FRACTRAN

...een tamelijk nutteloze programmeertaal

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Theoretische Informatica

Universiteit Leiden 23.11'16

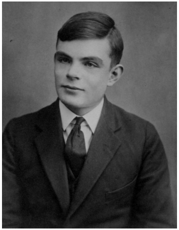
Leidsche Flesch Lunsch

"And I thought Brainf*ck was weird" – R.H.

$$\left(\frac{17}{91}, \frac{78}{85}, \frac{19}{51}, \frac{23}{38}, \frac{29}{33}, \frac{77}{29}, \frac{95}{23}, \frac{77}{19}, \frac{1}{17}, \frac{11}{13}, \frac{13}{11}, \frac{15}{14}, \frac{15}{2}, \frac{55}{1}\right)$$

models for computability

- Alan Turing (1912–1954)
  "On Computable Numbers, with an Application to the Entscheidungsproblem" (1937)
- Emil Post (1897–1954)
  "Finite Combinatory Processes - Formulation 1" (1936)
- Marvin Minsky (1927–2016)
  counter machine / register machine (1961)



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Life, Death and the Monster - Numberphile (2014)

John Horton Conway (1937)

combinatorial game theory:
surreal numbers, game of Life

geometric topology:
knot polynomials

group theory: "A perfect group of order 8,315,553,613,086,720,000 . . ."

algorithmics: doomsday algorithm

theoretical physics:
" *if experimenters have free will, then so do elementary particles*"

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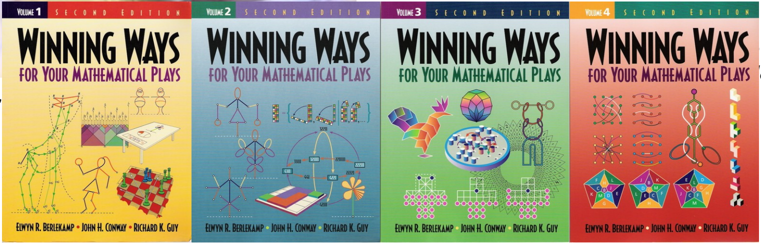

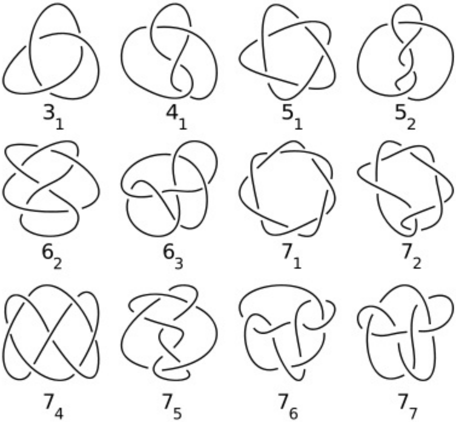

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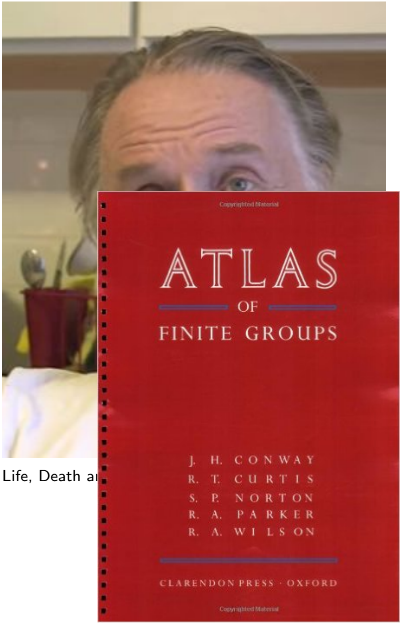

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2, 15, 825, 725, 1925, 2275, 425, 390, 330, 290, 770, 910, 170, 156, 132, 116, 308, 364, 68, 4, 30, 225, 12375, 10875, 28875, 25375, 67375, 79625, 14875, 13650, 2550, 2340, 1980, 1740, 4620, 4060, 10780, 12740, 2380, 2184, 408, 152, 92, 380, 230, 950, 575, 2375, 9625, 11375, 2125, 1950, 1650, 1450, 3850, 4550, 850, 780, 660, 580, 1540, 1820, 340, 312, 264, 232, 616, 728, 136, 8, 60, 450, 3375, 185625, 163125, 433125, 380625, 1010625, 888125, 2358125, 2786875, 520625, 477750, 89250, 81900, 15300, 14040, 11880, 10440, 27720, 24360, 64680, 56840, 150920, 178360, 33320, 30576, 5712, 2128, 1288, 5320, 3220, 13300, 8050, 33250, 20125, 83125, 336875, 398125, 74375, 68250, 12750, 11700, 9900, 8700, 23100, 20300, 53900, 63700, . . .

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$$\left(\frac{365}{46}, \frac{29}{161}, \frac{79}{575}, \frac{679}{451}, \frac{3159}{413}, \frac{83}{409}, \frac{473}{371}, \frac{638}{355}, \frac{434}{335}, \frac{89}{235}, \frac{17}{209}, \frac{79}{122}, \frac{31}{183}, \frac{41}{115}, \frac{517}{89}, \frac{111}{83}, \frac{305}{79}, \frac{23}{73}, \frac{73}{71}, \frac{61}{67}, \frac{37}{61}, \frac{19}{59}, \frac{89}{57}, \frac{41}{53}, \frac{833}{47}, \frac{53}{43}, \frac{86}{41}, \frac{13}{38}, \frac{23}{37}, \frac{67}{31}, \frac{71}{29}, \frac{83}{19}, \frac{475}{17}, \frac{59}{13}, \frac{41}{291}, \frac{1}{7}, \frac{1}{11}, \frac{1}{1024}, \frac{1}{97}, \frac{89}{1}\right)$$

started at 2^n the next power of 2 to appear is $2^{\pi(n)}$ where

$$\begin{array}{rccccccccccccccc} n = & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & \dots \\ \pi(n) = & 3 & 1 & 4 & 1 & 5 & 9 & 2 & 6 & 5 & 3 & 5 & 8 & 9 & \dots \end{array}$$

$$\frac{\pi}{2} = \frac{2}{1} \cdot \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdot \frac{6}{5} \cdot \frac{6}{7} \cdot \frac{8}{7} \cdot \frac{8}{9} \cdot \dots$$ (Wallis)

$$\left(\frac{17}{7\cdot13},\frac{2\cdot3\cdot13}{5\cdot17},\frac{19}{3\cdot17},\frac{23}{2\cdot19},\frac{29}{3\cdot11},\frac{7\cdot11}{29},\frac{5\cdot19}{23},\frac{7\cdot11}{19},\frac{1}{17},\frac{11}{13},\frac{13}{11},\frac{3\cdot5}{2\cdot7},\frac{3\cdot5}{2},\frac{5\cdot11}{1}\right)$$

$$2$$

$$\left(\frac{17}{7\cdot 13}, \frac{2\cdot 3\cdot 13}{5\cdot 17}, \frac{19}{3\cdot 17}, \frac{23}{2\cdot 19}, \frac{29}{3\cdot 11}, \frac{7\cdot 11}{29}, \frac{5\cdot 19}{23}, \frac{7\cdot 11}{19}, \frac{1}{17}, \frac{11}{13}, \frac{13}{11}, \frac{3\cdot 5}{2\cdot 7}, \frac{3\cdot 5}{2}, \frac{5\cdot 11}{1}\right)$$

$2,\ 3\cdot 5$

$$\left(\frac{17}{7\cdot13}, \frac{2\cdot3\cdot13}{5\cdot17}, \frac{19}{3\cdot17}, \frac{23}{2\cdot19}, \frac{29}{3\cdot11}, \frac{7\cdot11}{29}, \frac{5\cdot19}{23}, \frac{7\cdot11}{19}, \frac{1}{17}, \frac{11}{13}, \frac{13}{11}, \frac{3\cdot5}{2\cdot7}, \frac{3\cdot5}{2}, \frac{5\cdot11}{1}\right)$$

$2,\ 3\cdot5,\ 3\cdot5^2\cdot11$

$$\left(\frac{17}{7\cdot 13}, \frac{2\cdot 3\cdot 13}{5\cdot 17}, \frac{19}{3\cdot 17}, \frac{23}{2\cdot 19}, \frac{29}{3\cdot 11}, \frac{7\cdot 11}{29}, \frac{5\cdot 19}{23}, \frac{7\cdot 11}{19}, \frac{1}{17}, \frac{11}{13}, \frac{13}{11}, \frac{3\cdot 5}{2\cdot 7}, \frac{3\cdot 5}{2}, \frac{5\cdot 11}{1}\right)$$

$2,\ 3\cdot 5,\ 3\cdot 5^2\cdot 11,\ 5^2\cdot 29$

$$\left(\frac{17}{7\cdot13}, \frac{2\cdot3\cdot13}{5\cdot17}, \frac{19}{3\cdot17}, \frac{23}{2\cdot19}, \frac{29}{3\cdot11}, \frac{7\cdot11}{29}, \frac{5\cdot19}{23}, \frac{7\cdot11}{19}, \frac{1}{17}, \frac{11}{13}, \frac{13}{11}, \frac{3\cdot5}{2\cdot7}, \frac{3\cdot5}{2}, \frac{5\cdot11}{1}\right)$$

$2,\ 3\cdot5,\ 3\cdot5^2\cdot11,\ 5^2\cdot29,\ 5^2\cdot7\cdot11$

$$\left(\frac{17}{\color{blue}{7\cdot 13}}, \frac{2\cdot 3\cdot 13}{5\cdot 17}, \frac{19}{3\cdot 17}, \frac{23}{2\cdot 19}, \frac{29}{3\cdot 11}, \frac{7\cdot 11}{29}, \frac{5\cdot 19}{23}, \frac{7\cdot 11}{19}, \frac{1}{17}, \frac{11}{13}, \frac{13}{11}, \frac{3\cdot 5}{2\cdot 7}, \frac{3\cdot 5}{2}, \frac{5\cdot 11}{1}\right)$$

$$2,\ 3\cdot 5,\ 3\cdot 5^2\cdot 11,\ 5^2\cdot 29,\ 5^2\cdot 7\cdot 11,\ 5^2\cdot 7\cdot 13$$

$$\left(\frac{17}{7\cdot 13}, \frac{2\cdot 3\cdot 13}{5\cdot 17}, \frac{19}{3\cdot 17}, \frac{23}{2\cdot 19}, \frac{29}{3\cdot 11}, \frac{7\cdot 11}{29}, \frac{5\cdot 19}{23}, \frac{7\cdot 11}{19}, \frac{1}{17}, \frac{11}{13}, \frac{13}{11}, \frac{3\cdot 5}{2\cdot 7}, \frac{3\cdot 5}{2}, \frac{5\cdot 11}{1}\right)$$

$$2,\ 3\cdot 5,\ 3\cdot 5^2\cdot 11,\ 5^2\cdot 29,\ 5^2\cdot 7\cdot 11,\ 5^2\cdot 7\cdot 13,\ 5^2\cdot 17,\ 2\cdot 3\cdot 5\cdot 13,\ 2\cdot 3\cdot 5\cdot 11,$$
$$2\cdot 5\cdot 29, 2\cdot 5\cdot 7\cdot 11, 2\cdot 5\cdot 7\cdot 13, 2\cdot 5\cdot 17, 2^2\cdot 3\cdot 13, 2^2\cdot 3\cdot 11, 2^2\cdot 29, 2^2\cdot 7\cdot 11,$$
$$2^2\cdot 7\cdot 13, 2^2\cdot 17, 2^2, 2\cdot 3\cdot 5, 3^2\cdot 5^2, 3^2\cdot 5^3\cdot 11, 3\cdot 5^3\cdot 29, 3\cdot 5^3 7\cdot 11, \ldots$$

$$\left(\frac{2}{3}\right)$$

$${}_{2}5 \cdot {}_{3}3$$

$$\left(\frac{2}{3}\right)$$

$$2^5 \cdot 3^3 \rightsquigarrow 2^6 \cdot 3^2 \rightsquigarrow 2^7 \cdot 3^1 \rightsquigarrow 2^8 \cdot 3^0$$

$$\left(\frac{2}{3}\right)$$

$${}_{2}5 {\cdot}_{3}3 \leadsto {}_{2}6 {\cdot}_{3}2 \leadsto {}_{2}7 {\cdot}_{3}1 \leadsto {}_{2}8 {\cdot}_{3}0$$

$${}_{2}a {\cdot}_{3}b \leadsto {}_{2}a + b$$

$$\left(\frac{2}{3}\right)$$

$$2^5 \cdot 3^3 \rightsquigarrow 2^6 \cdot 3^2 \rightsquigarrow 2^7 \cdot 3^1 \rightsquigarrow 2^8 \cdot 3^0$$

$$2^a \cdot 3^b \rightsquigarrow 2^{a+b}$$

```
while b>0 do
    a++; b--;
od

        // a += b;
        // b = 0;
```

$$\left(\frac{5}{2\cdot 3}, \frac{1}{2}, \frac{1}{3}\right)$$

$2^5 \cdot 3^3$

$$\left(\frac{5}{2\cdot 3}, \frac{1}{2}, \frac{1}{3}\right)$$

$${}_2 5 \cdot {}_3 3 \rightsquigarrow {}_2 4 \cdot {}_3 2 \cdot {}_5 1 \rightsquigarrow {}_2 3 \cdot {}_3 1 \cdot {}_5 2 \rightsquigarrow {}_2 2 \cdot {}_3 0 \cdot {}_5 3 \rightsquigarrow {}_2 1 \cdot {}_5 3 \rightsquigarrow {}_5 3$$

$$\left(\frac{5}{2\cdot 3}, \frac{1}{2}, \frac{1}{3}\right)$$

$${}_{2}5 \cdot {}_{3}3 \leadsto {}_{2}4 \cdot {}_{3}2 \cdot {}_{5}1 \leadsto {}_{2}3 \cdot {}_{3}1 \cdot {}_{5}2 \leadsto {}_{2}2 \cdot {}_{3}0 \cdot {}_{5}3 \leadsto {}_{2}1 \cdot {}_{5}3 \leadsto {}_{5}3$$

$${}_{2}a \cdot {}_{3}b \leadsto {}_{5}\min(a, b)$$

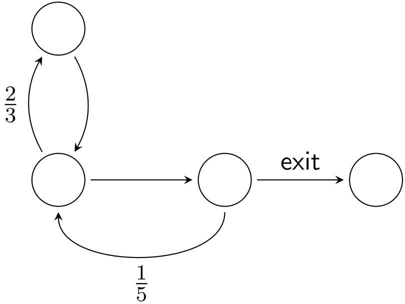
```
while a·b>0 do
    a--; b--; c++
od
          // a=0 || b=0
while a>0 do
    a--;
od
          // a=0
while b>0 do
    b--;
od
```

multiplication $a = b \cdot c$

registers a,b,c,d $2, 3, 5, 7$
states $11, 13, 17, 19, 23$

value $2^a \cdot 3^b \cdot 5^c \cdot 7^d \cdot s$

```
// (repeat c times) a += b;
while c>0 do
    while b>0 do
        a++; b--;
        d++
    od
    while d>0 do
        d--; b++;
    od
    c--;
od
// ready:  'exit'
```



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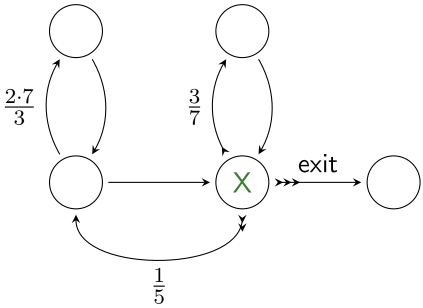

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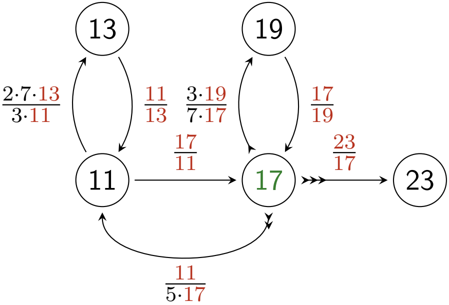
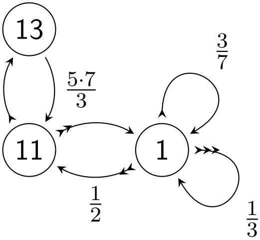


$$\left(\ldots, \frac{3\cdot 19}{7\cdot 17}, \frac{11}{5\cdot 17}, \frac{23}{17}, \ldots\right)$$

d > 0	d = 0 & c > 0	d = c = 0
$\frac{1}{7}$	$\frac{1}{5}$	

multiplication

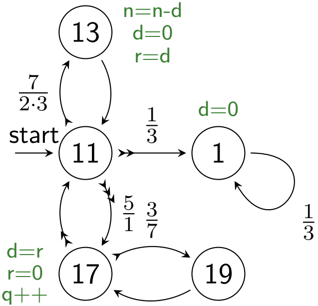
$$\left(\frac{5\cdot 7\cdot 13}{3\cdot 11}, \frac{11}{13}, \frac{1}{11}, \frac{3}{7}, \frac{11}{2}, \frac{1}{3}\right)$$



$$2^{a}\cdot 3^{b} \rightsquigarrow 5^{a\cdot b}$$

division

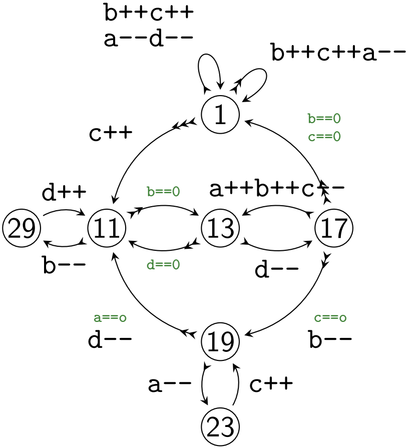
$$\left(\frac{7\cdot 13}{2\cdot 3\cdot 11}, \frac{11}{13}, \frac{1}{3\cdot 11}, \frac{5\cdot 17}{11}, \frac{3\cdot 19}{7\cdot 17}, \frac{17}{19}, \frac{11}{17}\cdot\frac{1}{3}\right)$$



$$2^{n}\cdot 3^{d}\cdot 11 \rightsquigarrow 5^{q}\cdot 7^{r}$$

$$(n = q\cdot d + r,\ 0 \leq r < d)$$

$$\left(\frac{17}{7\cdot 13}, \frac{2\cdot 3\cdot 13}{5\cdot 17}, \frac{19}{3\cdot 17}, \frac{23}{2\cdot 19}, \frac{29}{3\cdot 11}, \frac{7\cdot 11}{29}, \frac{5\cdot 19}{23}, \frac{7\cdot 11}{19}, \frac{1}{17}, \frac{11}{13}, \frac{13}{11}, \frac{3\cdot 5}{2\cdot 7}, \frac{3\cdot 5}{2}, \frac{5\cdot 11}{1}\right)$$



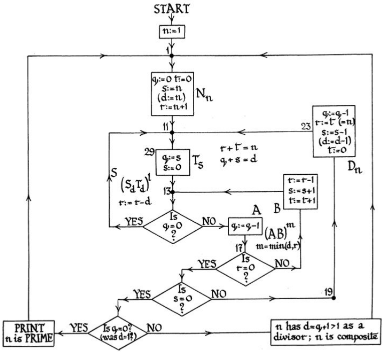


FIGURE 10. A Prime Printing Program.

R.K. Guy: Conway's prime producing machine, Mathematics Magazine (1983)

models for computability

- Alan Turing (1912–1954)
  "On Computable Numbers, with an Application to the Entscheidungsproblem" (1937)
- Emil Post (1897–1954)
  "Finite Combinatory Processes - Formulation 1" (1936)
- Marvin Minsky (1927–2016)
  counter machine / register machine (1961)

#z: increment counter i, goto #x

#z: if counter i zero, goto #x,
else decrement and goto #y

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  counter machine / register machine (1961)

#z: increment counter i, goto #x $\quad \frac{i \cdot x}{z}$

#z: if counter i zero, goto #x,
else decrement and goto #y $\quad \frac{y}{i \cdot z}, \frac{x}{z}$

Only FRACTRAN has these star qualities!

- *Makes workday really easy!*
  no complicated programming manual
- *Gets those functions really clean!*
  configuration in a single integer, no other foreign concepts
- *Matches any machine on the market*
  easy to write a FRACTRAN program to simulate other machines
- *Astoundingly simple universal program!*

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$$\left(\frac{583}{559}, \frac{629}{551}, \frac{437}{527}, \frac{82}{517}, \frac{615}{329}, \frac{371}{129}, \frac{1}{115}, \frac{53}{86}, \frac{43}{53}, \frac{23}{47}, \frac{341}{46}, \frac{41}{43}, \frac{47}{41}, \frac{29}{37}, \frac{37}{31}, \frac{299}{29}, \frac{47}{23}, \frac{161}{15}, \frac{527}{19}, \frac{159}{7}, \frac{1}{17}, \frac{1}{13}, \frac{1}{3}\right)$$

The End.

J.H. Conway.
Unpredictable Iterations.
In: Proc. 1972 Number Theory Conference, Boulder Colorado, 1972.

J.H. Conway.
FRACTRAN: A simple universal programming language for arithmetic
In: Open Problems in Communication and Computation, Springer, 1983.

Julian Havil.
Nonplussed!
Princeton University Press, 2007

H.J. Hoogeboom.
Carriers and Counters –
P Systems with Carriers vs. (Blind) Counter Automata
Developments in Language Theory, LNCS 2450, 2003.

(connection between counter automata and computational models)