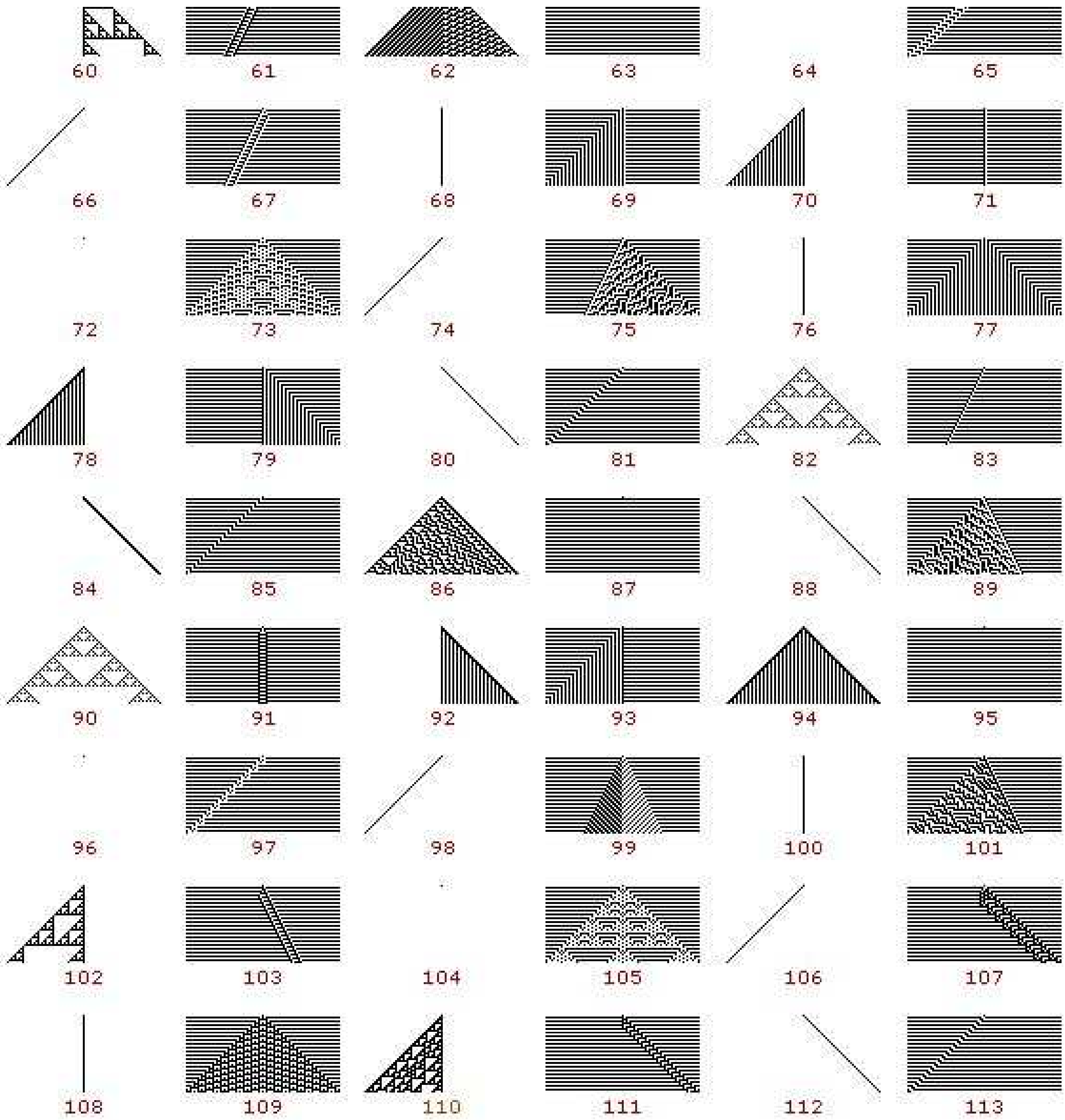

1 1 1
0
1 1 0
1
1 0 1
0
1 0 0
1
0 1 1
1
0 1 0
0
0 0 1
1
0 0 0
0




1 1 1
0
1 1 0
0
1 0 1
0
1 0 0
1
0 1 1
1
0 1 0
1
0 0 1
1
0 0 0
0



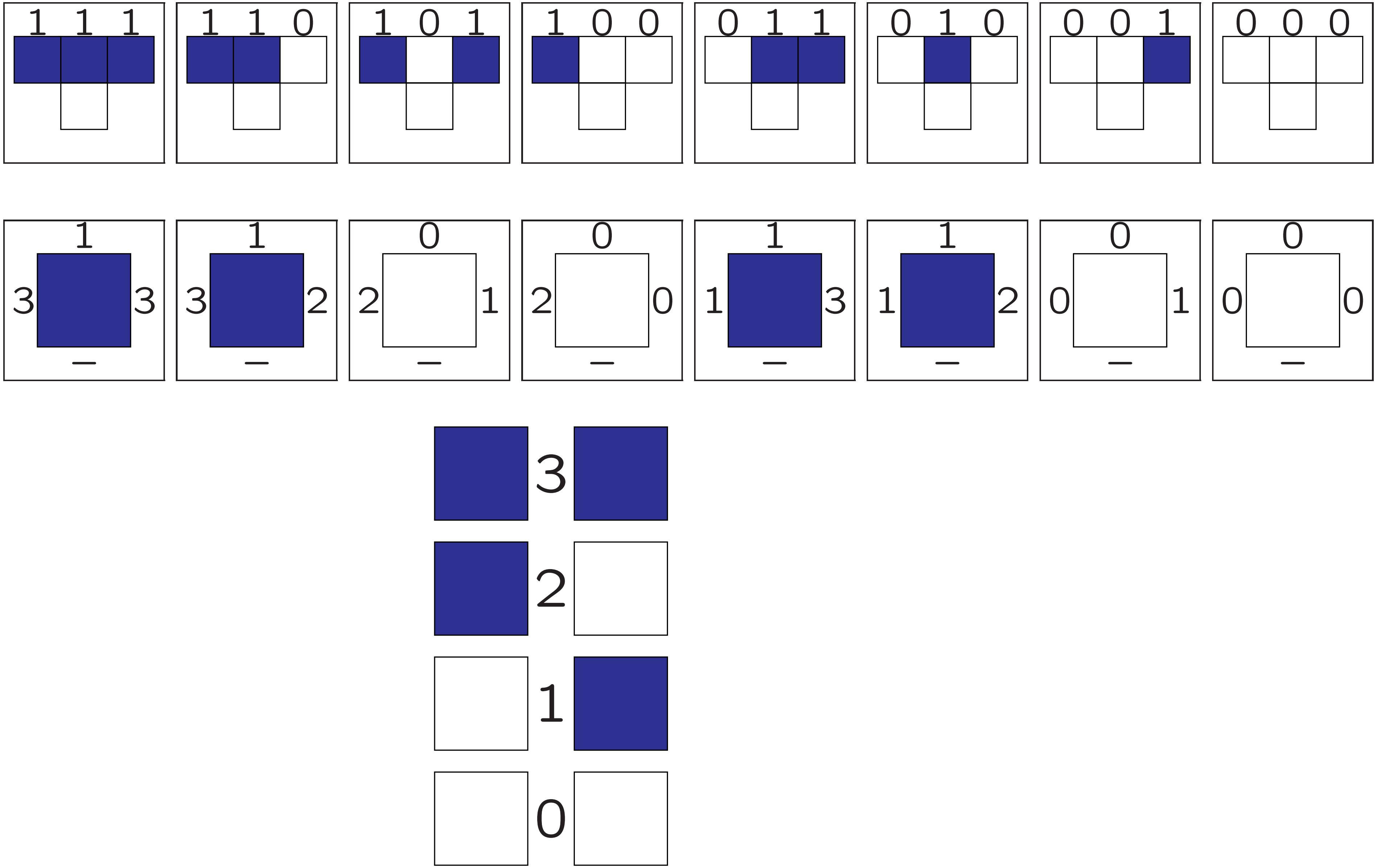


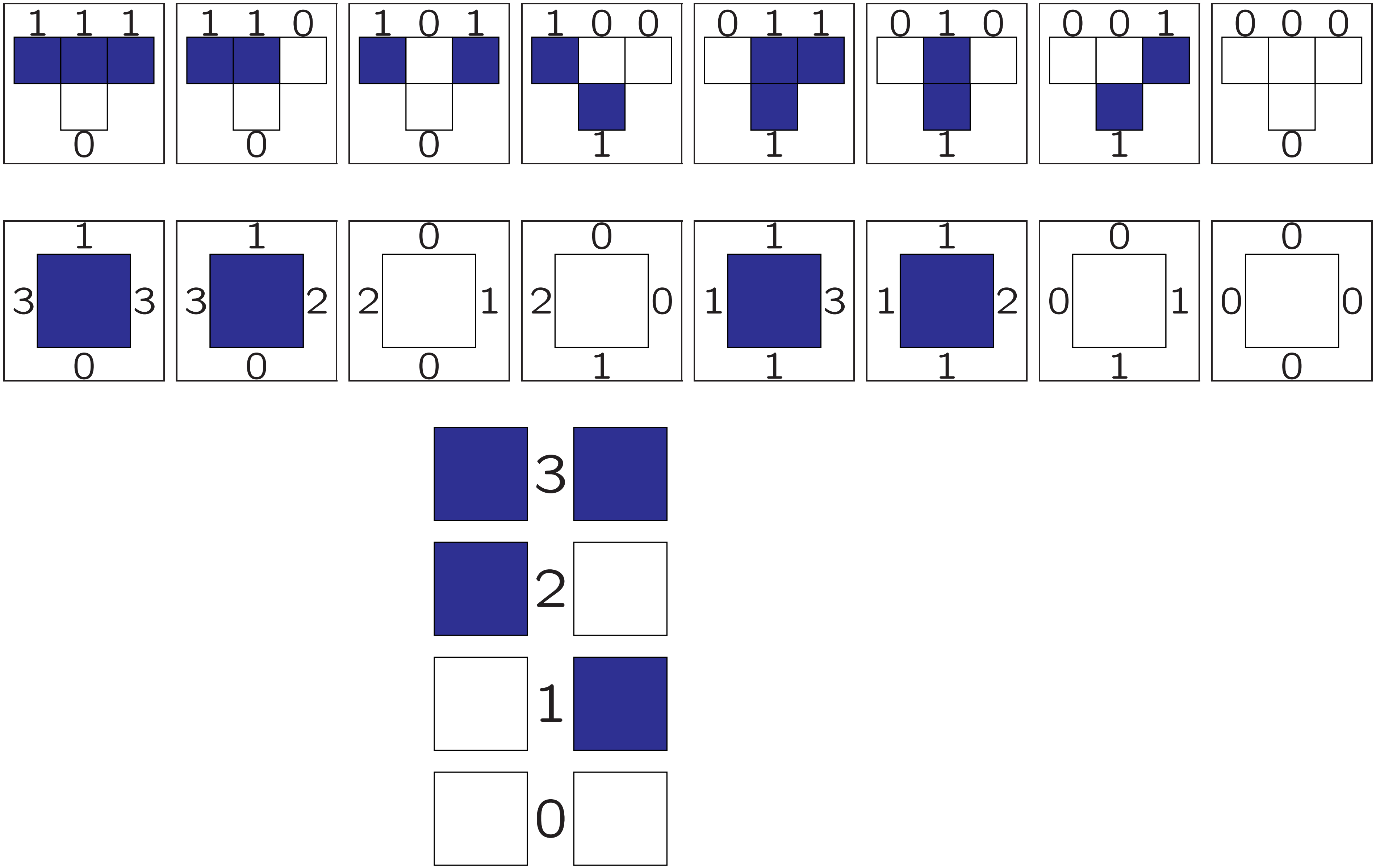

60 61 62 63 64 65
66 67 68 69 70 71
72 73 74 75 76 77
78 79 80 81 82 83
84 85 86 87 88 89
90 91 92 93 94 95
96 97 98 99 100 101
102 103 104 105 106 107
108 109 110 111 112 113

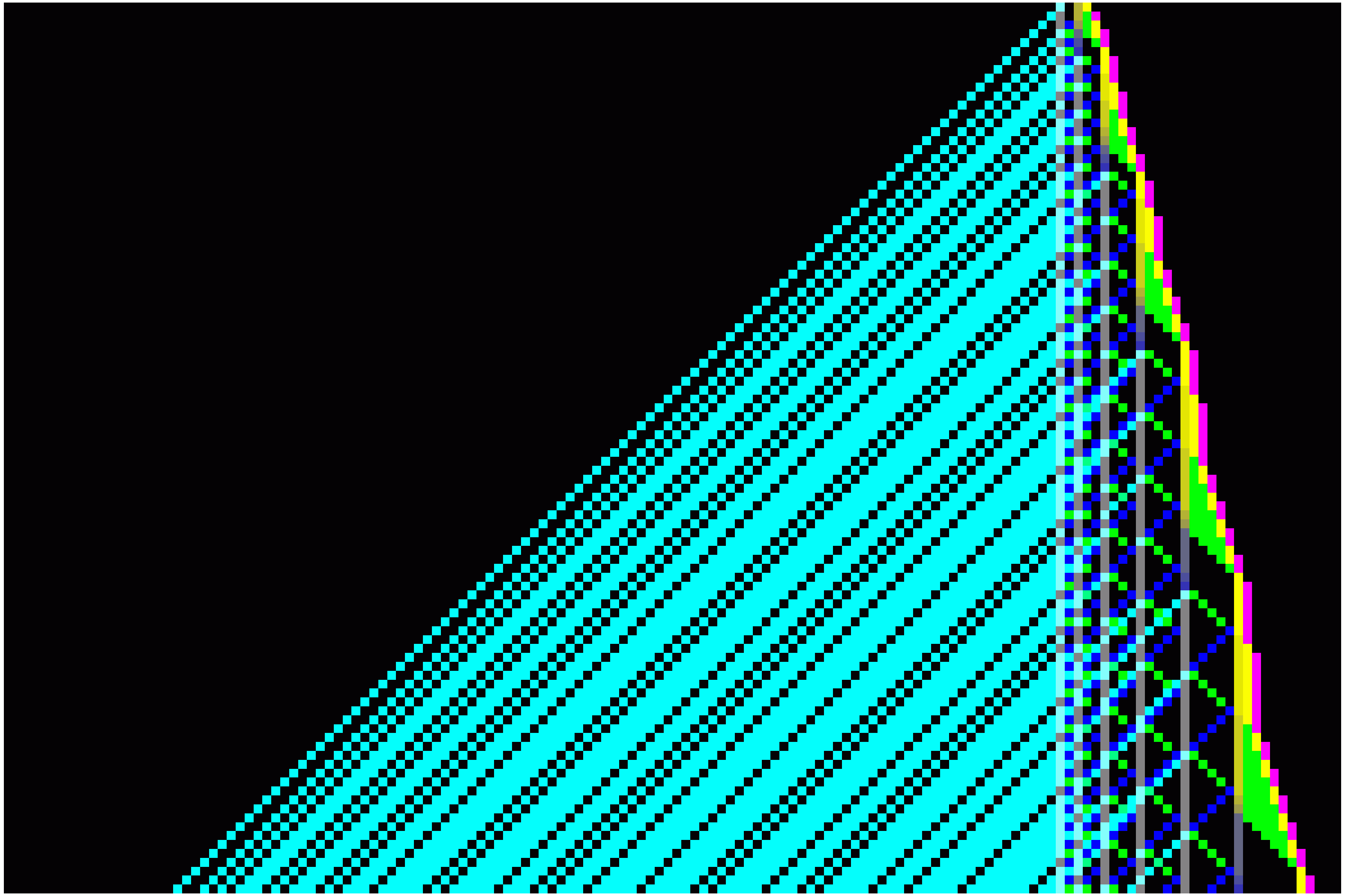
en.wikipedia.org

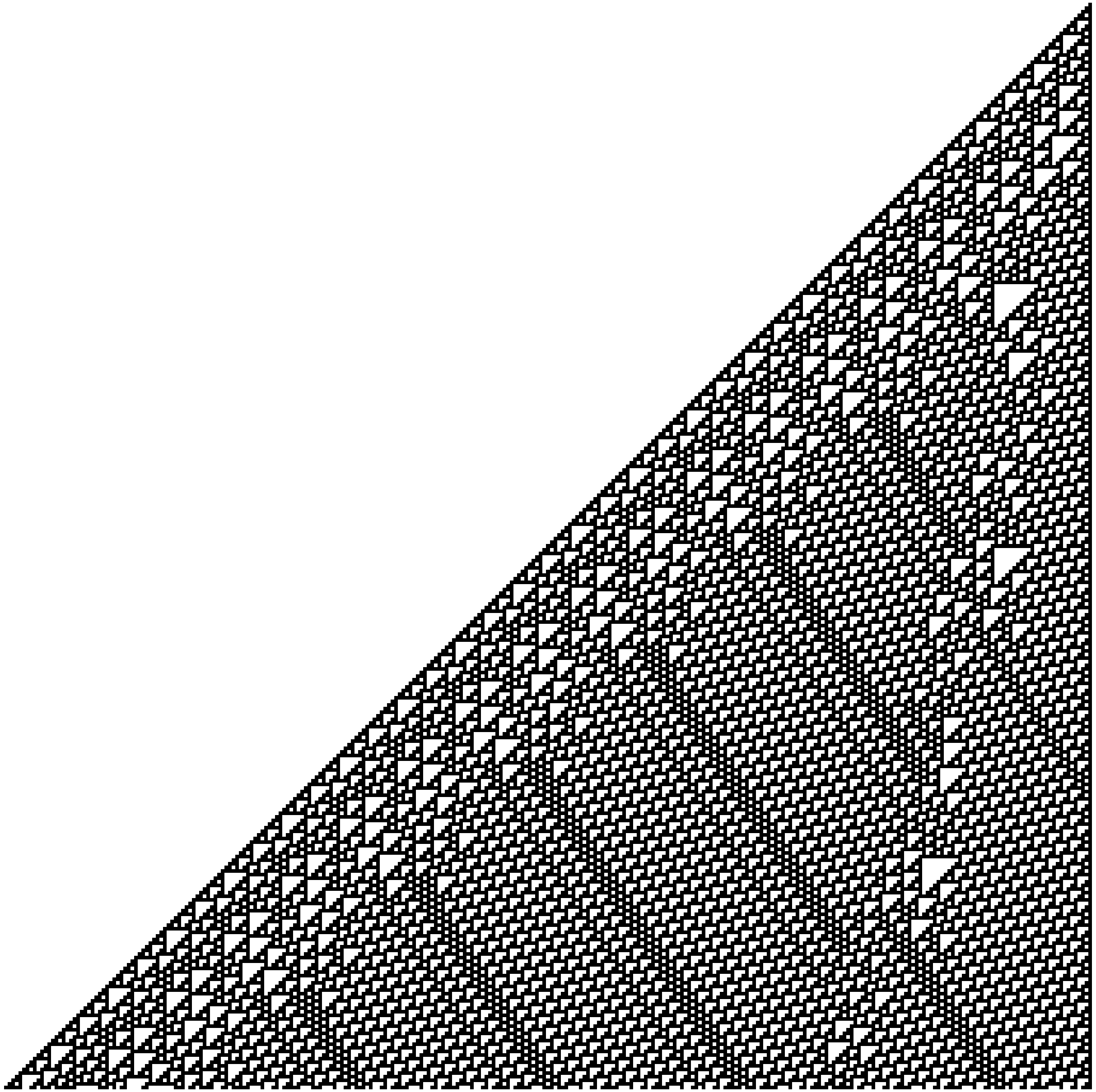
Conway's game of life

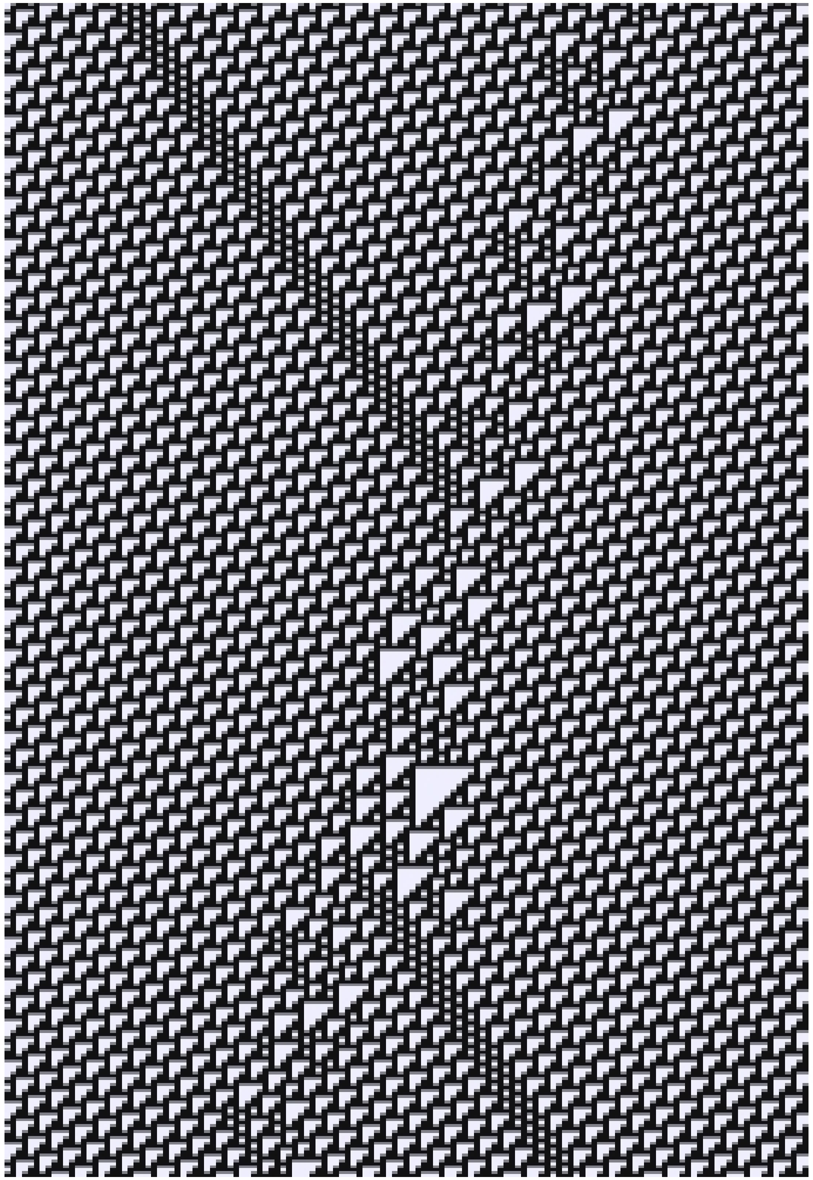
2-dim cellulaire automaat

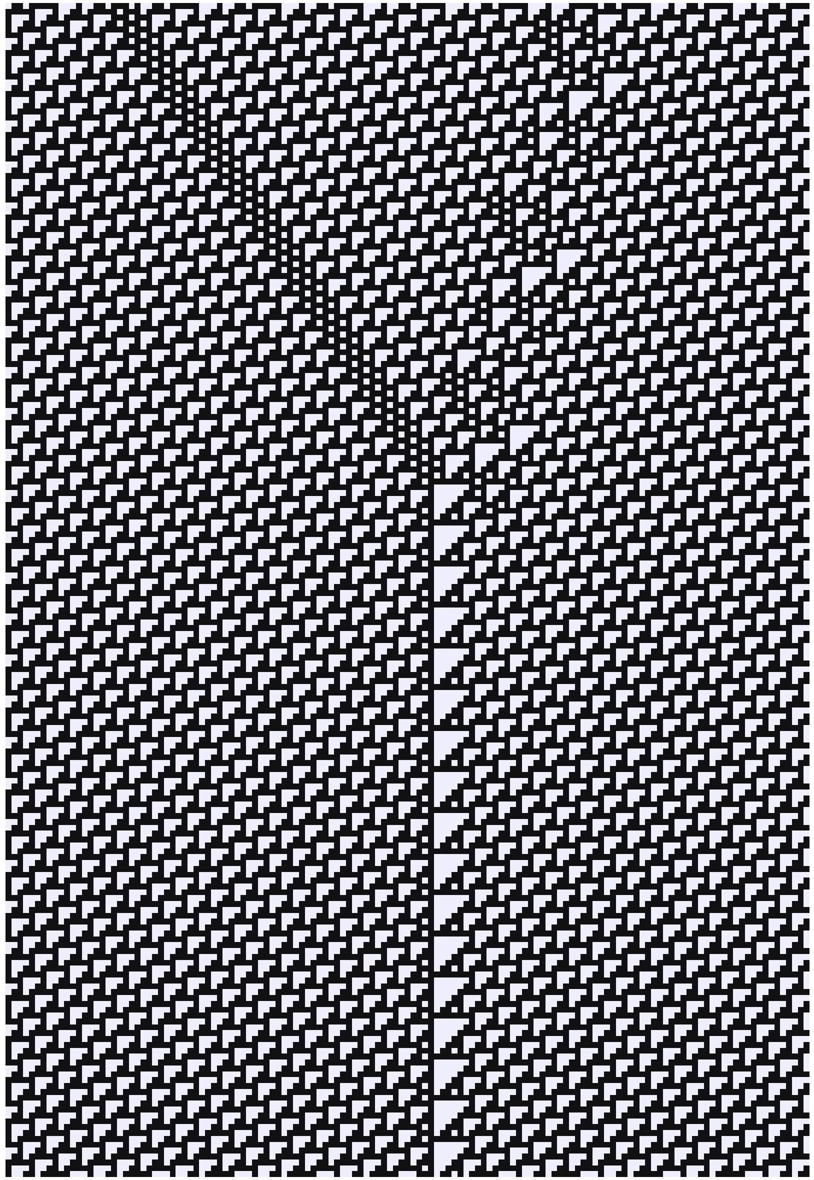

1 1 1
1 1 0
1 0 1
1 0 0
0 1 1
0 1 0
0 0 1
0 0 0
3 1 3
3 1 2
2 0 1
2 0 0
1 1 3
1 1 2
0 0 1
0 0 0
3
2
1
0


1 1 1 / 0
1 1 0 / 0
1 0 1 / 0
1 0 0 / 1
0 1 1 / 1
0 1 0 / 1
0 0 1 / 1
0 0 0 / 0
1 3 3 0
1 3 2 0
0 2 1 0
0 2 0 1
1 1 3 1
1 1 2 1
0 0 1 1
0 0 0 0
3
2
1
0









complex patterns
with simple rules

■ Turing and his Machine

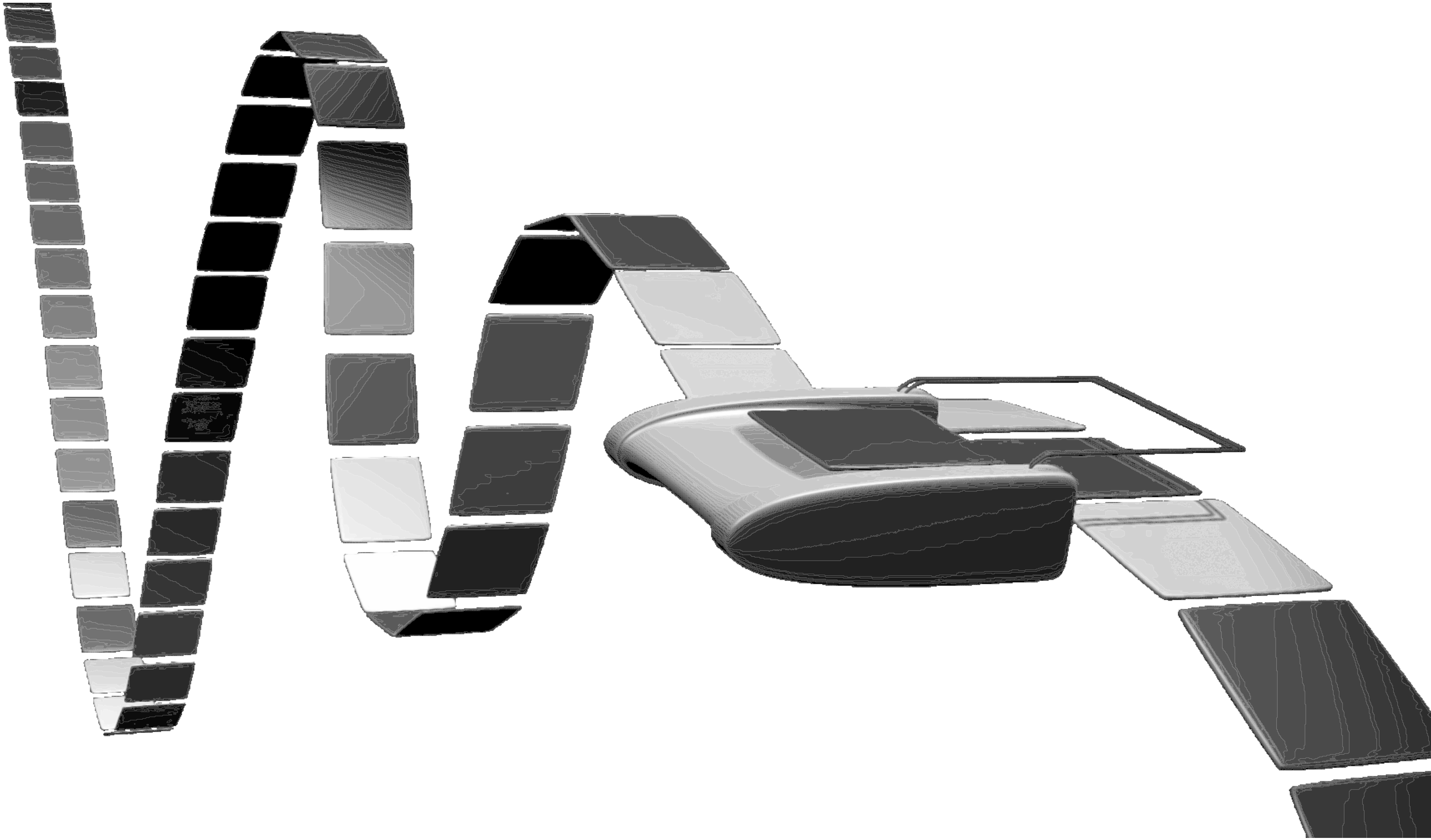
Alan Mathison Turing,
FRS OBE, 1912 – 1954

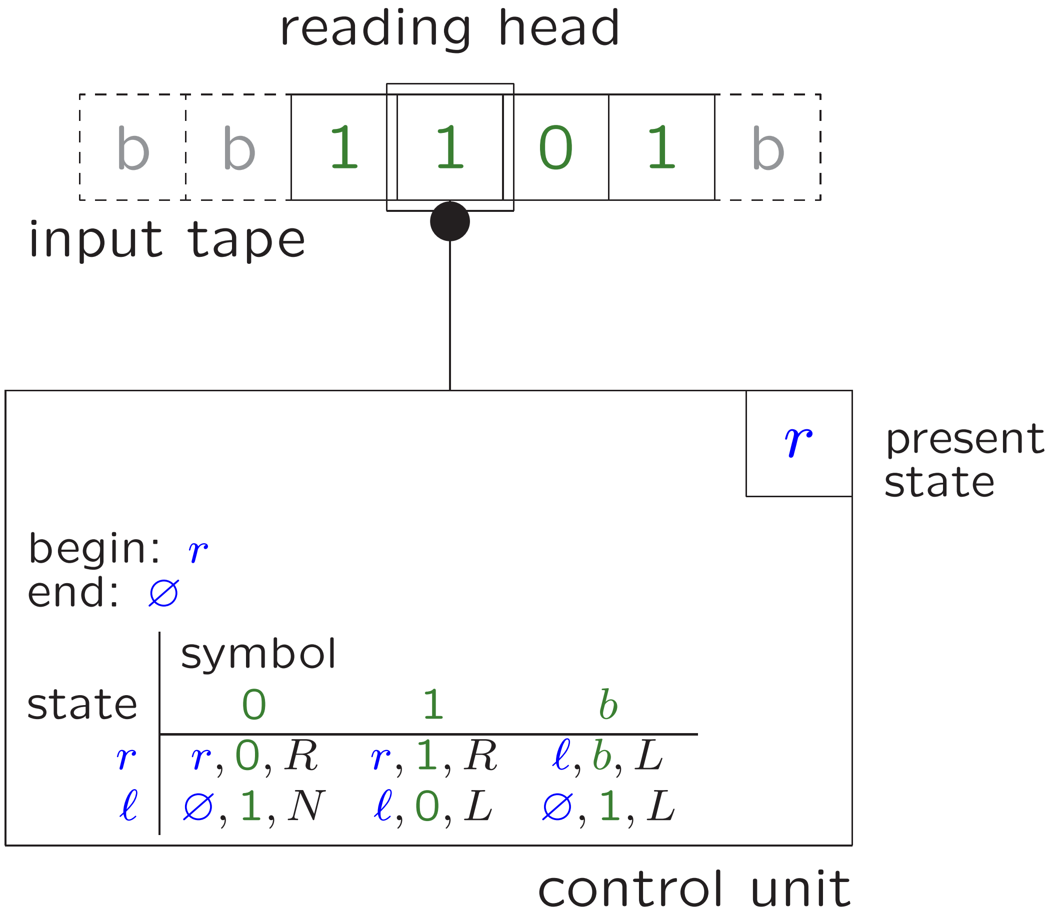
computability
whát can we compute?
Turing machine

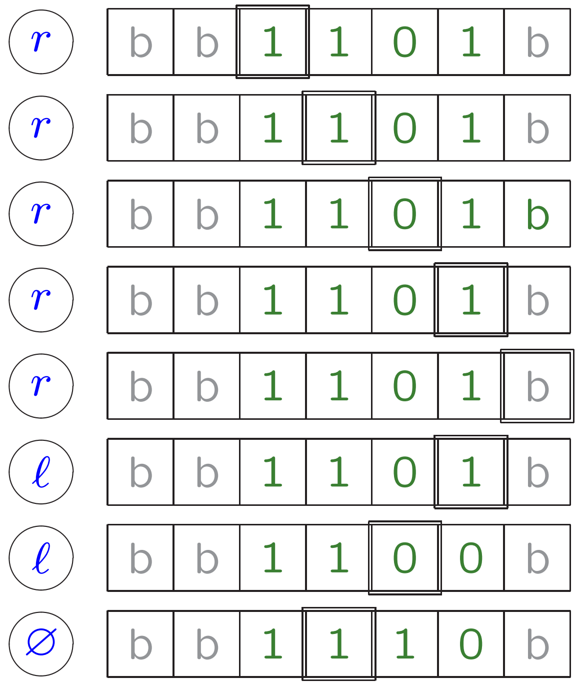
Enigma
breaking the code

artificial intelligence
Turing test

morphogenesis
pattern formation in biology







Turing, A. M. *On Computable Numbers, with an Application to the Entscheidungsproblem.* Proc. London Math. Soc. Ser. 2 42, 230-265, 1937.

$$\begin{aligned} &\text{Champernowne constant} \\ &\quad = 0.12345678910111213141516171819202122\ldots \end{aligned}$$

Sloane A033307

$$\sqrt{2} = 1.41421356237309504880168872420969807\ldots$$

Sloane A002193

$$\pi = 3.14159265358979323846264338327950288\ldots$$

Sloane A000796

is every number computable?

does every number have a name?

“It is possible to invent a single machine which can be used to compute any computable sequence. If this machine $\mathcal{U}$ is supplied with a tape on the beginning of which is written the S.D. [=description] of some computing machine $\mathcal{M}$, then $\mathcal{U}$ will compute the same sequence as $\mathcal{M}$.”

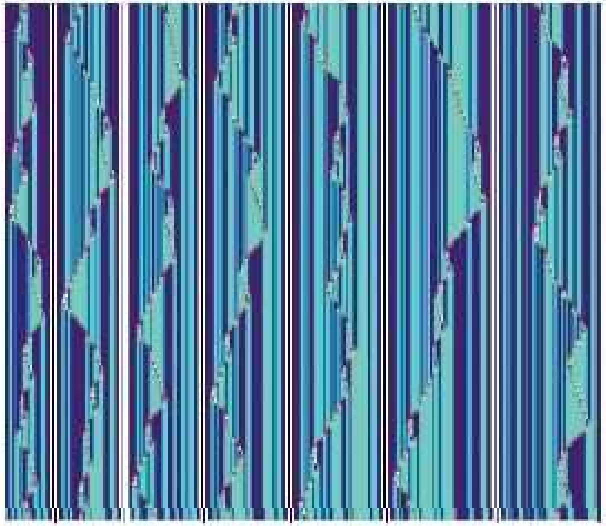
NRC 30.10.07

Student wint prijs met bewijs

Het was een probleem voor de fijnproevers onder de informatici: wat is de kleinste universele Turing-machine? Een student gaf antwoord, in 44 pagina's.

Door Ionica Smeets

ROTTERDAM, 30 OKT. Alex Smith, een twintigjarige student informatica en elektronica uit Birmingham, heeft een openstaand probleem uit de informatica opgelost. Hij won daarmee vorige week 25.000 dollar. Smith bewees dat een zeer eenvoudige machine elke mogelijke berekening kan uitvoe-

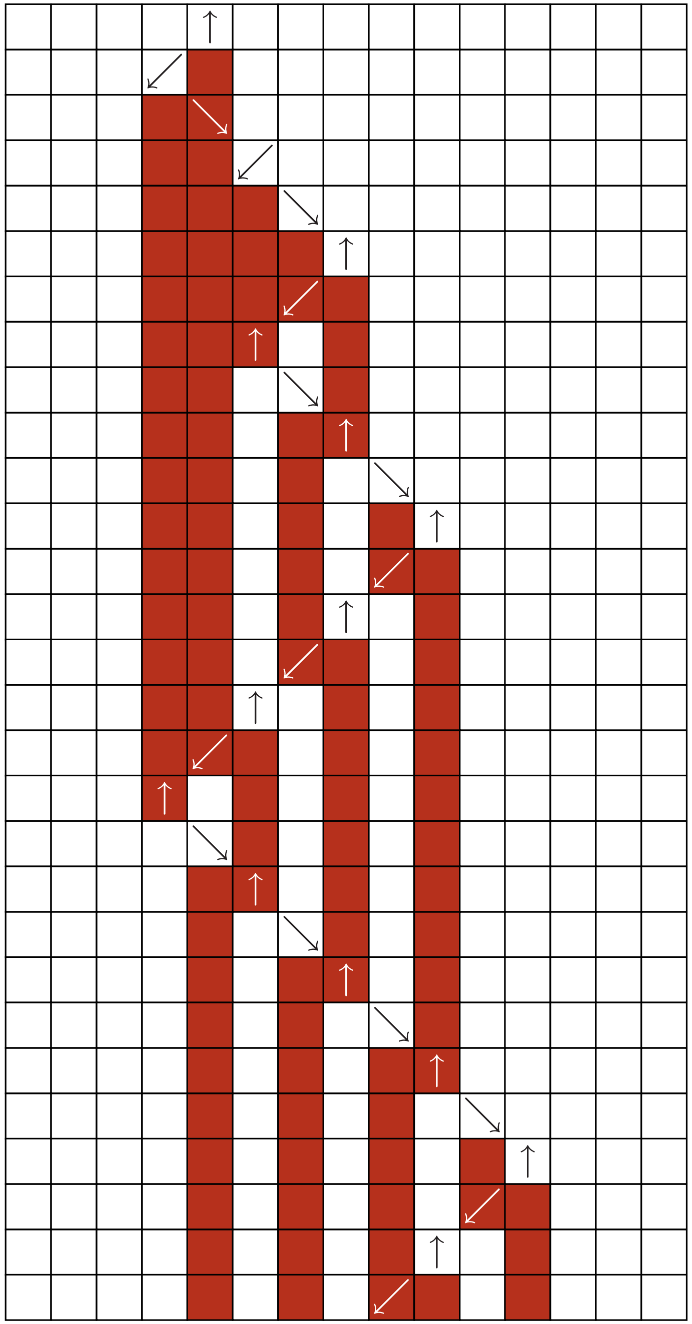


Schematische weergave van complexe berekening met een simpele Turing-machine. (Foto WI)

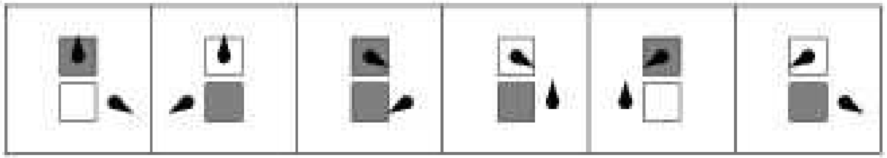
bepalen samen met de symbolen op de strook de stappen die de machine maakt.

Zo'n machine, schreef Wolfram in 2002, zou de kleinst mogelijk universele Turing-machine zijn. En in een ruim veertig pagina's tellend bewijs toont student Smith nu, tot zijn eigen verrassing, aan dat die bewering inderdaad juist is. In eerste instantie geloofde Smith juist dat hij kon bewijzen dat de machine niet universeel is, zo zei hij in een telefoongesprek met Wolfram.

Aan de informaticapraktijk verandert dit bewijs niet zoveel, denkt Peter van Emde Boas, lector in de mathematische informatica aan het Institute for Logic, Language en Computation van de Universiteit van Amsterdam. „Mooi dat die jongen dit probleem heeft



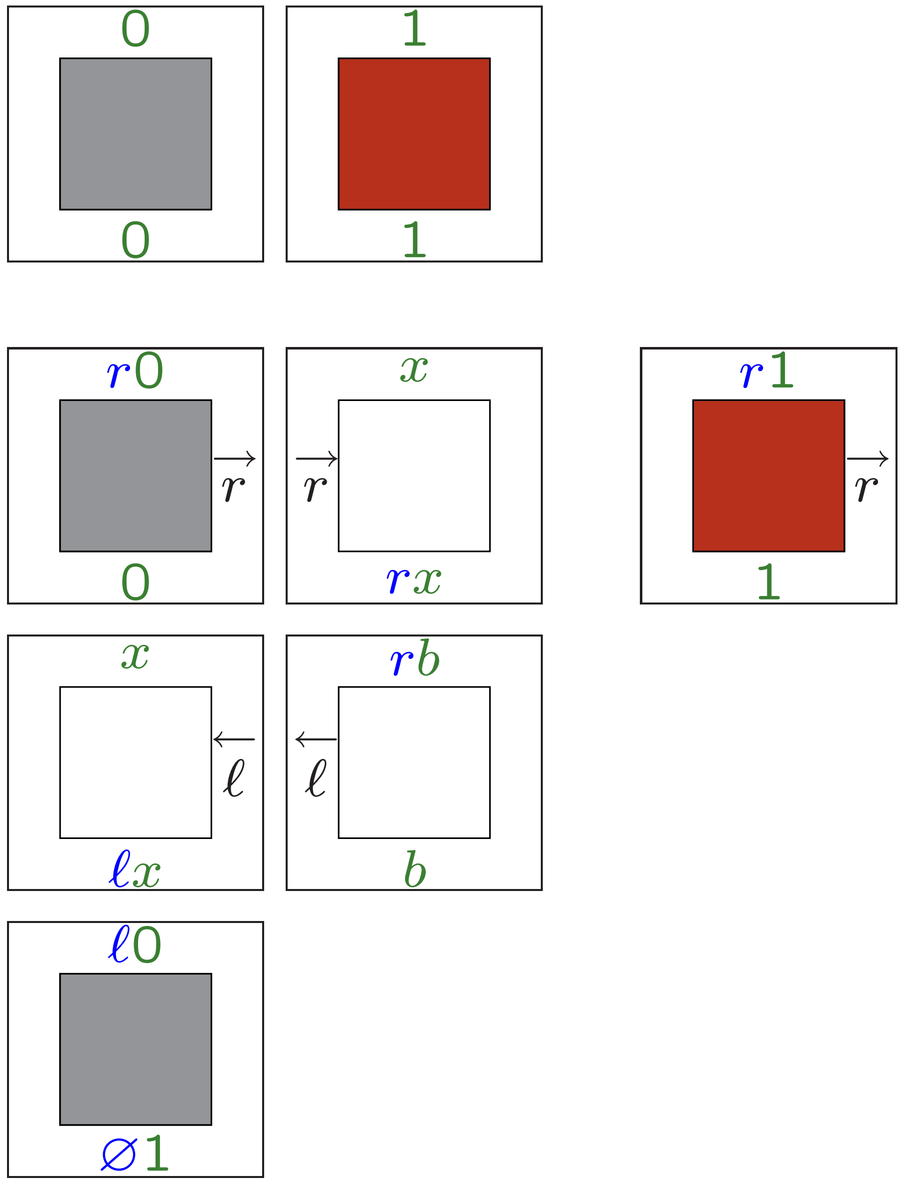
3 states,
2 symbols (colours)



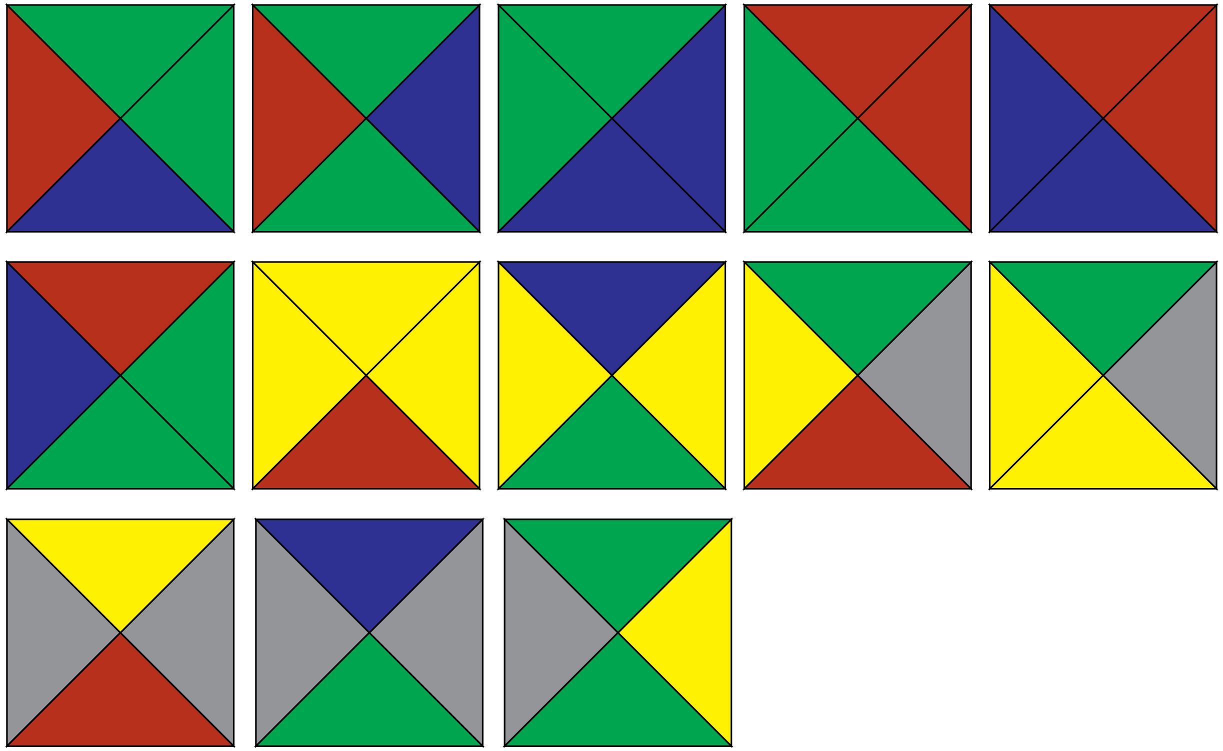
	0	1
$\uparrow$	$\swarrow, 1, L$	$\searrow, 0, R$
$\swarrow$	$\searrow, 1, R$	$\uparrow, 0, L$
$\searrow$	$\uparrow, 1, R$	$\swarrow, 1, R$

r0	1	0	1	1	B
0	r1	0	1	1	B
0	1	r0	1	1	B
0	1	0	r1	1	B
0	1	0	1	r1	B
0	1	0	1	1	rB
0	1	0	1	ℓ1	B
0	1	0	ℓ1	0	B
0	1	ℓ0	0	0	B
0	1	1	0	0	B

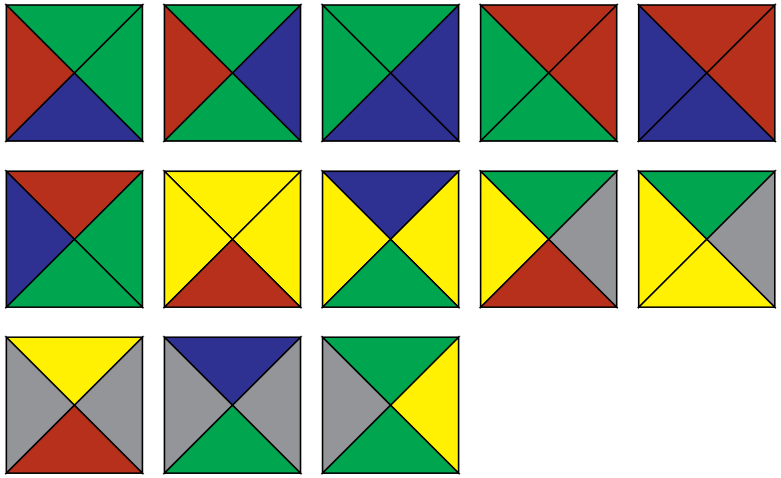
	0	1	b
r	$r, 0, R$	$r, 1, R$	ℓ, b, L
ℓ	$\varnothing, 1, N$	$\ell, 0, L$	$\varnothing, 1, L$



tiles are programmable



Wang tiles, 1961



pattern without regularity (hard)

Karel Culik II, 1996

input: set of tiles

question: is there a proper tiling of the plane (of a rectangle) ?

there is no algorithm that solves this problem

Berger 1966

(really!)

tiles are unpredictable

Bedankt!