

Fundamentele Informatica 3

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<http://www.liacs.leidenuniv.nl/~vlietrvan1/fi3/>

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7. Turing Machines

7.4. Combining Turing Machines

7.5. Multitape Turing Machines

7.4. Combining Turing Machines

Example.

A TM for $f(x) = a^{n_a(x)}$

$x = aababba$

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$x = aababba$

| | | | | | | | |
|----------|---|---|----------|----------|-----|----|---|
| <u>△</u> | a | a | b | a | b | b | a |
| △ | a | a | <u>△</u> | a | b | b | a |
| △ | a | a | <u>a</u> | b | b | a | △ |
| △ | a | a | a | <u>△</u> | b | a | △ |
| △ | a | a | a | <u>b</u> | a | △△ | |
| △ | a | a | a | <u>△</u> | a | △△ | |
| △ | a | a | a | <u>a</u> | △△△ | | |
| <u>△</u> | a | a | a | a | △△△ | | |

Many notations for composition

Example 7.17. Finding the Next Blank or the Previous Blank

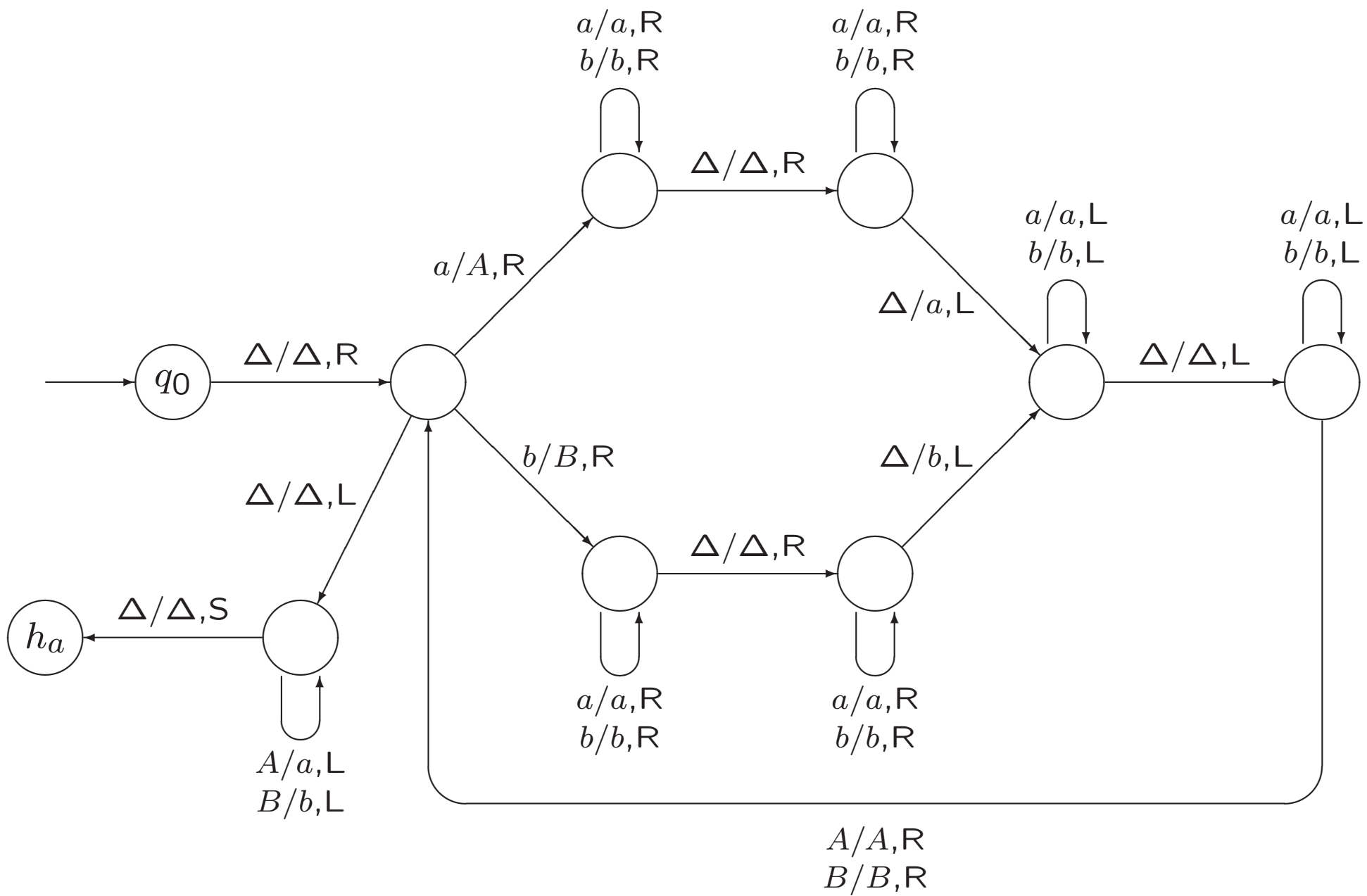
NB

PB

Example 7.18. Copying a String

Copy: from $\underline{\Delta}x$ to $\underline{\Delta}x\underline{\Delta}x$

$x = abaa$



A slide from lecture 2

Example 7.10. The Reverse of a String

Δ a a b a b
Δ A a b a b
Δ A a b a A
Δ B a b a A
Δ B A b a A
Δ B A b A A
Δ B A b A A
Δ B A B A A
Δ b a b a a

Example 7.24. Comparing Two Strings

Equal: accept $\underline{\Delta}x\Delta y$ if $x = y$,
and reject if $x \neq y$

Exercise 7.17.

For each case below, draw a TM that computes the indicated function.

- e. $E : \{a, b\}^* \times \{a, b\}^* \rightarrow \{0, 1\}$
defined by $E(x, y) = 1$ if $x = y$, $E(x, y) = 0$ otherwise.

Example 7.25. Accepting the Language of ...

Copy \rightarrow *NB* \rightarrow *R* \rightarrow *PB* \rightarrow *Equal*

Example 7.25. Accepting the Language of Palindromes

Copy \rightarrow *NB* \rightarrow *R* \rightarrow *PB* \rightarrow *Equal*

Example 7.20. Inserting and Deleting a Symbol

Delete: from $y\underline{\sigma}z$ to $y\underline{z}$

Insert(σ): from $y\underline{z}$ to $y\underline{\sigma}z$

N.B.: z does not contain blanks

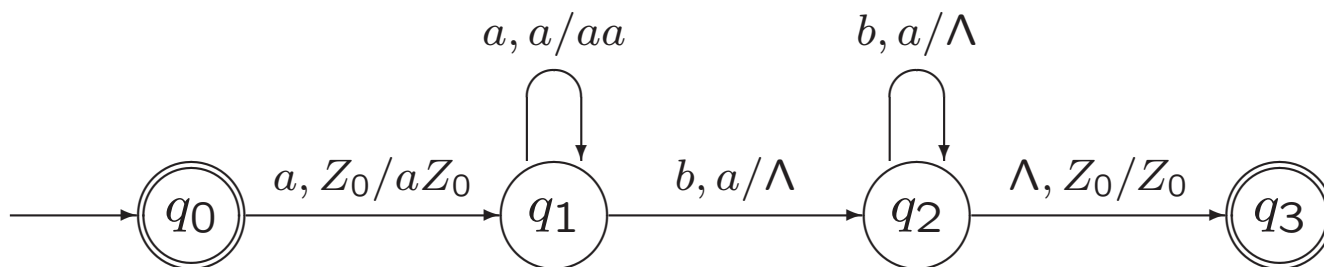
Example 7.21. Erasing the Tape

From the current position to the right

7.5. Multitape Turing Machines

Example 5.3. A PDA Accepting the Language $AnBn$

$$AnBn = \{a^i b^i \mid i \geq 0\}$$



Exercise 7.4.

For each of the following languages, draw a transition diagram for a Turing machine that accepts that language.

a. $AnBn = \{a^i b^i \mid i \geq 0\}$

With two tapes...

Exercise 7.4.

For each of the following languages, draw a transition diagram for a Turing machine that accepts that language.

a. $AnBn = \{a^i b^i \mid i \geq 0\}$

We could also use the portion of the tape to the right of the input, to simulate the stack of a deterministic pushdown automaton (works for any deterministic PDA!)

Example 7.24. Comparing Two Strings

Equal: accept $\underline{\Delta}x\Delta y$ if $x = y$,
and reject if $x \neq y$

2-tape TM...

A slide from lecture 2

Definition 7.1. Turing machines

A Turing machine (TM) is a 5-tuple $T = (Q, \Sigma, \Gamma, q_0, \delta)$, where

Q is a finite set of states. The two *halt* states h_a and h_r are not elements of Q .

Σ , the input alphabet, and Γ , the tape alphabet, are both finite sets, with $\Sigma \subseteq \Gamma$. The *blank* symbol Δ is not an element of Γ .

q_0 , the initial state, is an element of Q .

δ is the transition **function**:

$$\delta : Q \times (\Gamma \cup \{\Delta\}) \rightarrow (Q \cup \{h_a, h_r\}) \times (\Gamma \cup \{\Delta\}) \times \{R, L, S\}$$

2-Tape TM $T = (Q, \Sigma, \Gamma, q_0, \delta)$, where

$$\delta : Q \times (\Gamma \cup \{\Delta\})^2 \rightarrow (Q \cup \{h_a, h_r\}) \times (\Gamma \cup \{\Delta\})^2 \times \{R, L, S\}^2$$

Combination of two slides from lecture 2

Notation:

description of tape contents: $x\underline{\sigma}y$ or $x\underline{y}$

configuration $xqy = xqy\Delta = xqy\Delta\Delta$

initial configuration corresponding to input x : $q_0\Delta x$

In the third edition of the book, a configuration is denoted as (q, \underline{xy}) or $(q, \underline{x\sigma y})$ instead of xqy or $xq\sigma y$.

This old notation is also allowed for *Fundamentele Informatica 3*.

Configuration of 2-tape TM is

$$(q, x_1 \underline{a_1} y_1, x_2 \underline{a_2} y_2)$$

Initial configuration corresponding to input string x is

$$(q_0, \underline{\Delta} x, \underline{\Delta})$$

Output will appear on first tape.

Theorem 7.26.

For every 2-tape TM $T = (Q, \Sigma, \Gamma, q_0, \delta)$, there is an ordinary 1-tape TM $T_1 = (Q_1, \Sigma, \Gamma_1, q_1, \delta_1)$ with $\Gamma \subseteq \Gamma_1$, such that

1. For every $x \in \Sigma^*$, T accepts x if and only if T_1 accepts x , and T rejects x if and only if T_1 rejects x .
(In particular, $L(T) = L(T_1)$.)

2. For every $x \in \Sigma^*$, if

$$(q_0, \underline{\Delta}x, \underline{\Delta}) \vdash_T^* (h_a, y\underline{a}z, u\underline{b}v)$$

for some strings $y, z, u, v \in (\Gamma \cup \{\Delta\})^*$ and symbols $a, b \in \Gamma \cup \{\Delta\}$, then

$$q_1 \Delta x \vdash_{T_1}^* y h_a a z \quad \text{i.e., } (q_1, \underline{\Delta}x) \vdash_{T_1}^* (h_a, y\underline{a}z)$$

Proof...

Simulating two tapes on one

| | | | | | | |
|----------|----------|----------|----------|----------|----------|---------|
| Δ | <u>1</u> | Δ | 0 | 1 | Δ | \dots |
| 0 | 1 | 0 | <u>0</u> | Δ | Δ | \dots |

If $\delta(p, 1, 0) = (q, \Delta, 1, L, R) \dots$

Simulating two tape heads

Simulating move of 2-tape TM T by 1-tape TM T_1

1. Move left to \$, right to σ' , back to \$

Simulating move of 2-tape TM T by 1-tape TM T_1

1. Move left to \$, right to σ' , back to \$

2. Move right to τ'

Let $\delta(p, \sigma, \tau) = (q, \sigma_1, \tau_1, D_1, D_2)$

If $q = h_r$, reject

Otherwise, $\tau' \rightarrow \tau_1$ and move D_2

Simulating move of 2-tape TM T by 1-tape TM T_1

1. Move left to \$, right to σ' , back to \$

2. Move right to τ'

Let $\delta(p, \sigma, \tau) = (q, \sigma_1, \tau_1, D_1, D_2)$

If $q = h_r$, reject

Otherwise, $\tau' \rightarrow \tau_1$ and move D_2

3. If \$, reject

Otherwise, (if #, move #) place ' and back to \$

Simulating move of 2-tape TM T by 1-tape TM T_1

1. Move left to \$, right to σ' , back to \$

2. Move right to τ'

Let $\delta(p, \sigma, \tau) = (q, \sigma_1, \tau_1, D_1, D_2)$

If $q = h_r$, reject

Otherwise, $\tau' \rightarrow \tau_1$ and move D_2

3. If \$, reject

Otherwise, (if #, move #) place ' and back to \$

4. Move right to σ' , $\sigma' \rightarrow \sigma_1$ and move D_1

Simulating move of 2-tape TM T by 1-tape TM T_1

1. Move left to \$, right to σ' , back to \$

2. Move right to τ'

Let $\delta(p, \sigma, \tau) = (q, \sigma_1, \tau_1, D_1, D_2)$

If $q = h_r$, reject

Otherwise, $\tau' \rightarrow \tau_1$ and move D_2

3. If \$, reject

Otherwise, (if #, move #) place ' and back to \$

4. Move right to σ' , $\sigma' \rightarrow \sigma_1$ and move D_1

5. If \$, reject

Otherwise, (if #, move #) place '

If T accepts, then...

6. Delete second track

If T accepts, then...

6. Delete second track

7. Delete \$ and #

If T accepts, then...

6. Delete second track
7. Delete \$ and #
8. Find σ' , unprime, halt in h_a

Corollary 7.27.

Every language that is accepted by a 2-tape TM can be accepted by an ordinary 1-tape TM,
and every function that is computed by a 2-tape TM can be computed by an ordinary TM.

This generalizes to k -tape TMs for $k \geq 3$.