# Quantum Algorithms lecture

## Quantum algorithm for Topological data analysis

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What is topological data analysis?

The topological data analysis pipeline

- Going from a dataset to a topological object
- Studying topological object using homology

What can quantum computers bring to the table?

The Lloyd, Garnerone & Zanardi algorithm

# What is topological data analysis?

studying the shape of a dataset

Topological data analysis studies the *topology* of your dataset.

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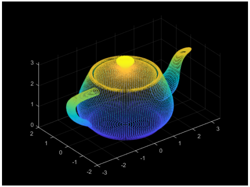
Typically, the dataset is a point cloud in a $d$-dimensional space such as $\mathbb{R}^d$.

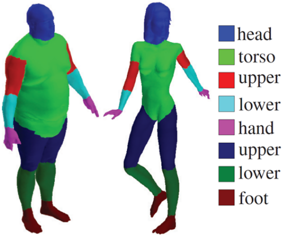


- E.g, a point cloud sampled from a 3d object.

# What can topological data analysis be used for?

an example of an application of topological data analysis

Suppose we would like to classify parts of the human body.



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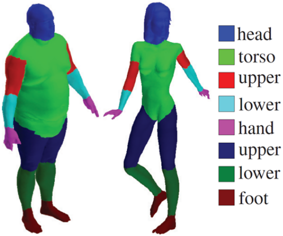
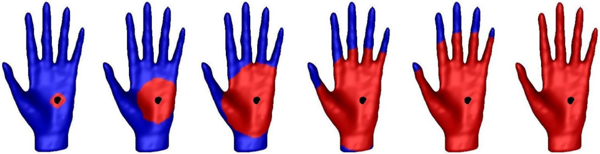



TDA does this by studying *persistent topological* features.



- I.e., features of neighbourhoods around points at different scales.

# What topological features do we study? How do we study them?

an example of an important topological feature

Important topological feature to study: *the number of holes*.

(a) One hole

(b) Two holes

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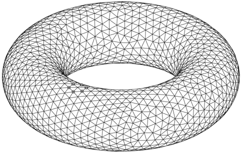
**Algebraic topology:** use algebra to compute topological features.



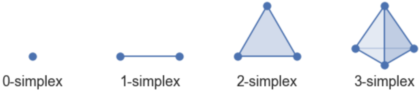
- Triangulate object and study connections between triangles using algebra.

# Going from a dataset to a simplicial complex

## The Vietoris-Rips complex

**1st step TDA pipeline:** converting point-cloud to a *simplicial complex*.

- A collection of points, lines, triangles and their $k$-dimensional counterparts.



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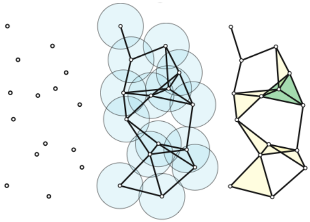
**Vietoris-Rips complex**: draw circles with radius $\epsilon > 0$ around data-points and connect data-points if their circles overlap.



- *Persistent topology:* how does topology change if we vary $\epsilon$?

# Studying topology of simplicial complex using homology

turning simplicial complex into a complex vector space

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- A tool from algebraic topology to extract topological features.

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First, for a point cloud $\mathcal{D} = \{v_i\}_{i=1}^n$ encode the $k$-simplices of the Vietoris-Rips complex as $n$-bit strings.

$\mathrm{VR}(\mathcal{D}, \epsilon)_k := \{j \in \{0,1\}^n \mid j$ has $k$ ones and $\forall j_i, j_l = 1$ we have $||v_i - v_l|| < \epsilon\}$.

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Next, we turn this into a complex vector space

$$\mathcal{H}_k^{\epsilon} := \mathsf{span}_{\mathbb{C}}\{|j\rangle \mid j \in \mathsf{VR}(\mathcal{D}, \epsilon)_k\} \subset \left(\mathbb{C}^2\right)^{\otimes n}.$$

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We now have an algebraic object to study!

# Studying topology of simplicial complex using homology



In order to study $\mathcal{H}_k^\epsilon$ we consider the so-called $k$*-th boundary map*

$$\partial_k^\epsilon : \mathcal{H}_k^\epsilon \to \mathcal{H}_{k-1}^\epsilon$$

$$|j\rangle \mapsto \sum_{i=1}^{k} (-1)^i \, |j(\hat{i})\rangle \,,$$

where $j(\hat{i})$ is obtained from $j$ by setting $i$-th 1 to 0.

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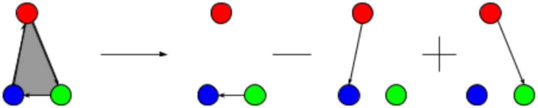

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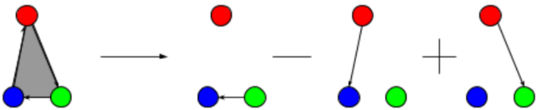

Using this map we can study how simplices are connected to eachother!

# Studying topology of simplicial complex using homology

## the homology group

Important object to study is the so-called *$k$-th homology group* defined as

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Its dimension, called the $k$*-th betti-number*, is an important topological feature.

$$\beta_k^\epsilon = \dim H_k^\epsilon.$$

- Turns out: $\beta_k^\epsilon$ is equal to number of $k$-dimensional holes at scale $\epsilon$.

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**Goal in TDA:** compute $\beta_k^\epsilon$ to extract topological features of your point cloud.

To compute $\beta_k^\epsilon$ we consider the so-called $k$*-th combinatorial Laplacian*

$$\Delta_k^\epsilon := \partial_k^T \partial_k + \partial_{k+1} \partial_{k+1}^T : \mathcal{H}_k^\epsilon \to \mathcal{H}_k^\epsilon.$$

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**Question:** can we use quantum computers to efficiently compute this dimension?

# What can quantum computers bring to the table?

Quantum linear algebra: Hamiltonian simulation and quantum phase estimation

Quantum computers are generally very good at doing linear algebra.

- $\Delta_k^\epsilon$ is Hermitian $\implies$ Hamiltonian simulation can implement $U = e^{i\Delta_k^\epsilon}$!
- Also, can use quantum phase estimaton to investigate eigenvalues of $U$.

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We are interested in the number of eigenvalues that are equal to 0 since

$$\beta_k^\epsilon = \dim\left(\ker \Delta_k^\epsilon\right) = \#\{j : \lambda_j(\Delta_k^\epsilon) = 0\},$$

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**LGZ algorithm:** combine Hamiltonian simulation and QPE to estimate number of eigenvalues that are equal to 0.

# The Lloyd, Garnerone & Zanardi algorithm

an overview of the algorithm

**Goal:** estimate $\beta_k^\epsilon = \dim\left(\ker \Delta_k^\epsilon\right) = \#\{j : \lambda_j(\Delta_k^\epsilon) = 0\}$.

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1. Use Grover's algorithm to prepare the *simplex-state* given by

$$|\psi_k\rangle := \frac{1}{\sqrt{\dim \mathcal{H}_k^\epsilon}} \sum_{j \in \mathsf{VR}(\mathcal{D},\epsilon)_k} |j\rangle \,.$$

   - I.e., the uniform superposition over the complex vector space $\mathcal{H}_k^\epsilon$.

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4. Estimate the number of eigenvalues $\lambda_j(\Delta_k^\epsilon) = 0$.
   - Either using quantum counting or Monte-Carlo estimation.

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For $j \in \{0,1\}^n$ we can efficiently check whether $j \in \mathsf{VR}(\mathcal{D},\epsilon)_k$.

- First, check whether $j$ contains exactly $k$ ones.
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  - Requires a total of $O(k^2)$ computations of pairwise distances $||v_i - v_j||$.

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Allows us to efficiently implement a quantum oracle to the membership function

$$f : \{0,1\}^n \to \{0,1\}$$

$$f(j) = \begin{cases} 1 \text{ if } j \in \mathsf{VR}(\mathcal{D},\epsilon)_k, \\ 0 \text{ else.} \end{cases}$$

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Therefore, Grover's algorithm allows us to implement the mapping

$$\frac{1}{\sqrt{2^n}} \sum_{j \in \{0,1\}^n} |j\rangle \mapsto |\psi_k\rangle = \frac{1}{\sqrt{\dim \mathcal{H}_k^\epsilon}} \sum_{j \in \mathsf{VR}(\mathcal{D},\epsilon)_k} |j\rangle .$$

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using Hamiltonian simulation and QPE to uniformly sample from the eigenvalues of $\Delta_k^\epsilon$

We would like to use HS and QPE to sample from the eigenvalues of $e^{i\Delta_k^\epsilon}$.

- Similar to Problem 3 of last tutorial, this gives us the eigenvalues of $\Delta_k^\epsilon$.
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From the simplex-state $|\psi_k\rangle$ we can efficiently prepare the mixed state

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So, the mixed state $P_{\mathcal{H}_k^\epsilon}$ is the $j$-th eigenstate $|\psi_j\rangle$ with probability $\frac{1}{\dim \mathcal{H}_k^\epsilon}$.
Let's use this state as input to the HS + QPE routine!

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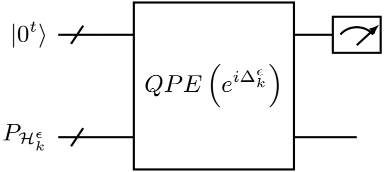
the circuit and the output distribution

Altogether, after the Grover-step the LGZ algorithm runs the following circuit



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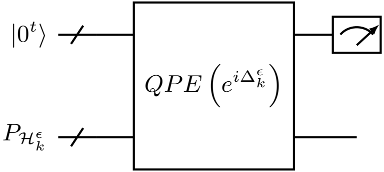


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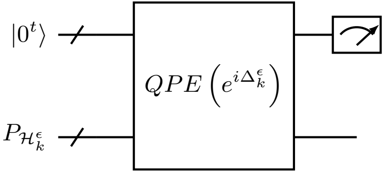


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In particular, the probability of measuring $\lambda = 0$ is given bu

$$\mathsf{Pr}\left(0\right) = \frac{\#\{j \mid \lambda_j = 0\}}{\dim \mathcal{H}_k^\epsilon} = \frac{\beta_k^\epsilon}{\dim \mathcal{H}_k^\epsilon}.$$

Using quantum counting or Monte-Carlo estimation, we can now estimate $\beta_k^\epsilon$.

# References

where to find the papers

- Nature: https://www.nature.com/articles/ncomms10138
- Arxiv: https://arxiv.org/pdf/1408.3106.pdf
- Nice review paper: https://arxiv.org/pdf/1906.07673.pdf

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- **Question:** can we show classical/quantum seperation?
  - Does it solve a hard problem for complexity class with (conjectured) quantum advantage?
- **Question:** can we use samples from the eigenvalues of $\Delta_k^\epsilon$ for something else than computing Betti numbers?

# My research

why am I looking at this algorithm? what are the interesting questions?

- **Motivation:** dequantizations of Ewin Tang et al.
- **Question:** can we show classical/quantum seperation?
  - Does it solve a hard problem for complexity class with (conjectured) quantum advantage?
- **Question:** can we use samples from the eigenvalues of $\Delta_k^\epsilon$ for something else than computing Betti numbers?
  - So-called applications of "simplicial spectral theory".