Taken from: Logic in Computer Science (M. Ruth and M. Ryan) Cambridge University Press

1) Express in propositional logic.

- **a.** If the sun shines today, then it won't shine tomorrow.
- **b.** If a request occurs, then it will eventually be acknowledged, or the requesting process won't ever be able to make progress.
- **c.** Today it will rain or shine, but not both.
- **d.** If Dick met Jane yesterday, then they had a cup of coffee together, or they took a walk in the park.

2) Connectives usually have a *binding priority*: $\neg$ binds more thightly than $\wedge$ and $\vee$, and the latter two bind more thightly than $\rightarrow$. Hence $\neg p \vee q \rightarrow \neg p$ denotes $((\neg p) \vee q) \rightarrow (\neg p)$, forgetting outer brackets. Implication is *right-associative*, expressions of the form $p \rightarrow q \rightarrow r$ denote $p \rightarrow (q \rightarrow r)$.

Write the following expressions in full.

$\neg p \rightarrow p \wedge r$
$(p \rightarrow q) \rightarrow (r \rightarrow s \vee t)$
$p \vee q \rightarrow \neg p \vee r$
$p \vee q \wedge r$

3) Compute truth tables of the formulas

- **a.** $((p \rightarrow q) \rightarrow p) \rightarrow p$.
- **b.** $p \rightarrow (q \rightarrow r)$ and $(p \rightarrow q) \rightarrow r$.
- **c.** $p \vee (\neg(q \wedge (r \rightarrow q)))$.

4) Use the predicates
$A(x, y)$: x admires y,
$B(x, y)$: x attended y
$P(x)$: x is a professor
$S(x)$: x is a student
$L(x)$: x is a lecture
and the constant m: Mary

to translate the following ito predicate logic:

- **a.** Mary admires every professor —the answer is not $(\forall x)A(m, P(x))$.
- **b.** Some professor admires Mary.
- **c.** Mary admires herself.
- **d.** No student attended every lecture.
- **e.** No lecture was attended by any student.

5) Let $M(x, y)$ mean that x is the mother of y. Similarly $F(x, y)$, $H(x, y)$, $S(x, y)$ and $B(x, y)$ say that x os the father, husband, sister, brother of y respectively. You may use constants to denote individuals, like 'Ed'. Translate into predicate logic

a. Everybody has a mother.

b. Everybody has a father and a mother.

c. Whoever has a mother has a father.

d. Ed is a grandfather.

e. All fathers are parents.

f. All husbands are spouses.

g. No uncle is an aunt.

h. All brothers are siblings.

i. Nobody's grandmother is anybody's father.

j. Ed and Patsy are husband and wife.

k. Carl is Monique'brother-in-law.

6) Let ϕ be the formula $(\forall x)(\forall y)(\exists z)(R(x, y) \to R(y, z))$.

Let $A = \{a, b, c, d\}$ be the domain, and let R be interpreted as the relation $\{(b, c), (b, b), (b, a)\}$. Is ϕ true under this interpretation?

Now, let $B = \{a, b, c\}$ be the domain, and let R be interpreted as the relation $\{(b, c), (a, b), (c, b)\}$. Is ϕ true under this interpretation?