Logica (I&E)

najaar 2017

`http://liacs.leidenuniv.nl/~vlietrvan1/logica/`

Rudy van Vliet
kamer 140 Snellius, tel. 071-527 2876
rvvliet(at)liacs(dot)nl

college 11, maandag 20 november 2017

2. Predicate logic
2.3. Proof theory of predicate logic

Als je sneller wilt spelen kun je wel harder lopen maar in weze bepaalt de bal de snelheid van het spel.

A slide from lecture 10:

Analogy $\wedge$ and $\forall$

elimination		introduction
$\dfrac{\phi \wedge \psi}{\phi}\ \wedge e_R$	$\dfrac{\phi \wedge \psi}{\psi}\ \wedge e_L$	$\dfrac{\phi \quad \psi}{\phi \wedge \psi}\ \wedge i$
$\dfrac{\forall x \phi}{\phi[t/x]}\ \forall x\ e$		$\dfrac{\boxed{\begin{array}{l} x_0 \\ \quad \vdots \\ \quad \phi[x_0/x] \end{array}}}{\forall x \phi}\ \forall x\ i$

A slide from lecture 10:

Analogy $\vee$ and $\exists$

elimination	introduction
$\dfrac{\phi\vee\psi \qquad \boxed{\begin{array}{c}\phi\\ \vdots\\ \chi\end{array}} \qquad \boxed{\begin{array}{c}\psi\\ \vdots\\ \chi\end{array}}}{\chi}\ \vee\mathrm{e}$	$\dfrac{\phi}{\phi\vee\psi}\ \vee\mathrm{i}_R \qquad \dfrac{\psi}{\phi\vee\psi}\ \vee\mathrm{i}_L$
$\dfrac{\exists x\phi \qquad \boxed{\begin{array}{cc}x_0 & \phi[x_0/x]\\ & \vdots\\ & \chi\end{array}}}{\chi}\ \exists x\ \mathrm{e}$	$\dfrac{\phi[t/x]}{\exists x\phi}\ \exists x\ \mathrm{i}$

Example.

$$\exists x P(x),\ \forall x \forall y (P(x) \rightarrow Q(y)) \vdash \forall y Q(y)$$

Proof. . .

Why fresh variables

Example.

$$\exists x P(x),\ \forall x(P(x) \to Q(x)) \vdash \forall y Q(y)$$

‘Proof’...

2.3.2. Quantifier equivalences

Is

$$\forall x \forall y \phi \dashv\vdash \forall y \forall x \phi$$

valid?

Is

$$(\forall x \phi) \wedge (\forall x \psi) \dashv\vdash \forall x (\phi \wedge \psi)$$

valid?

Is

$$(\forall x \phi) \wedge \psi \dashv\vdash \forall x (\phi \wedge \psi)$$

valid?

Example 2.12.
Not all birds can fly.

$$\neg\forall x(B(x) \to F(x))$$

$$\exists x(B(x) \land \neg F(x))$$

Theorem 2.13. Let ϕ and ψ be formulas of predicate logic.

1.(a) $\neg\forall x\phi \dashv\vdash \exists x\neg\phi$

(b) $\neg\exists x\phi \dashv\vdash \forall x\neg\phi$

Proof. . .

Theorem 2.13. Let ϕ and ψ be formulas of predicate logic.

2. Assuming that x is not free in ψ:

   (a) $\forall x \phi \land \psi \dashv\vdash \forall x(\phi \land \psi)$

   (b) $\forall x \phi \lor \psi \dashv\vdash \forall x(\phi \lor \psi)$

   (c) . . . (h)

Proof. . .

Theorem 2.13. Let ϕ and ψ be formulas of predicate logic.

3.(a) $\forall x \phi \wedge \forall x \psi \dashv\vdash \forall x(\phi \wedge \psi)$

(b) $\exists x \phi \vee \exists x \psi \dashv\vdash \exists x(\phi \vee \psi)$

Proof. . .

Theorem 2.13. Let ϕ and ψ be formulas of predicate logic.

4.(a) $\forall x \forall y \phi \dashv\vdash \forall y \forall x \phi$

(b) $\exists x \exists y \phi \dashv\vdash \exists y \exists x \phi$

Proof...

Study this proof yourself.