

introducing

Membrane Computing

Models of Computation
Utrecht 7 june 2012

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Leiden Center for
Natural Computing

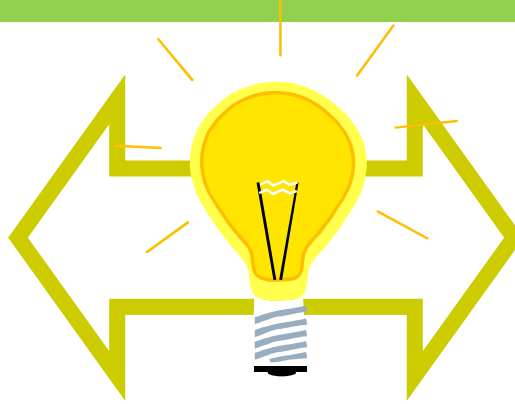
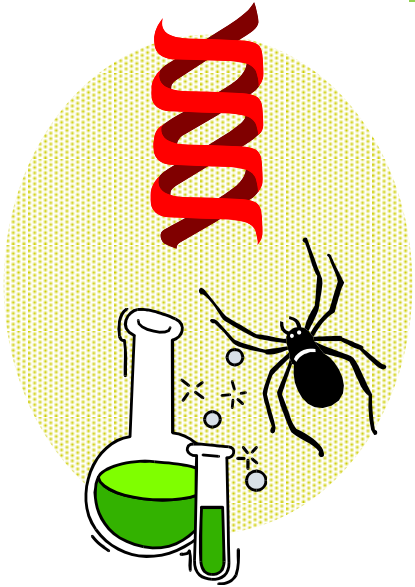
natural computation

dna sorting &
protein networks

understanding nature
as a computational process

neural netw &
genetic alg

bio inspired computing



bio hardware

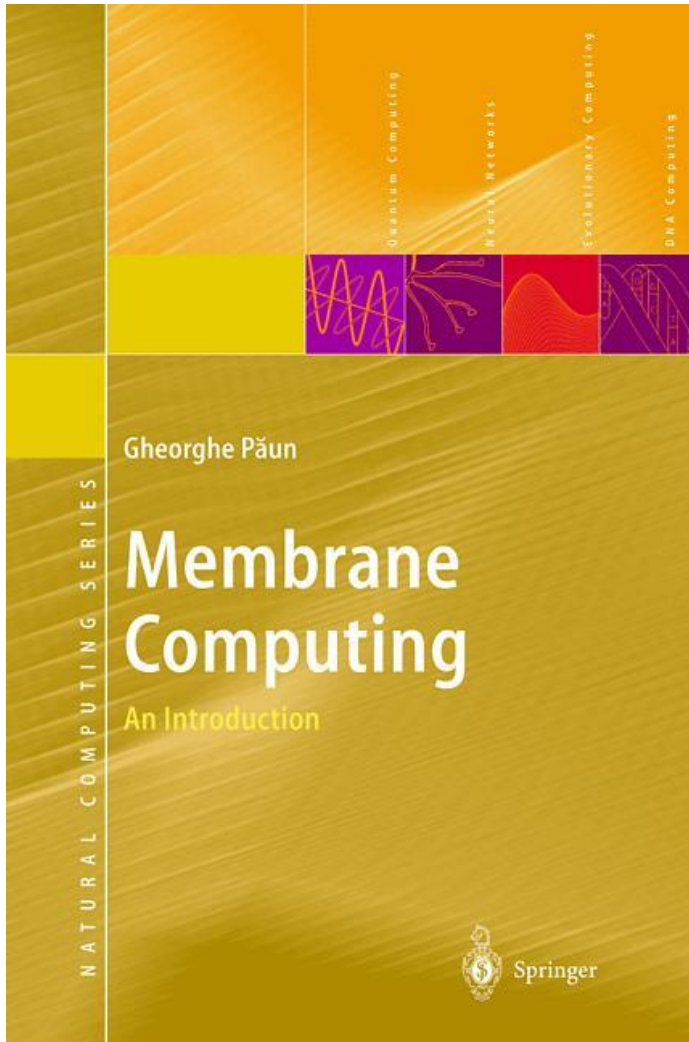
DNA computing

bio-informatics

gene finding

Membrane Computing

introducing some features
(almost) no formalisms
no (real) proofs



Gheorghe Păun

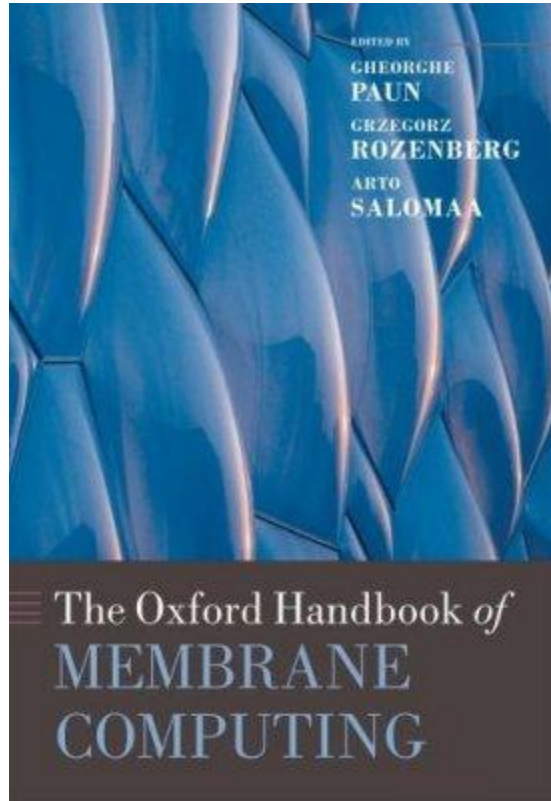
Membrane Computing

An Introduction

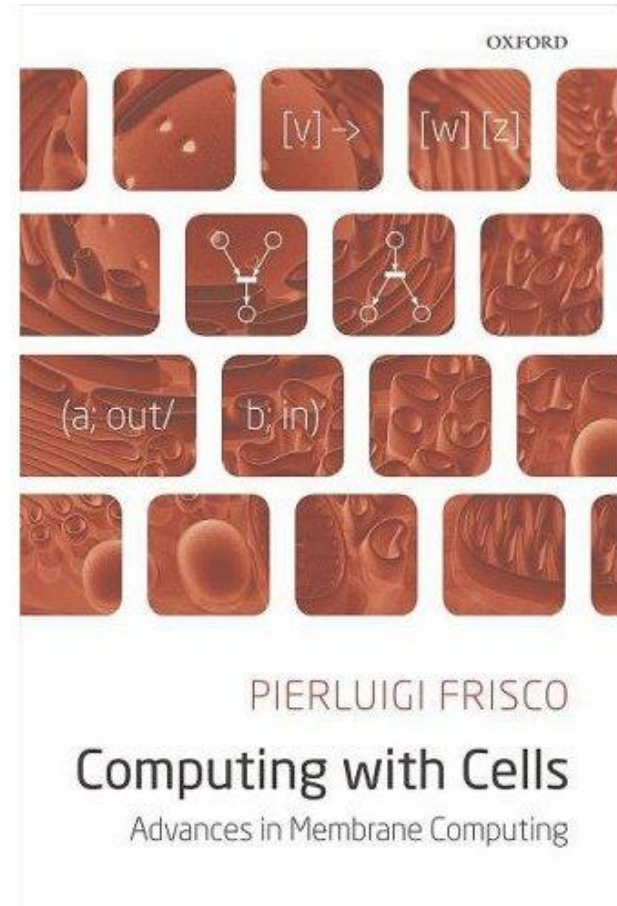
Natural Computing Series

Springer 2002

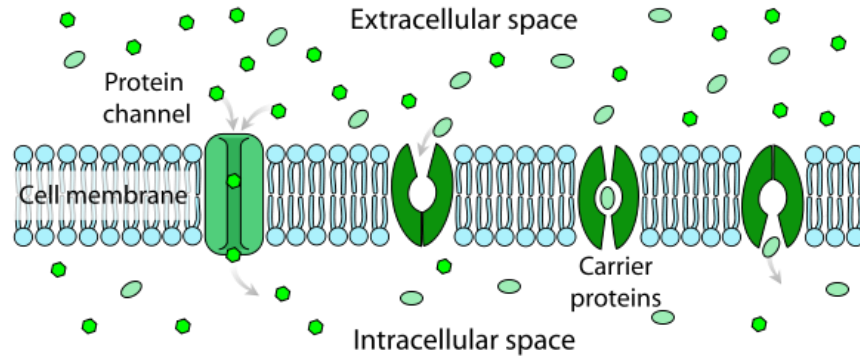
ISBN 3-540-42601-4



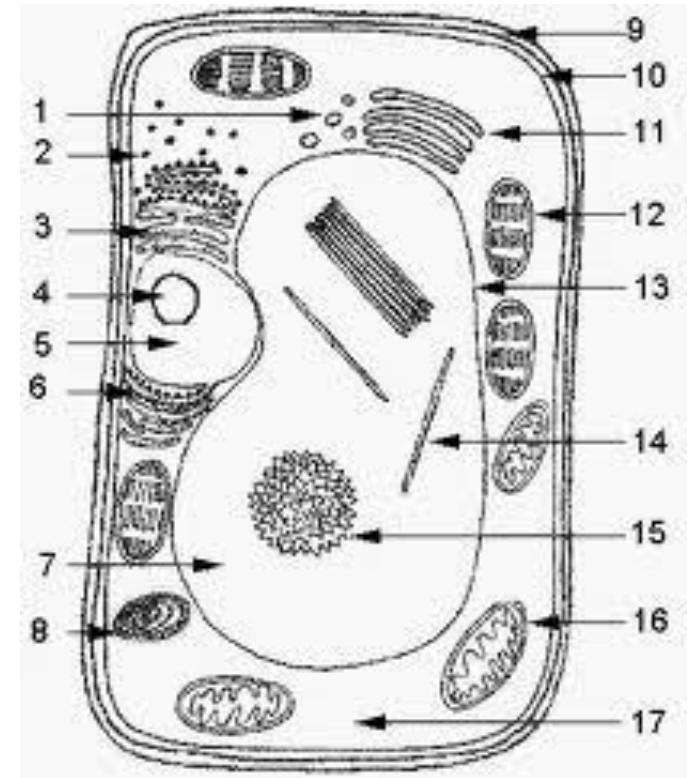
(2010)



(2009)



nested compartments
- information
membranes - communication



Places & Tokens

before that we had Petri nets

petri nets

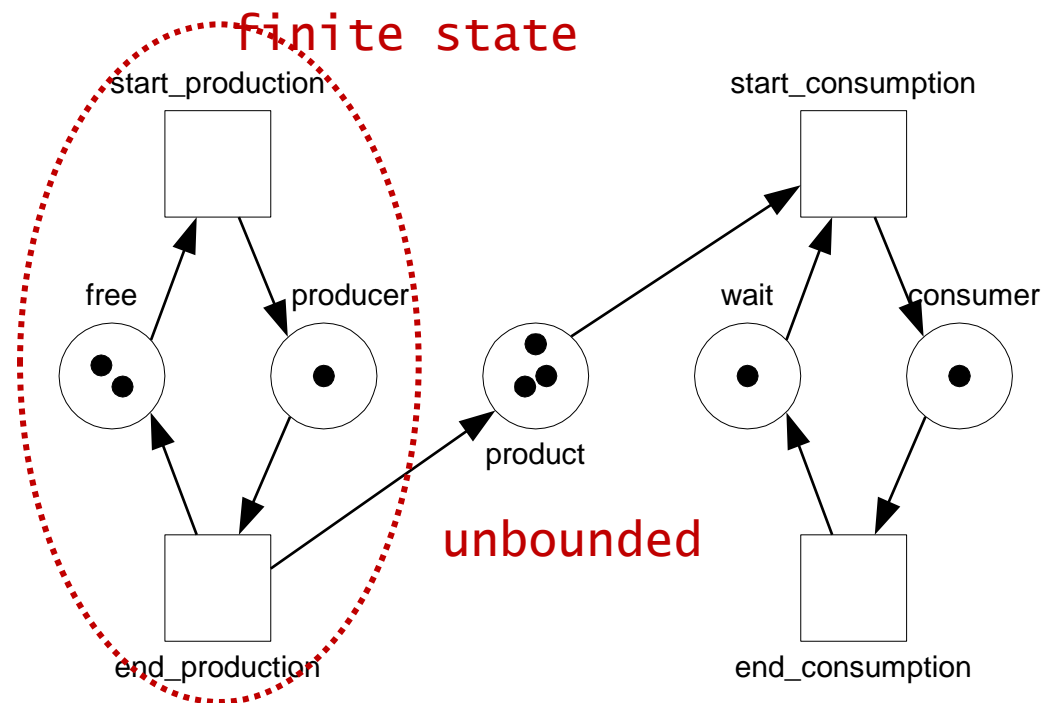
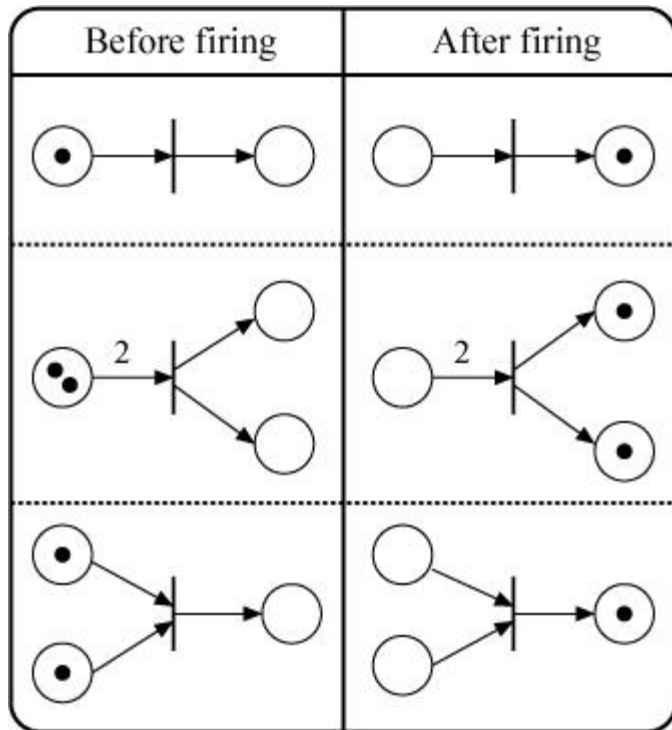
circles
boxes

places + tokens
transitions

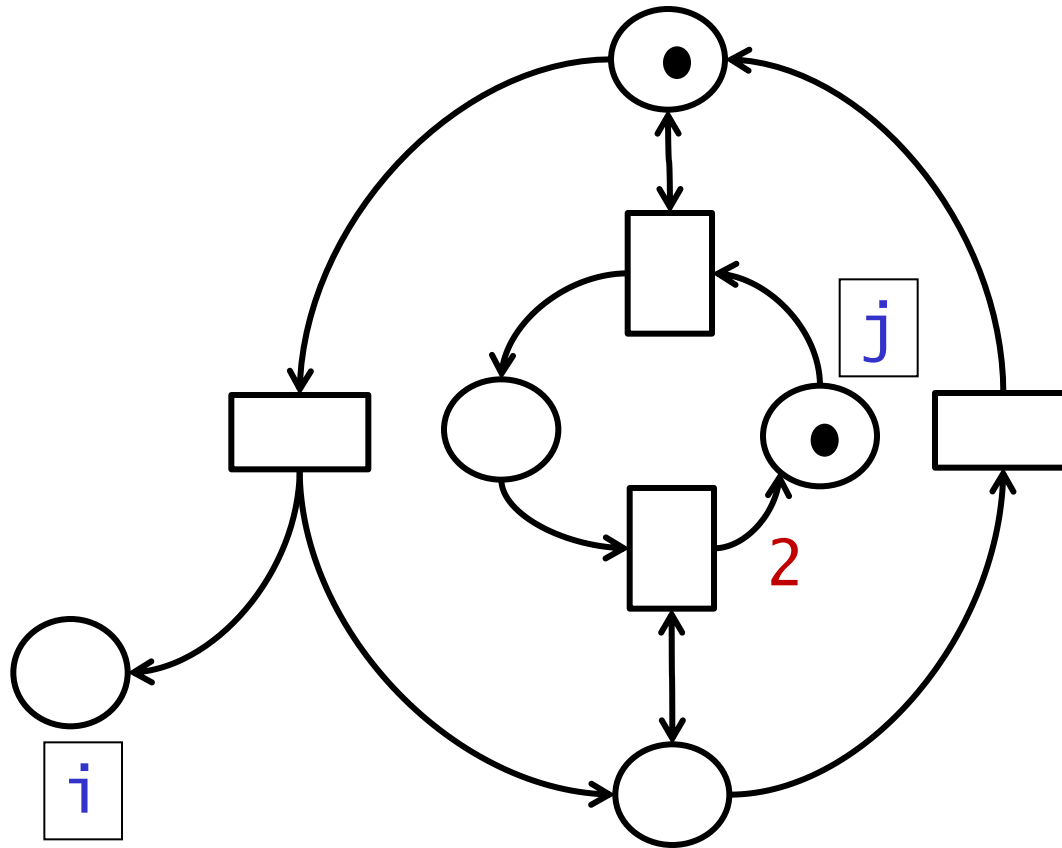
resources
actions

producer
three units

consumer
two units

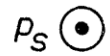


computing with petri net

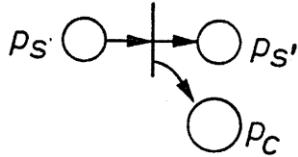


$$\{ (i, j) \mid j \leq 2^i \}$$

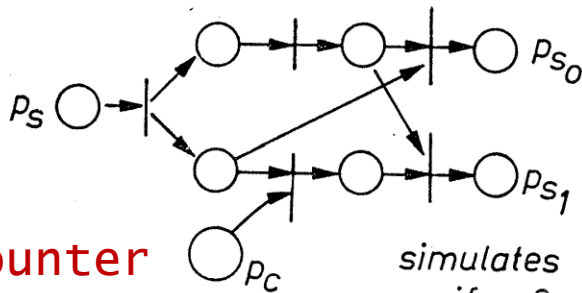
maximal strategy



simulates "start in state s "

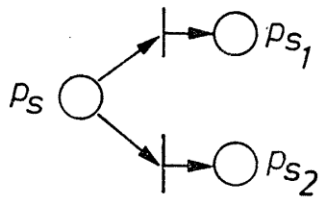


simulates
" $s: c := c + 1; \underline{\text{goto}} s';$ "



simulates zero-testing:
 $s: \underline{\text{if}} c = 0 \underline{\text{then}} \underline{\text{goto}} s_0$
 $\underline{\text{else}} c := c - 1; \underline{\text{goto}} s_1;$

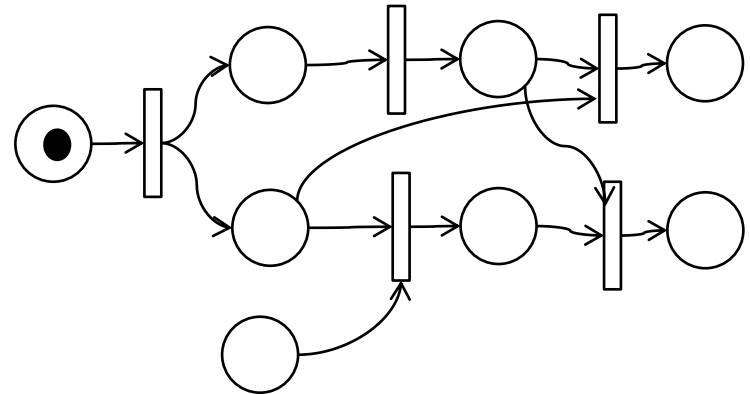
counter



simulates choice:
" $s: \underline{\text{goto}} s_1 \underline{\text{or}} s_2;$ "



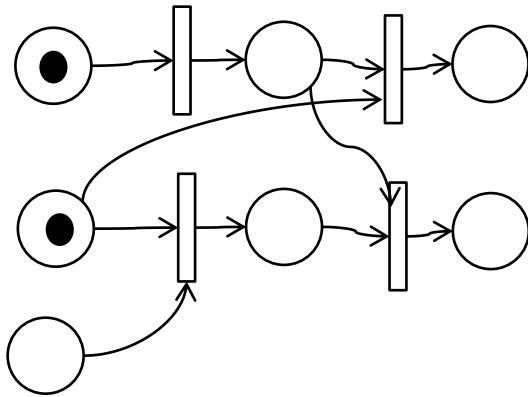
simulates " $s: \underline{\text{halt}}$ "
(no transition is allowed
to take a token from
the place p_{HALT})



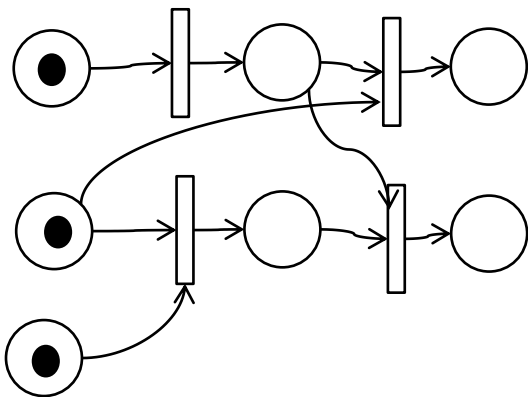
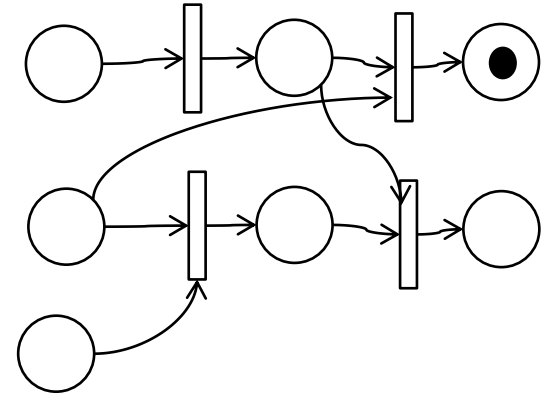
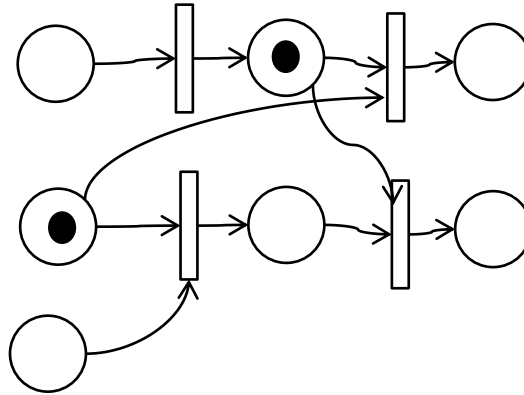
counter

Burkhard – Ordered firing in Petri Nets, EIK 1981

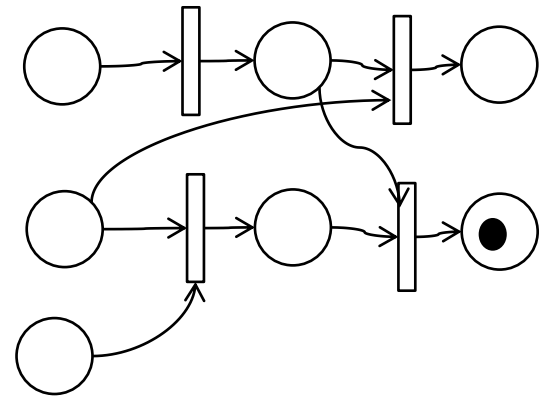
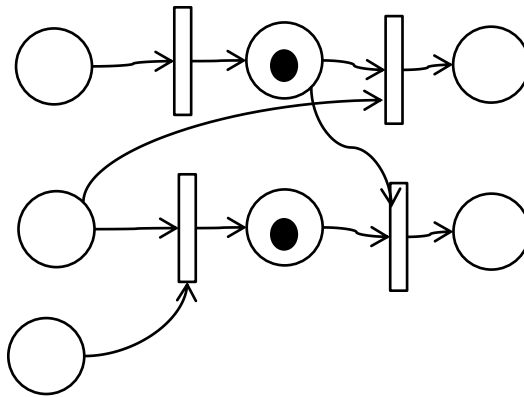
forced steps



counter empty

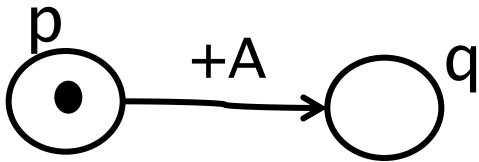


counter
at least one token

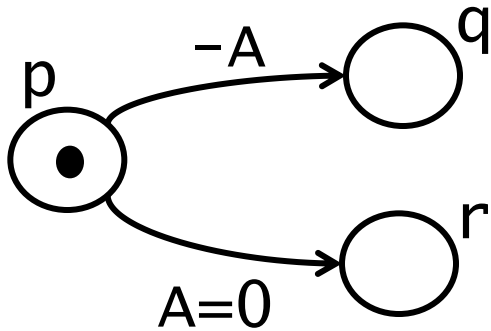


where is max par important?

about counters



counter automaton =
register machine
Minski, Fischer
FRACTRAN

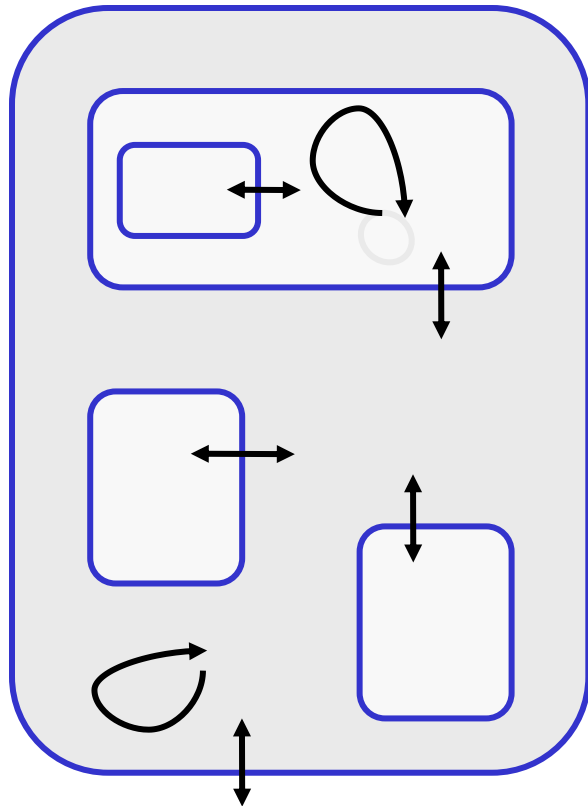


states + instructions
 $+A$, $-A$, $A=0$ zero test

petri net =
'partially blind' counter automaton
(no zero test)
Greibach, Petri

Membranes & Objects

Membrane Computing



nested membranes

objects

- strings
- unstructured

rules

- communication

$ac \rightarrow ba_{out}c_{in}$

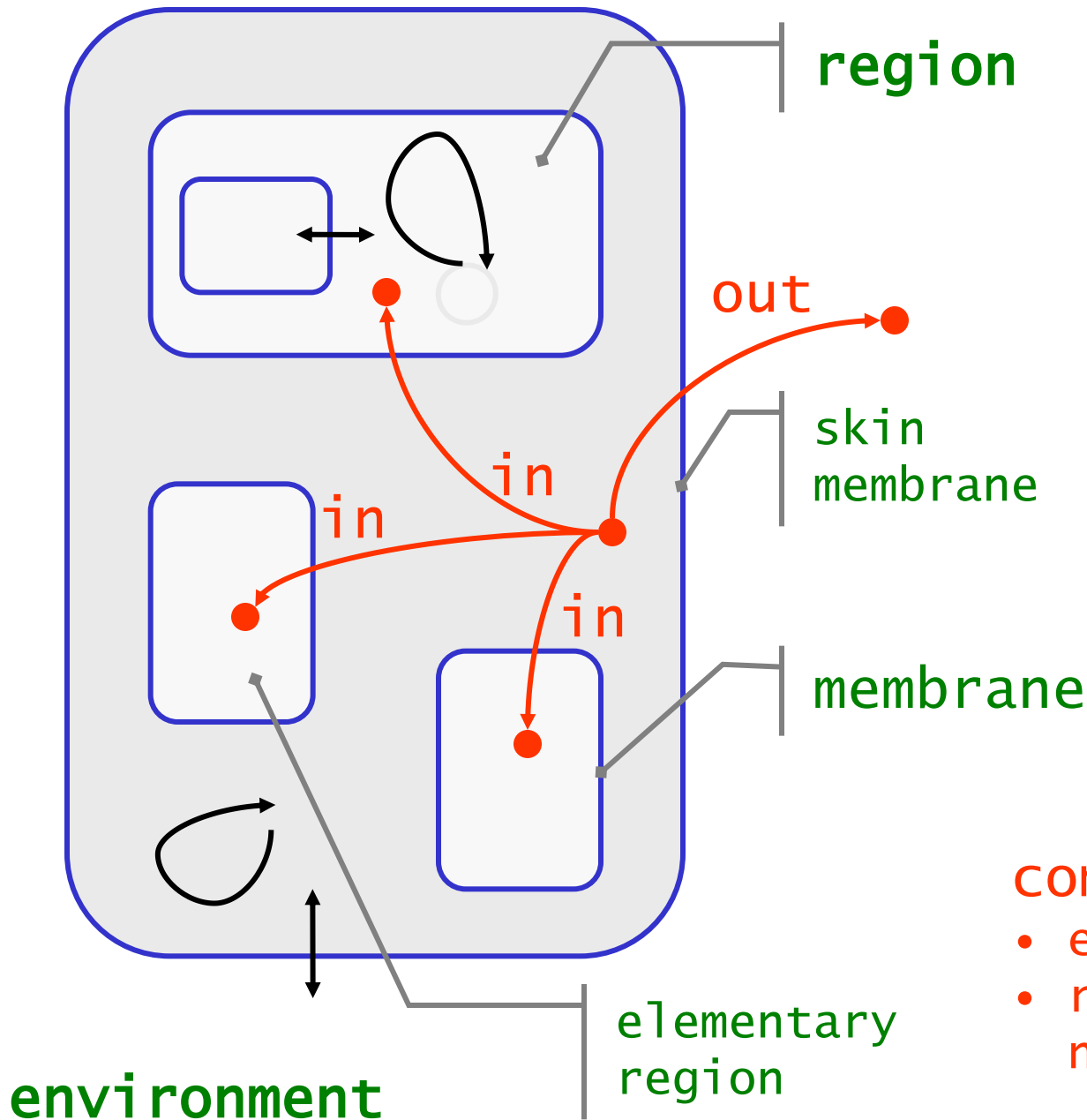
- evolution

rewriting

$(A \rightarrow aAb)_{out}$

splicing

Structure



- communication**
- explicit addressing
 - relative in/out
 - non-deterministic

what is generated?

where is the input/output?

objects inside membrane:
(vectors of) natural numbers

- acceptance of input (by halting)
- generating output 'enumerable'
needs nondeterminism

Carriers are usually saturable monomeric proteins, which bind the transported solutions with high specificity, and move them at low flow rates. **Channels** are usually oligomeric complexes (with helical transmembrane domains or barrel-shaped structures), which show less stereospecificity than carriers but larger transport rates.

(p.38) Cell Biology for Membrane Computing

Carriers are usually saturable monomeric proteins, which bind the transported solutions with high specificity, and move them at low flow rates. **Channels** are usually oligomeric complexes (with helical transmembrane domains or barrel-shaped structures), which show less stereospecificity than carriers but larger transport rates.

(p.38) Cell Biology for Membrane Computing

Carriers & Objects

pure communication
maximal parallelism

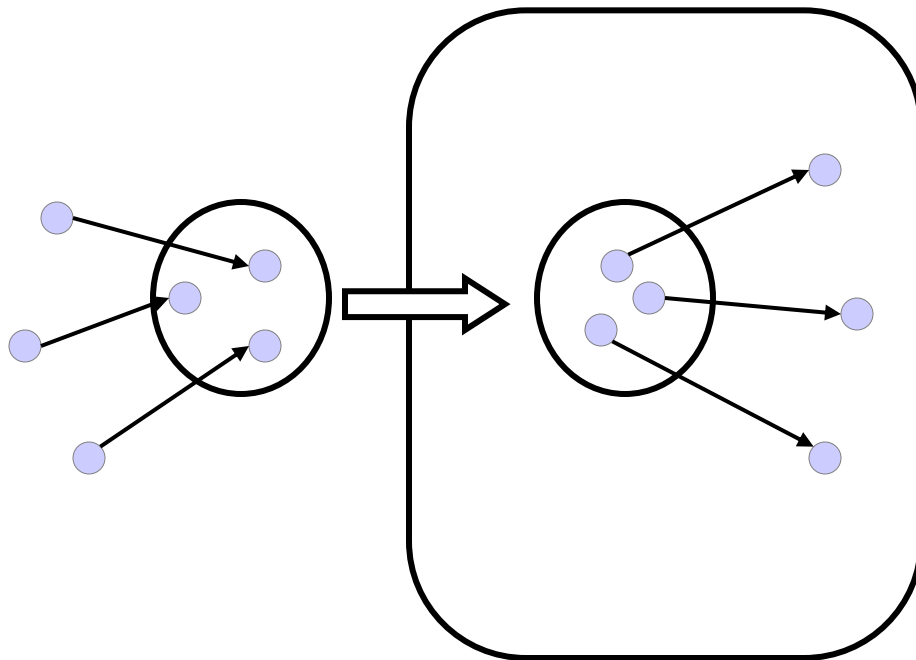
Carriers and Counters (DLT2002)

P systems with Carriers vs. (Blind) Counter Automata

P systems with carriers

Martín-Vide Păun Rozenberg

Contents



• objects

multiset symbols
infinite supply
in environment

• carriers

finite number

Rules

$va_1 \dots a_k \rightarrow v[a_1 \dots a_k]$

attaching

$v[a_1 \dots a_k] \rightarrow in$

carry in

$v[a_1 \dots a_k] \rightarrow out$

carry out

$v[a_1 \dots a_k] \rightarrow va_1 \dots a_k$

detaching

$v a_1 \dots a_k \leftrightarrow v[a_1 \dots a_k]$
 $v[a_1 \dots a_k] \rightarrow \text{in/out}$

Computations

evolving multisets
infinite supply environment
fixed carriers

maximal parallelism
halting by 'blocking'
counting objects

'output' membrane

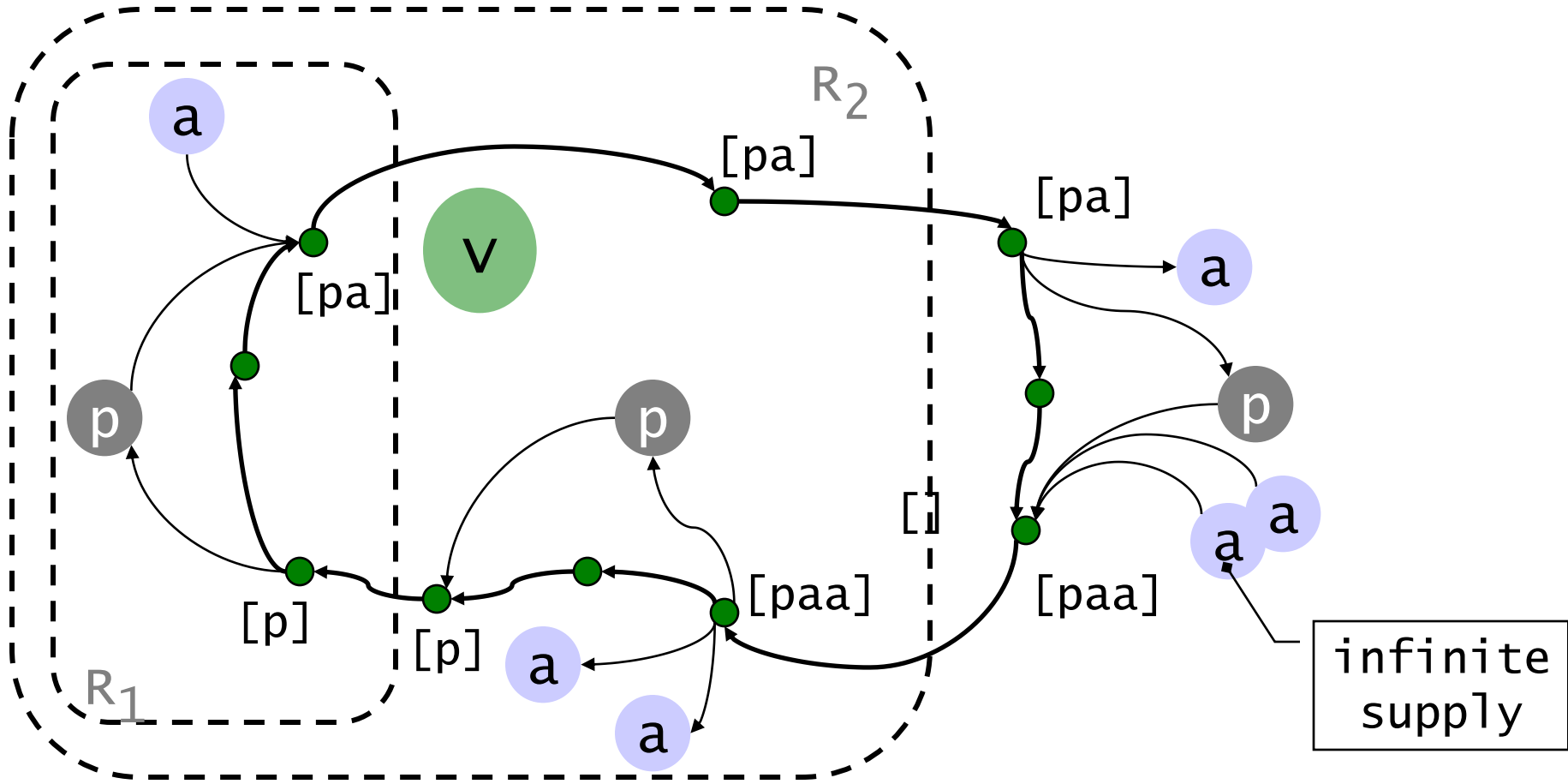
$\mathbb{N}^k \text{CP}_m(c, p)$

- membranes
- carriers
- passengers

(here $k=1$)

P systems with carriers

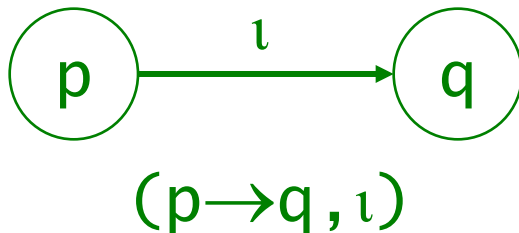
example



E	R ₂	R ₁
$v[pa] \rightarrow vpa$	$vp \rightarrow v[p] \rightarrow in$	$v[p] \rightarrow vp$
$vpaa \rightarrow v[paa] \rightarrow in$	$v[pa] \rightarrow out$	$vpa \rightarrow v[pa] \rightarrow out$
	$v[paa] \rightarrow vpaa$	

counter automata

Minsky, Fischer



ε	nil
$+A$	add one
$-A$	subtract one
$A=0$	zero test

several counters
acceptance by final state
& empty counters

NRE

output counter

Recursively Enumerable sets (of numbers)

blind counter automata

Greibach

no zero test,
except final test for acceptance
Petri nets

NBC

DLT'02 paper

MaVi-Pău-Roz '02

$$\mathbb{NRE} = \mathbb{NCP}_2(3, 3)$$

$$\begin{aligned}\mathbb{NRE} &= \mathbb{NCP}_1(2, 3) \\ &= \mathbb{NCP}_1(*, 2)\end{aligned}$$

$$\begin{aligned}\mathbb{NBC} &= \mathbb{NCP}_*(1, *) \\ &= \mathbb{NCP}_1(1, 3)\end{aligned}$$

$$\mathbb{NRE} = \mathbb{NCP}_*(*, 1)$$

$\mathbb{NCP}_m(c, p)$

- membranes
- carriers
- passengers

1. single membrane

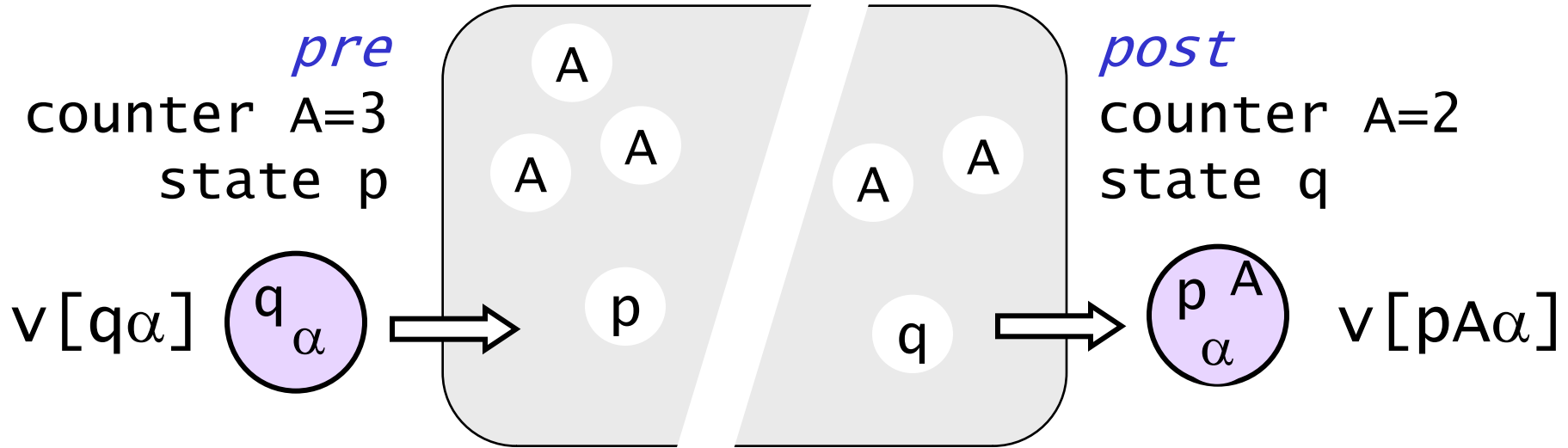
2. single carrier

3. single passenger

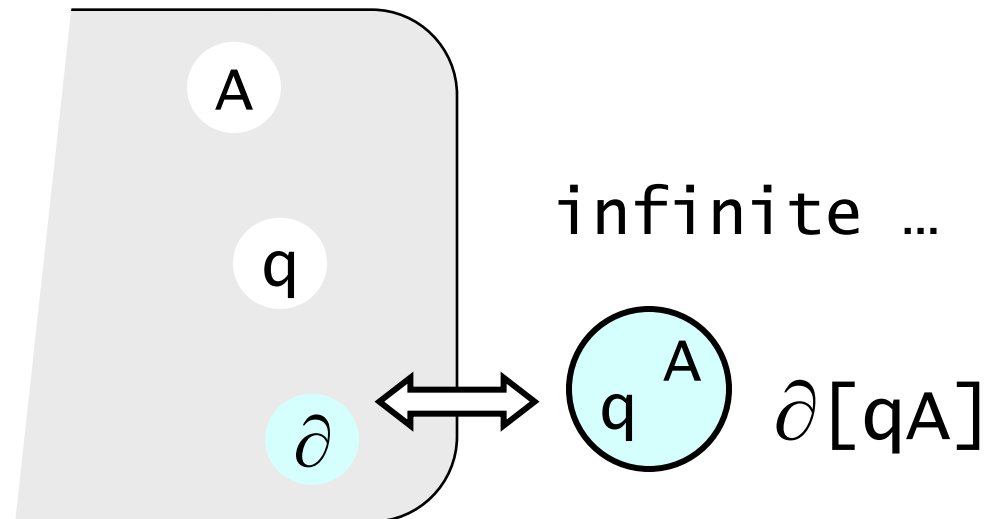
1. single membrane

$$\mathbb{NRE} \subseteq \mathbb{NCP}_1(2, 3)$$

$$\alpha: (p \rightarrow q, -A)$$



$$\alpha: (p \rightarrow q, A=0)$$



2. single carrier

P without parallelism

carriers + objects

P system halting:

no applicable rules

blind counter aut.

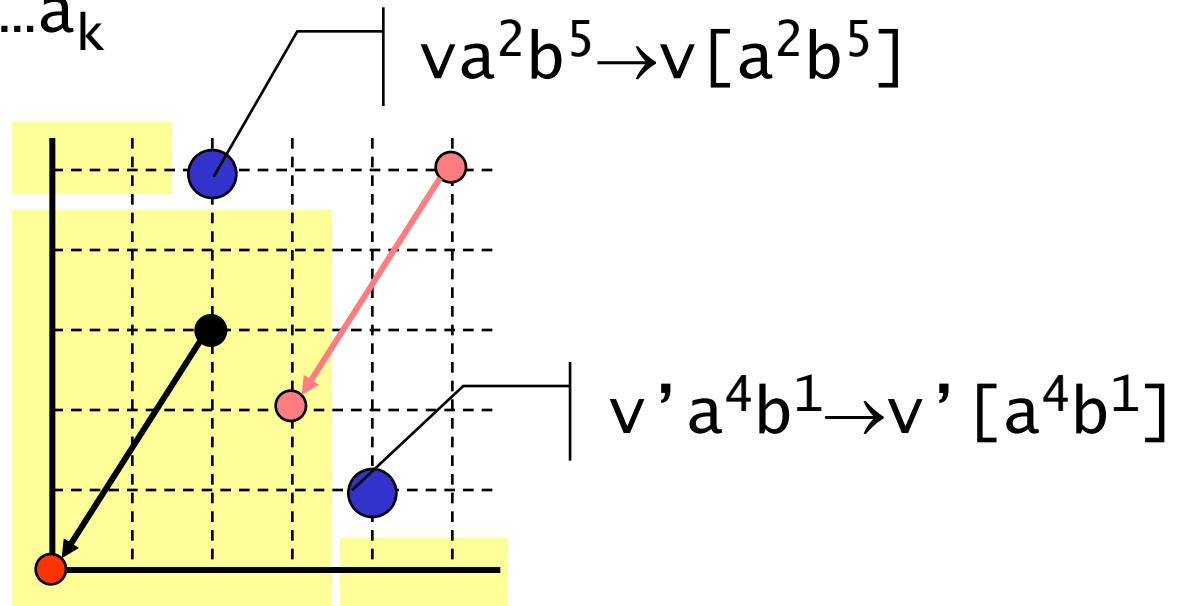
state + counters

final state

& empty counters

- $va_1 \dots a_k \rightarrow v[a_1 \dots a_k]$
- $v[a_1 \dots a_k] \rightarrow \text{in/out}$
- $v[a_1 \dots a_k] \rightarrow va_1 \dots a_k$

guess vector
& test by zero
acceptance



2. single carrier

$$\mathbb{N}BC = \mathbb{N}CP_1(1, 3)$$

$$\mathbb{N}BC \subseteq \mathbb{N}CP_1(1, 3)$$

$$\mathbb{N}RE \subseteq \mathbb{N}CP_1(2, 3)$$

forget about 'zero test' $\hat{\partial}$

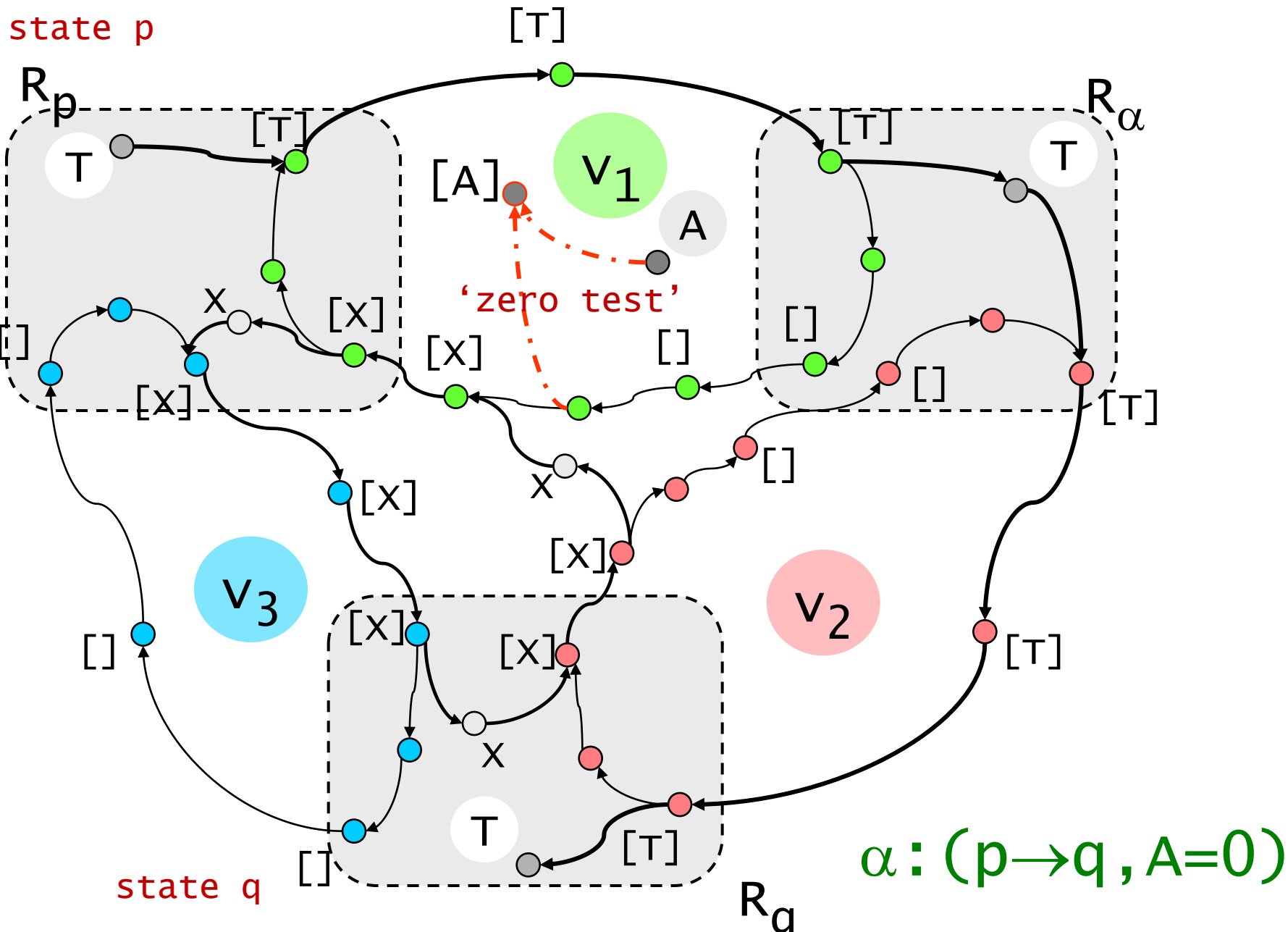
$$\mathbb{N}BC \supseteq \mathbb{N}CP_*(1, *)$$

no parallelism !

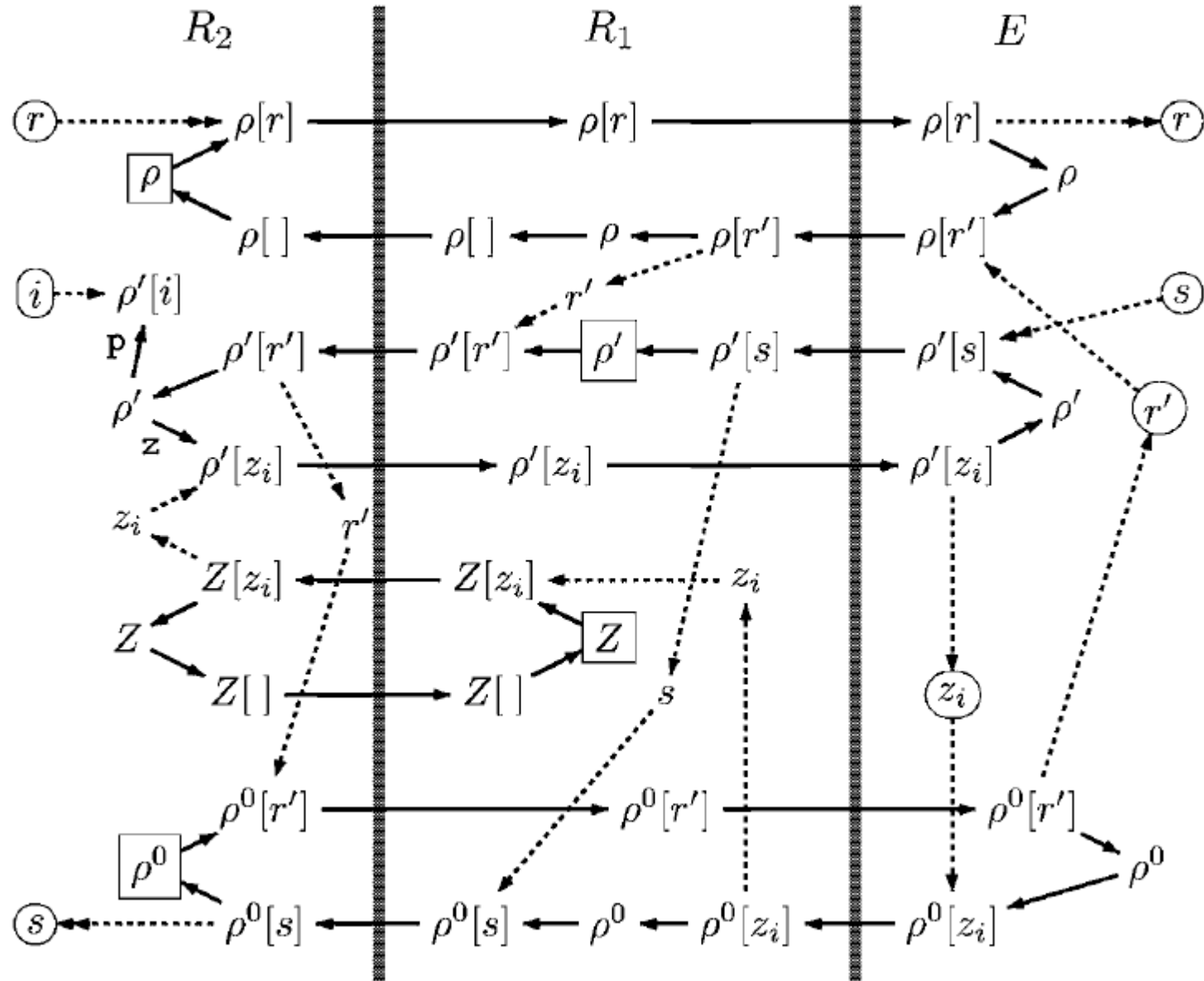
$\mathbb{N}BC$ single object semilinear \rightarrow regular
also with more objects $\{ (i, j) \mid j \leq 2^i \}$

3. single passenger

$$\mathbb{NRE} \subseteq \mathbb{NCP}_*(*, 1)$$



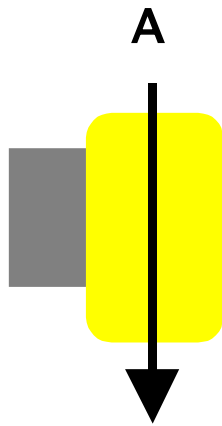
two membranes is ok



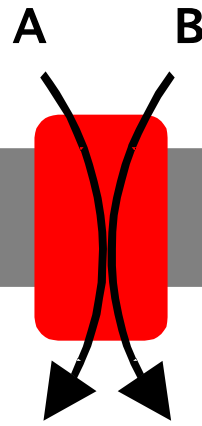
$$\mathbb{NRE} = \text{NCP}_2(*, 1)$$

Symport & Antiport

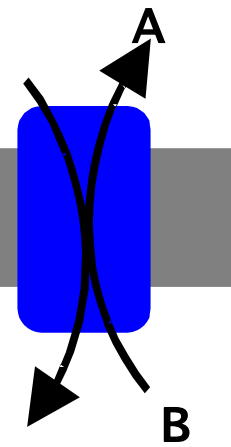
Uniport



Symport

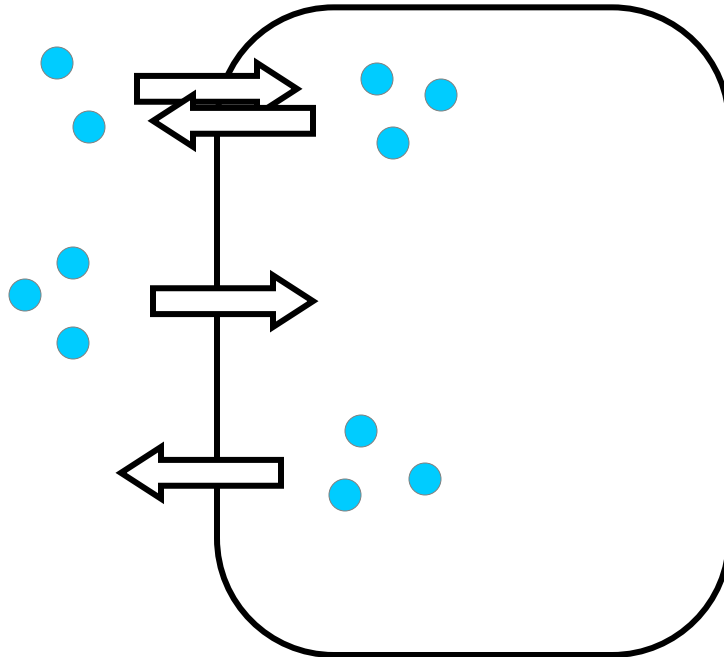


Antiport



P systems with symport/antiport

Păun & Păun



Contents

- objects
multiset symbols
infinite supply
in environment

Rules

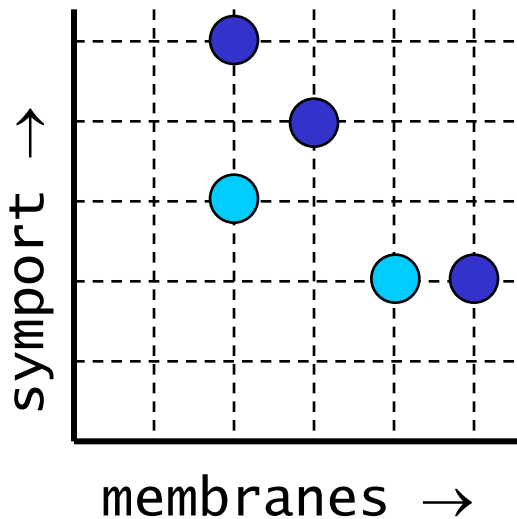
$(a_1 \dots a_k, \text{in}; b_1 \dots b_\ell, \text{out})$ antiport
 $(a_1 \dots a_k, \text{in})$ symport
 $(a_1 \dots a_k, \text{out})$

WMC paper (& Frisco)

Pău-Pău '02

$$\text{NRE} = \text{NPP}_2(2, 2)$$

$$\text{NRE} = \text{NPP}_1(1, 2)$$


$$\text{NPP}_m(s, a)$$

- membranes
- symport
- antiport

1. single membrane

2. symport only

$$\text{NRE} = \text{NPP}_1(1, 2)$$

good vs. bad ?
infinite
blocking



conflicting counters A & A'

$(p \rightarrow q, A=0)$

$(p \rightarrow r, +A')$

$(r \rightarrow r', \epsilon)$

$(r' \rightarrow q, -A')$

$(\#, \text{in}; AA', \text{out})$

$(\#, \text{in}; \#, \text{out})$

counter aut

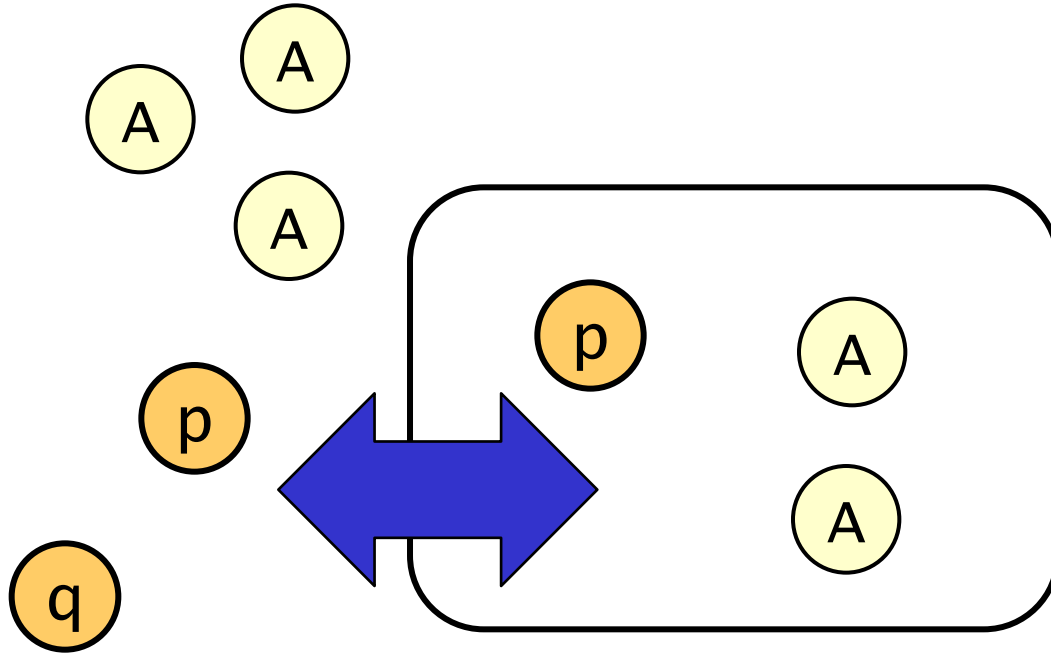
/ new

antiport

max parallelism: forced move

single membrane will do

sadly enough



#

#

$(p \rightarrow q, +A)$
 $(p \rightarrow q, -A)$

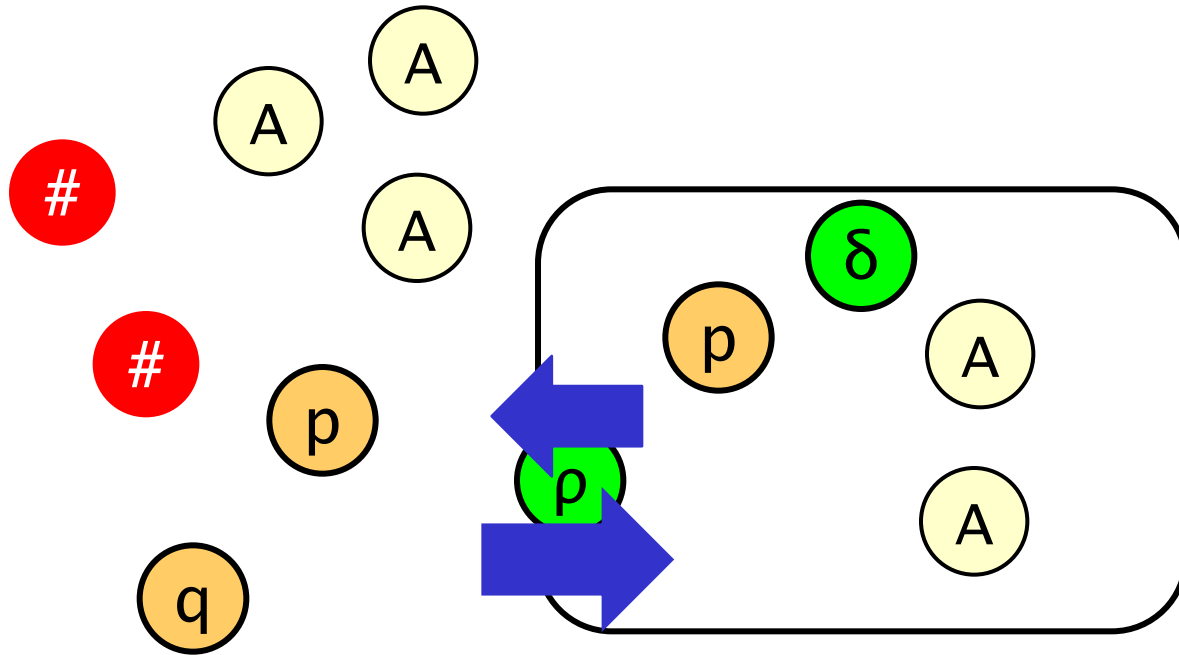
counter aut

$(qA, in; p, out)$
 $(q, in; pA, out)$

antiport

antiport to symport

add a 'carrier'



technical: halting state (extra symbol)

$(p \rightarrow q, +A)$
 $(p \rightarrow q, -A)$

counter aut

$(qA, \text{in}; p, \text{out})$
 $(q, \text{in}; pA, \text{out})$

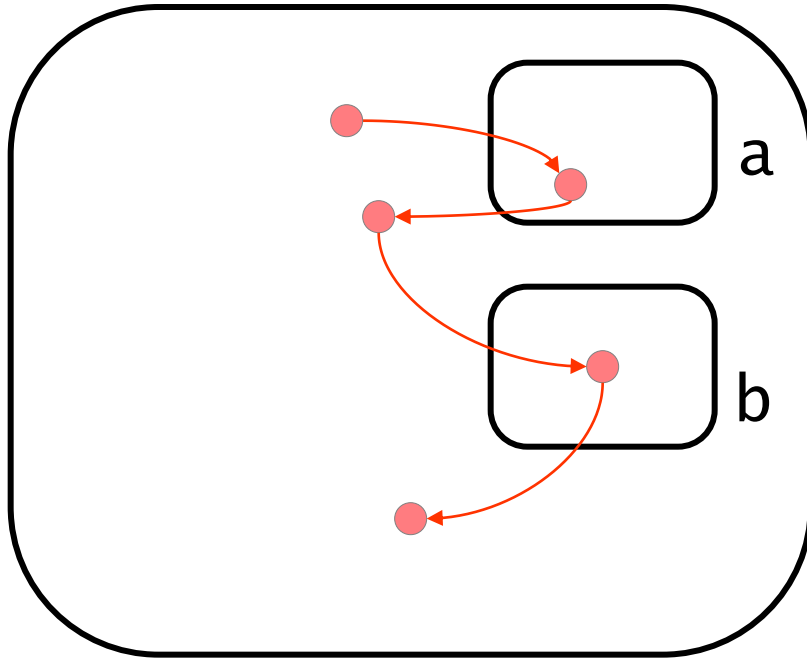
antiport

$(\rho qA, \text{in}) (\rho p, \text{out})$
 $(\rho q, \text{in}) (\rho pA, \text{out})$

symport

symport/antiport : following the traces

Ionescu, MartínVide, Păun & Păun



Contents

- **objects**
 - multiset symbols
 - infinite supply
 - in environment
- **traveller**

Rules

$(a_1 \dots a_k, \text{in}; b_1 \dots b_\ell, \text{out})$ **antiport**
 $(a_1 \dots a_k, \text{in})$ **symport**
 $(a_1 \dots a_k, \text{out})$

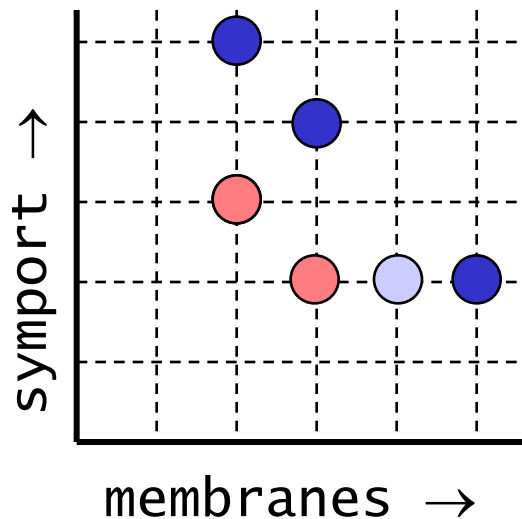
WMC paper

Ion-MaVi-Pău-Pău '02

$$1 \cdot \text{RE} = 1 \cdot \text{LP}_2(2, 2)$$

$$l \cdot \text{RE} = l \cdot \text{LP}_{l+1}(0, 2)$$

$$l \cdot \text{RE} = l \cdot \text{LP}_{l+1}(3, 0)$$



$l \cdot \text{LP}_m(s, a)$

- letters
- membranes
- symport
- antiport

1. two+ letters

2. single letter
symport only

conclusion ... after WMC'02 ...

carrier P systems \leftrightarrow **counter** automata
maximal parallelism \leftrightarrow zero test

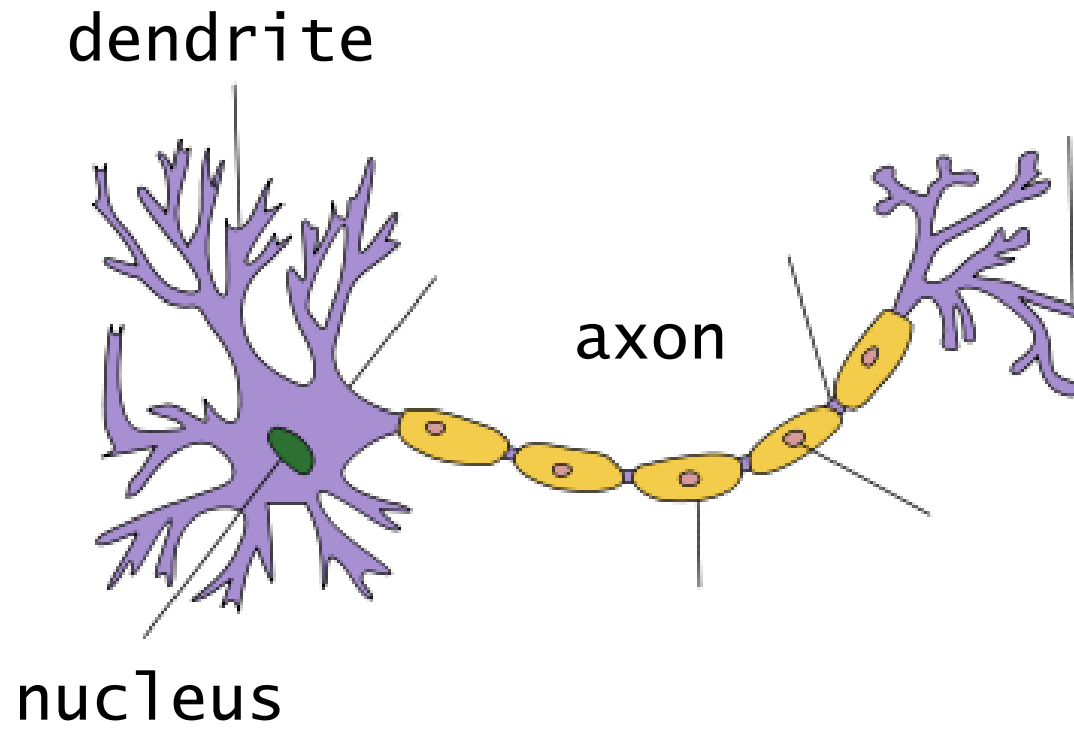
Petri nets!

single membrane	RE
single carrier	BC
single passenger	RE

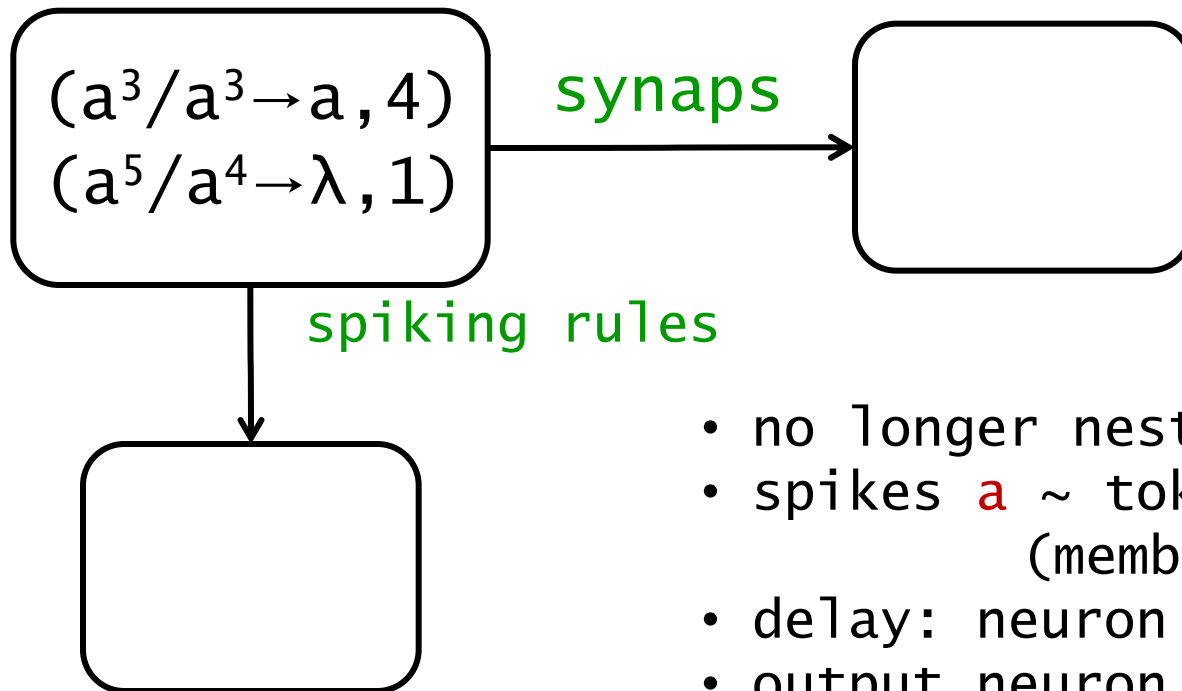
P systems with unstructured objects

- **catalysts & communicative** [Sosík]
- **P-automata** [Csuhaj-Varjú & Vaszil]
- **analysing systems** [Freund & Oswald]
- with **symport/antiport**
- following **traces**

Neurons & Spiking



neuron

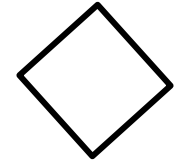


- no longer nested structure
 - spikes $a \sim$ tokens
(membrane potential)
 - delay: neuron closed
 - output neuron
 - time / synchronisation
-
- astrocytes
 - neuron division

astrocytes

block signals
at threshold

technical trick

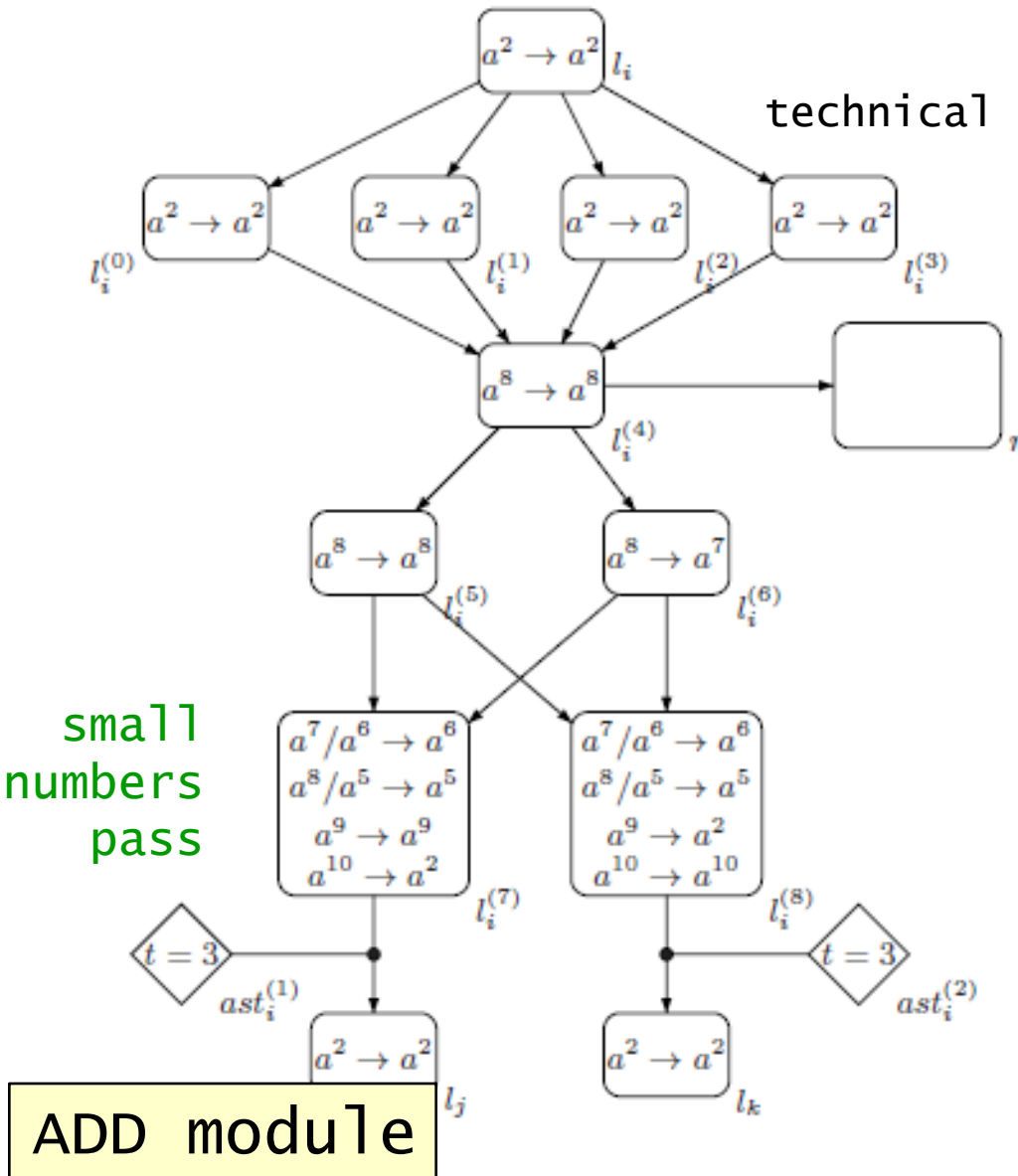


counter

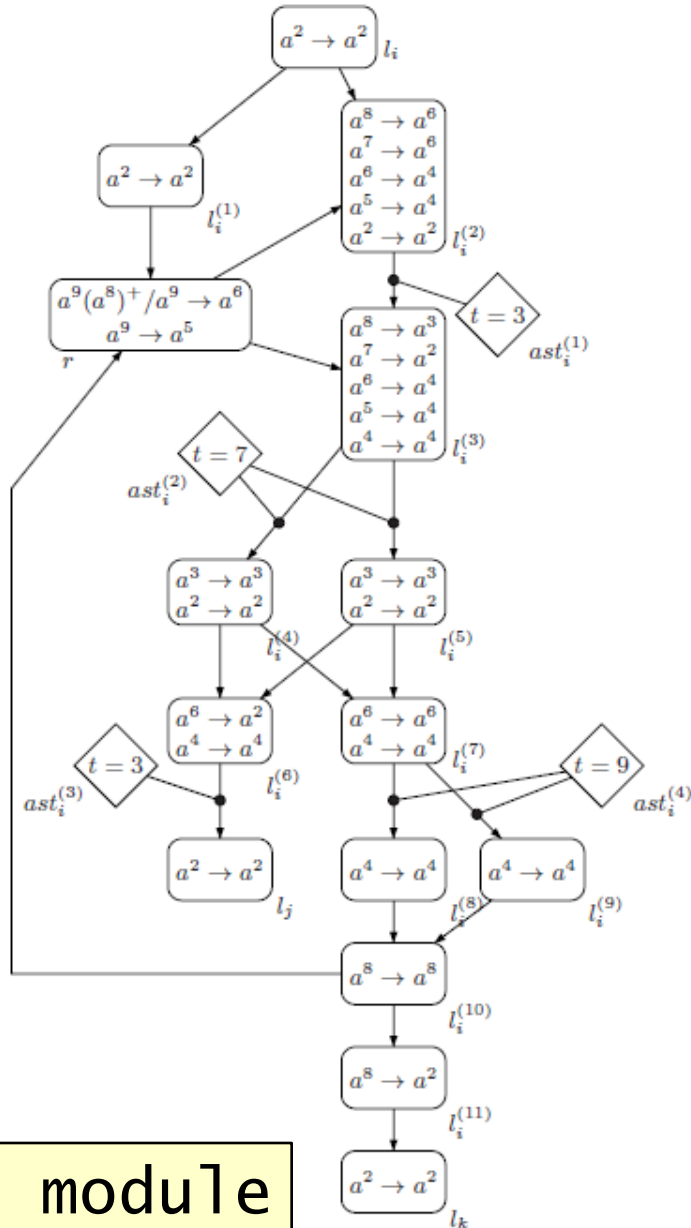
small
numbers
pass

nondeterminism
through timing!

no maximality,
i.e., asynchronous



module substract



rules in 'counter'-
membrane
never really empty

different instructions
same counter
[conflict!?!]

not *forced* to clear up
may block

acceptance by
'final state'

SUB module

'solving' NP complete problems

SAT satisfiability

input spike train = formula

transformed into neuron structure

linear in time,

exponential in size

initial system

depends on size

Input module

$$a^4$$

$$a^4/a^3 \rightarrow a^3; 4n$$

$$a \rightarrow a; nm-1$$

d_0

initial contents

$$[a^2]_{b_1} \rightarrow []_{d_1} || []_{b_2}$$

$$[a^2]_{e_1} \rightarrow []_{c_{x_1}} || []_{e_2}$$

e_1

b_1
label

Satisfiability checking module

deterministic

$$[a^2]_{g_1} \rightarrow []_{h_1} || []_{g_2}$$

g_1

forgetting

$$[a^2]_{f_1} \rightarrow []_{t_1} || []_{f_2}$$

f_1

$$a^7$$

$$a^7/a^2 \rightarrow a^2; 2n-3$$

$$a^5/a^2 \rightarrow a^2; 2n-1$$

$$a^3 \rightarrow a^3; nm+2$$

3

$$a^2$$

$$a \rightarrow a$$

$$a^2 \rightarrow a^2$$

$$a^3 \rightarrow \lambda$$

$$a^4 \rightarrow a$$

2

$$a^2 \rightarrow \lambda$$

$$a^3 \rightarrow a; 1$$

$$a^6 \rightarrow a^2; 1$$

4

$$a \rightarrow a$$

$$a^2 \rightarrow a^2$$

1

$$(a^2)^+ / a \rightarrow a$$

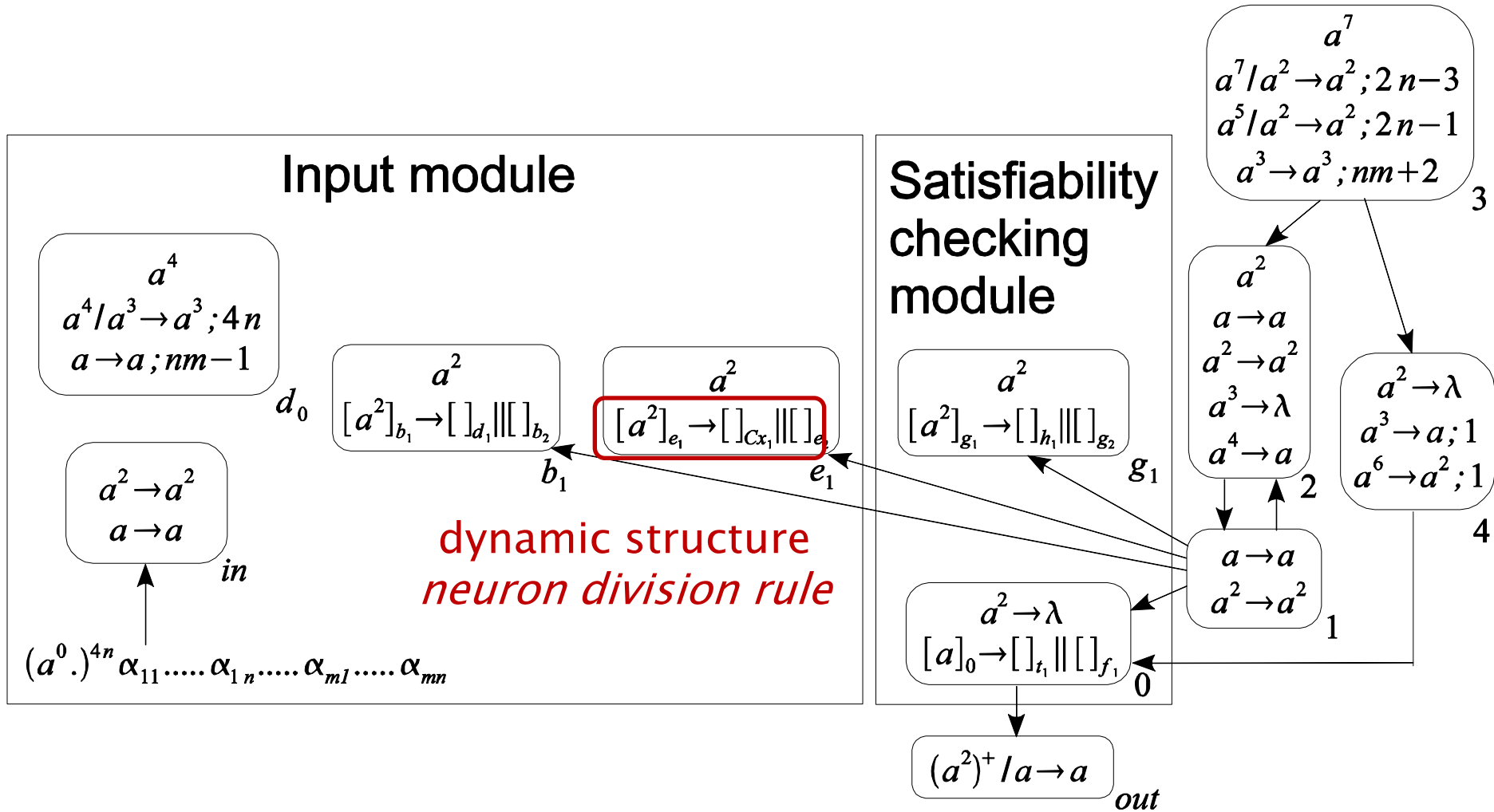
regular!

out output

$$(a^0 \cdot)^{4n} \alpha_{11} \dots \alpha_{1n} \dots \alpha_{m1} \dots \alpha_{mn}$$

input: problem instance
spike train

initial system



neuron division

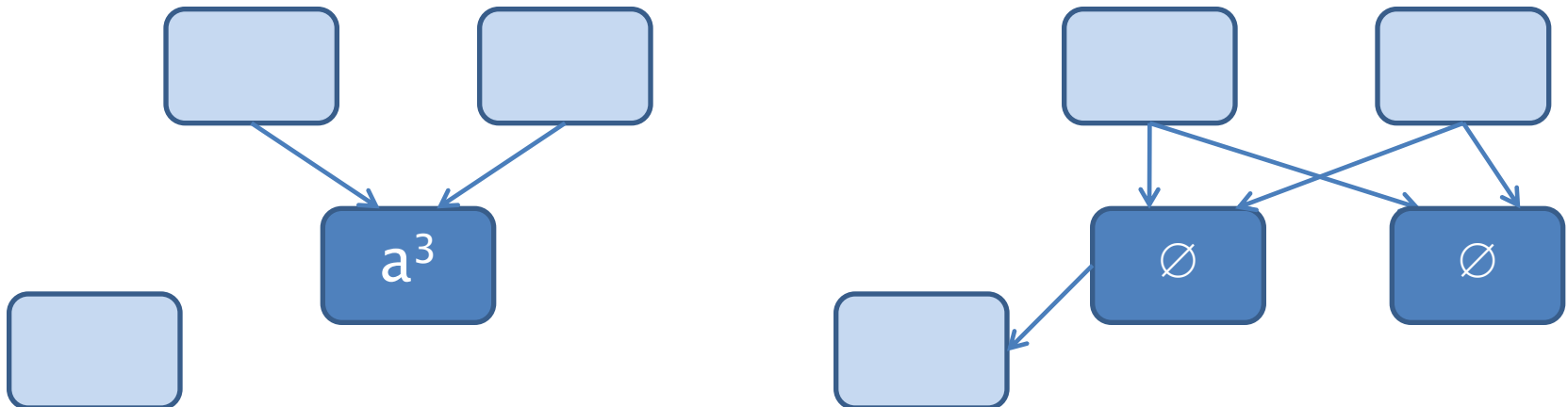
$$[E]_i \rightarrow []_j \parallel []_k$$

neuron division

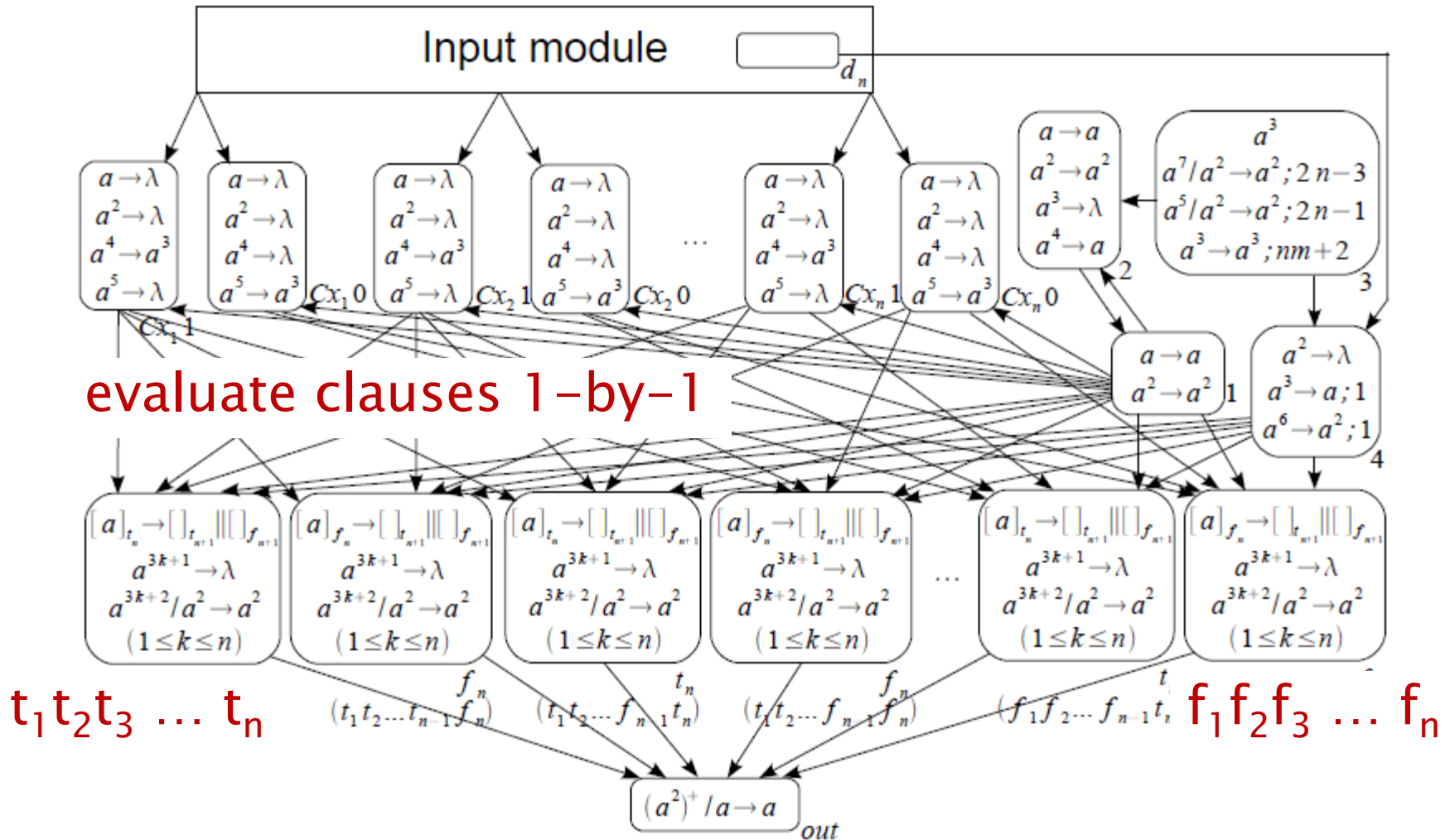
E regular expression 'test'

children inherit synapses
+ *synapse dictionary* (based on label)

no initial spikes



satisfiability and output



nature computes!

how?
what?